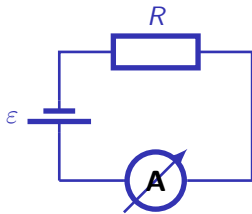


D.C. Circuits

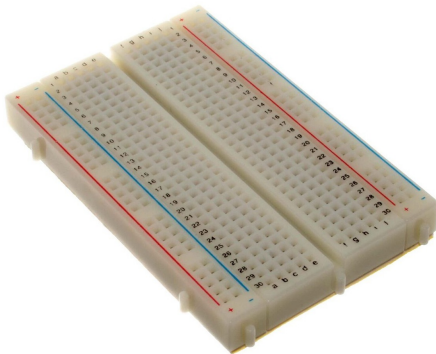
AS Physics ■ Topic 10

A-Level Physics



What we will cover

- 1 e.m.f. and p.d. 10.1
- 2 Internal resistance 10.1.1
- 3 Kirchhoff's laws 10.2
- 4 Solving circuits 10.2.1
- 5 Potential dividers 10.3
- 6 The potentiometer 10.3.1
- 7 Recap





Sheet 10.1 – what you need to know

You must be able to:

- define **e.m.f.** and **p.d.** as energy per unit charge
- use $W = EQ$ and $W = VQ$, and say where the energy goes
- draw circuits with the standard symbols
- connect an ammeter and a voltmeter, and say what makes each ideal

What the exam asks

“Define the electromotive force (e.m.f.) of a source.” **[2]**

“Define electric potential difference across a component.” **[1]**

“Calculate the time taken for an input energy of 1.2 kJ to be supplied to the motor.” **[2]**

Practise on exercise sheet 10.1, then try the real exam questions on past-paper sheet 10.1.

e.m.f. and p.d.

Both in volts: the **potential difference** is the energy each unit of charge gives up *inside* a component.

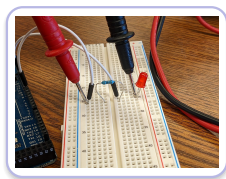
e.m.f. ϵ

electromotive force : energy
into the circuit per unit charge

p.d. V

energy **out** (resistor, lamp, motor)

Build and test on a **breadboard** ; measure with a **multimeter** .



Breadboard multimeter

Energy from an e.m.f.

e.m.f.: $E = \frac{W}{Q}$, the energy
given to each coulomb by the
source.

p.d.: $V = \frac{W}{Q}$, the energy
each coulomb gives up in a
component.

$$E = 9.0 \text{ V}, Q = 30 \text{ C}, V = 8.4 \text{ V}$$

given to the charge, from $E = \frac{W}{Q}$:
 $W = EQ = (9.0 \text{ V})(30 \text{ C}) = 270 \text{ J}$

given up outside, from $V = \frac{W}{Q}$:
 $W = VQ = (8.4 \text{ V})(30 \text{ C}) = 252 \text{ J}$

the rest heats the battery:

$$W = EQ - VQ = 270 \text{ J} - 252 \text{ J} = 18 \text{ J}$$

Circuit symbols



cell



battery



switch (open)



fuse



earth



fixed resistor



variable resistor



potentiometer



thermistor



light-dependent resistor



lamp



diode



light-emitting diode (LED)



capacitor



inductor



ammeter



voltmeter



galvanometer



motor



junction of conductors

Ideal **ammeter** : zero R ,
in **series**.

Ideal **voltmeter** : infinite
 R , in **parallel**.

Also on the sheet: the **fuse** , the **capacitor** , the variable resistor or **rheostat** , and the **light-dependent resistor** .



Sheet 10.1.1 – what you need to know

You must be able to:

- explain **internal resistance** and the lost volts Ir
- use $E = I(R + r)$ and $V = E - Ir$
- read E and r off a graph of V against I
- explain why the terminal p.d. falls as more current is drawn

What the exam asks

“Calculate the internal resistance r of the cell.” [3]

“Show that the terminal potential difference of the battery is 4.8 V.” [2]

“State and explain the effect on the terminal potential difference of the battery.” [3]

Practise on exercise sheet 10.1.1, then try the real exam questions on past-paper sheet 10.1.1.

Internal resistance

A real source has an **internal resistance** r : part of the energy each coulomb is given is lost **inside** it.

$$E = I(R + r) \quad V = E - Ir$$

$I = 0$: $V = E$. Short circuit, $R = 0$:

$I = \frac{E}{r}$, the largest current.

$$E = 1.5 \text{ V}, r = 0.50 \, \Omega, R = 2.5 \, \Omega$$

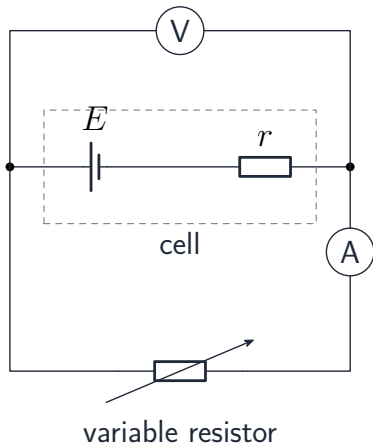
the e.m.f. drives I through **both** resistances:

$$I = \frac{E}{R + r} = \frac{1.5 \text{ V}}{2.5 \, \Omega + 0.50 \, \Omega} = 0.50 \text{ A}$$

the terminal p.d. is what is left after the lost volts:

$$V = E - Ir = 1.5 \text{ V} - (0.50 \text{ A})(0.50 \, \Omega) = 1.25 \text{ V}$$

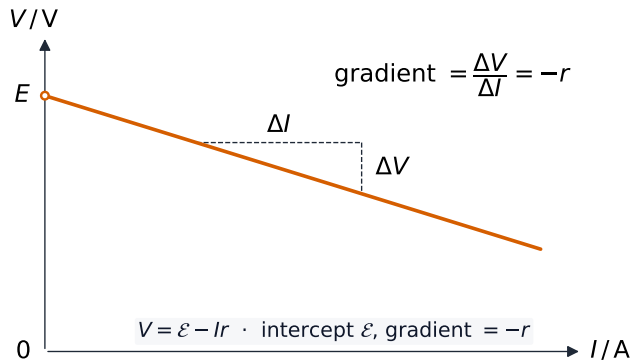
The cell drawn with its own resistance



A real cell is an e.m.f. E in series with r , so the terminal p.d. is $V = E - Ir$.

The terminal p.d. **falls** as more current is drawn, and the **lost volts** Ir heat the cell itself.

Internal resistance, in detail



Plot V against I and you get a straight line: the **intercept** is the e.m.f. E and the **gradient** is $-r$. That is how r is measured.

Reading E and r off the graph

Vary R , record V and I , plot V against I :

$$V = E - rI \quad \text{is} \quad y = c + mx$$

$$\text{intercept} = E; \quad \text{gradient} = -r$$

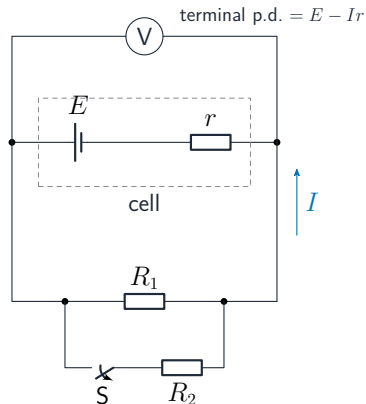
Two points on the line: (0.40 A, 2.80 V)
and (1.60 A, 2.20 V)

$$\text{gradient} = \frac{\Delta V}{\Delta I} = \frac{2.20 \text{ V} - 2.80 \text{ V}}{1.60 \text{ A} - 0.40 \text{ A}} = -0.50 \Omega$$

$$r = -\text{gradient} = 0.50 \Omega$$

$$E = V + Ir = 2.80 \text{ V} + (0.40 \text{ A})(0.50 \Omega) = 3.00 \text{ V}$$

Closing a switch: the terminal p.d. falls



Closing S puts R_2 in parallel with R_1 :

- 1 the external resistance **falls**
- 2 so the current $I = \frac{E}{R + r}$ **rises**
- 3 so the lost volts Ir **rise**
- 4 so the terminal p.d. $V = E - Ir$ **falls**

Each link is a mark: never jump from the switch straight to the answer.



Sheet 10.2 – what you need to know

You must be able to:

- state Kirchhoff's laws and the conservation law behind each
- derive the series and parallel formulae, naming the law at each step
- combine resistors in series and in parallel
- find the current and the p.d.s in a series circuit

What the exam asks

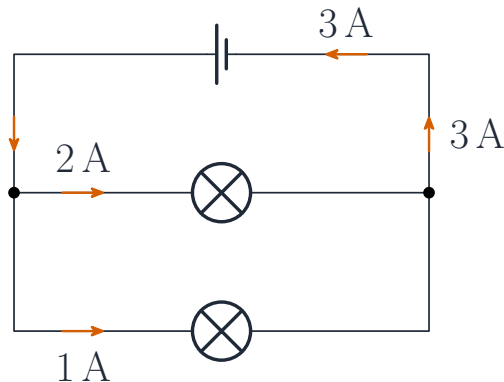
“State Kirchhoff's first law.” [1]

“State the conservation law that gives rise to Kirchhoff's second law.” [1]

“Use Kirchhoff's laws to show that the total resistance R_T of the external circuit is given by $R_T = R_1 + R_2$.” [2]

Practise on exercise sheet 10.2, then try the real exam questions on past-paper sheet 10.2.

Kirchhoff's two laws



at each junction: $3\text{ A} = 2\text{ A} + 1\text{ A}$

- **First law** (junction): current in = current out (**charge conserved**)
- **Second law** (loop): around **any closed loop**, $\sum \varepsilon = \sum V$ – this is **conservation of energy**

Deriving the series and parallel formulae

Series: the same current I

second law: $V = V_1 + V_2$

each p.d. is IR : $IR = IR_1 + IR_2$

divide by I : $R = R_1 + R_2$

Parallel: the same p.d. V

first law: $I = I_1 + I_2$

each current is $\frac{V}{R}$: $\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$

divide by V : $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

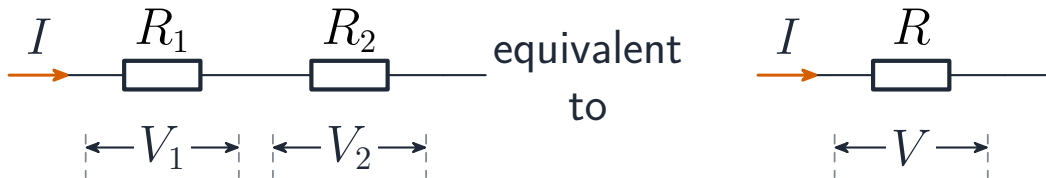
“Use Kirchhoff's laws to show that...”: the marks are for **naming the law** and for the step that divides out I or V .

Resistors in series

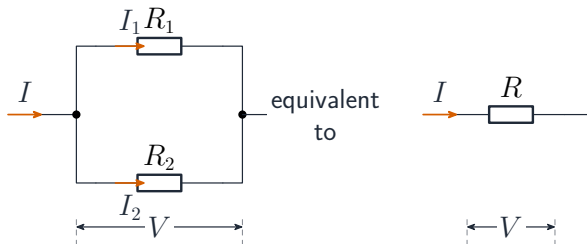
Same **current** ; p.d.s add:

$$R_{\text{series}} = R_1 + R_2 + \dots$$

Always **larger** than any single **resistor** .



Resistors in parallel



Same p.d.; the currents add:

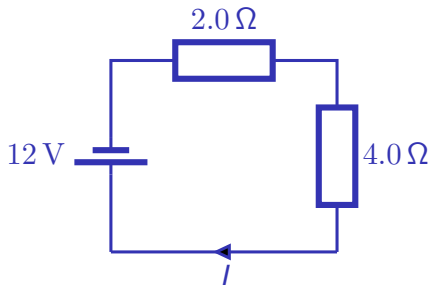
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

4.0 Ω and 12 Ω in parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4.0\,\Omega} + \frac{1}{12\,\Omega} = \frac{1}{3.0\,\Omega}$$

$R = 3.0\,\Omega$: less than the smaller resistor

Worked example: one loop



Find I and each p.d.

second law round the loop: $E = IR_1 + IR_2$

$$I = \frac{E}{R_1 + R_2} = \frac{12\text{ V}}{2.0\ \Omega + 4.0\ \Omega} = 2.0\text{ A}$$

$$V_1 = IR_1 = (2.0\text{ A})(2.0\ \Omega) = 4.0\text{ V}$$

$$V_2 = IR_2 = (2.0\text{ A})(4.0\ \Omega) = 8.0\text{ V}$$

Check: $V_1 + V_2 = 4.0\text{ V} + 8.0\text{ V} = 12\text{ V} = E$ ✓
– the p.d.s use up the e.m.f.



Sheet 10.2.1 – what you need to know

You must be able to:

- reduce a network one step at a time
- share the current at a junction and the p.d. round a loop
- decide which resistor dissipates the most **power**
- handle two cells in one loop

What the exam asks

“Show that the combined resistance R of the network is given by $R = R_1 + R_2 + R_3$.” [2]

“State and explain which resistor dissipates more power.” [3]

“State and explain the effect, if any, of this change on the current in the cell.” [2]

Practise on exercise sheet 10.2.1, then try the real exam questions on past-paper sheet 10.2.1.

Solving a circuit

1 label every current
& direction

2 junction rule at each node

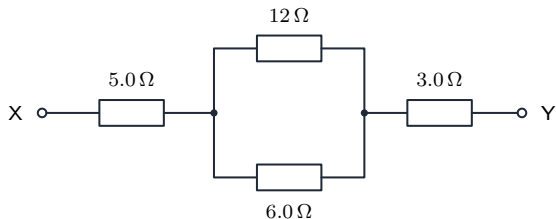
3 loop rule around each loop

4 $V = IR$ for each resistor

5. solve the equations together

In a **symmetric** network, spot branches carrying equal currents – and remember the branch with the most current dissipates the most **power** , since $P = I^2 R$.

Reduce a network one step at a time



1. The parallel pair

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{12\ \Omega} + \frac{1}{6.0\ \Omega} = \frac{3}{12\ \Omega},$$

so $R_p = 4.0\ \Omega$

2. Then the three in series

$$\begin{aligned} R_{XY} &= R_a + R_p + R_b \\ &= 5.0\ \Omega + 4.0\ \Omega + 3.0\ \Omega = 12\ \Omega \end{aligned}$$

Which resistor dissipates the most power?

In series: the same current

$$I = \frac{V}{R_4 + R_8} = \frac{12 \text{ V}}{4.0 \Omega + 8.0 \Omega} = 1.0 \text{ A}$$

$$P_4 = I^2 R_4 = (1.0 \text{ A})^2 (4.0 \Omega) = 4.0 \text{ W}$$

$$P_8 = I^2 R_8 = (1.0 \text{ A})^2 (8.0 \Omega) = 8.0 \text{ W}$$

Same I and $P = I^2 R$: the **larger** resistor wins.

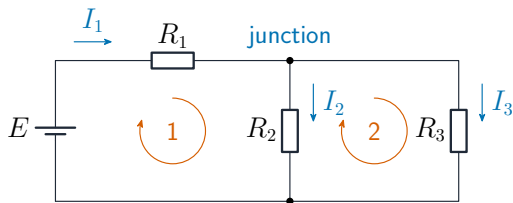
In parallel: the same p.d.

$$P_4 = \frac{V^2}{R_4} = \frac{(12 \text{ V})^2}{4.0 \Omega} = 36 \text{ W}$$

$$P_8 = \frac{V^2}{R_8} = \frac{(12 \text{ V})^2}{8.0 \Omega} = 18 \text{ W}$$

Same V and $P = \frac{V^2}{R}$: the **smaller** resistor wins.

Two loops: share the current, then the p.d.



$$E = 9.0 \text{ V}, R_1 = 2.0 \Omega, R_2 = R_3 = 6.0 \Omega$$

Find I_1 , I_2 and I_3

first law at the junction: $I_1 = I_2 + I_3$

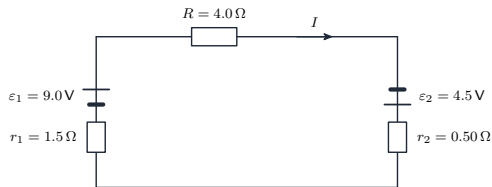
$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{2}{6.0 \Omega}, \text{ so } R_p = 3.0 \Omega$$

$$I_1 = \frac{E}{R_1 + R_p} = \frac{9.0 \text{ V}}{2.0 \Omega + 3.0 \Omega} = 1.8 \text{ A}$$

$$V_p = I_1 R_p = (1.8 \text{ A})(3.0 \Omega) = 5.4 \text{ V}$$

$$I_2 = \frac{V_p}{R_2} = \frac{5.4 \text{ V}}{6.0 \Omega} = 0.90 \text{ A} = I_3$$

Two cells that oppose



The second law, round the loop

$$\varepsilon_{\text{net}} = \varepsilon_1 - \varepsilon_2 = 9.0 \text{ V} - 4.5 \text{ V} = 4.5 \text{ V}$$

$$\begin{aligned} R_{\text{total}} &= r_1 + r_2 + R \\ &= 1.5 \Omega + 0.50 \Omega + 4.0 \Omega = 6.0 \Omega \end{aligned}$$

The current and a terminal p.d.

$$I = \frac{\varepsilon_{\text{net}}}{R_{\text{total}}} = \frac{4.5 \text{ V}}{6.0 \Omega} = 0.75 \text{ A}$$

$$\begin{aligned} V_1 &= \varepsilon_1 - Ir_1 \\ &= 9.0 \text{ V} - (0.75 \text{ A})(1.5 \Omega) = 7.9 \text{ V} \end{aligned}$$



Sheet 10.3 – what you need to know

You must be able to:

- explain the principle of a **potential divider**
- use the output form and the ratio form of the divider equation
- find a sensor's resistance from two p.d.s
- explain in three steps why a sensing circuit's output rises or falls

What the exam asks

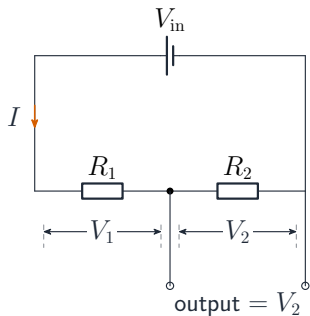
“Calculate the potential difference across the LDR.” [2]

“Calculate the resistance of the thermistor.” [2]

“Explain, without calculation, how the power dissipated in the wire changes as the temperature of the thermistor is increased.” [2]

Practise on exercise sheet 10.3, then try the real exam questions on past-paper sheet 10.3.

Potential dividers



The same current flows through both, so the p.d. is shared in proportion to R :

$$V_{\text{out}} = V_{\text{in}} \times \frac{R_2}{R_1 + R_2} \quad \frac{V_1}{V_2} = \frac{R_1}{R_2}$$

$$V_{\text{in}} = 9.0 \text{ V}, R_1 = 6.0 \text{ k}\Omega, R_2 = 3.0 \text{ k}\Omega$$

$$\begin{aligned} V_{\text{out}} &= V_{\text{in}} \times \frac{R_2}{R_1 + R_2} \\ &= 9.0 \text{ V} \times \frac{3.0 \text{ k}\Omega}{6.0 \text{ k}\Omega + 3.0 \text{ k}\Omega} = 3.0 \text{ V} \end{aligned}$$

Finding a sensor's resistance

A thermistor replaces R_1 . With the same 9.0 V supply, the output across $R_2 = 3.0 \text{ k}\Omega$ is now 6.0 V.

1. Its p.d.: the second law

$$V_T = V_{\text{in}} - V_{\text{out}} = 9.0 \text{ V} - 6.0 \text{ V} = 3.0 \text{ V}$$

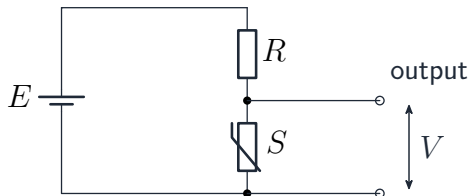
2. Its resistance: the ratio form

$$\frac{R_T}{R_2} = \frac{V_T}{V_{\text{out}}}, \text{ so}$$

$$R_T = R_2 \times \frac{V_T}{V_{\text{out}}} = 3.0 \text{ k}\Omega \times \frac{3.0 \text{ V}}{6.0 \text{ V}} = 1.5 \text{ k}\Omega$$

“Calculate the resistance of the thermistor”: find its own p.d. first, then share in proportion.

Sensor dividers



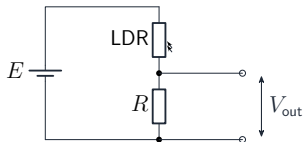
Swap one resistor for a **sensor** :

- **thermistor** : output changes with temperature
- **LDR** : output changes with light

Feed the output to a **transistor** base or a **comparator** to switch a load when the temperature or **light intensity** passes a **threshold** .

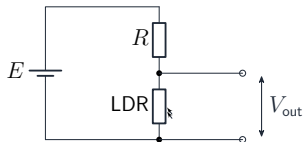
Why the output rises: three steps

A: output across the fixed resistor



brighter light $\Rightarrow V_{out}$ rises

B: output across the LDR



brighter light $\Rightarrow V_{out}$ falls

- 1 the **sensor's resistance** changes
- 2 so its **share** of the supply p.d. changes
- 3 so the p.d. across the **output component** changes

Thermistor: R falls as it warms.
LDR: R falls in brighter light. **Swap**
the output and the answer reverses.



Sheet 10.3.1 – what you need to know

You must be able to:

- explain why the p.d. along a uniform wire is proportional to its length
- compare two e.m.f.s from their **balance lengths**
- explain a **null method**, and why it measures an e.m.f.
- say why the driver cell must have the larger e.m.f.

What the exam asks

“Explain what is meant by a null method.” [1]

“Explain why the potential difference V between any two points on wire XY is proportional to the distance L between those points.” [2]

“Calculate the e.m.f. of cell X .” [2]

Practise on exercise sheet 10.3.1, then try the real exam questions on past-paper sheet 10.3.1.

A uniform wire is a potential divider

A uniform wire has $R = \frac{\rho L}{A}$ with ρ and A fixed, so $R \propto L$.

The same current flows all along it, so by $V = IR$ the p.d. across a length is **proportional to that length**:

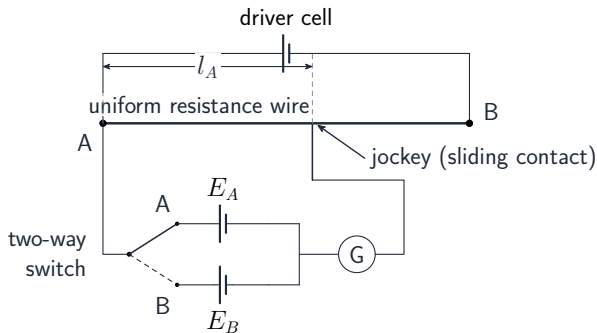
$$V = V_{\text{wire}} \times \frac{L}{L_{\text{total}}}$$

2.0 V across 100 cm of uniform wire

the p.d. across the first 30 cm:

$$V = V_{\text{wire}} \times \frac{L}{L_{\text{total}}} = 2.0 \text{ V} \times \frac{30 \text{ cm}}{100 \text{ cm}} = 0.60 \text{ V}$$

Potentiometer – the null method



Slide the contact until the **galvanometer** reads **zero**:

$$\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2}.$$

At balance no current flows, so the cell's internal resistance does not matter.

Comparing two e.m.f.s

A 1.02 V standard cell balances at 51.0 cm; a second cell at 75.0 cm

at balance each e.m.f. equals the p.d. across its balance length, so

$$\frac{E_2}{E_1} = \frac{\ell_2}{\ell_1}$$

$$E_2 = E_1 \times \frac{\ell_2}{\ell_1} = 1.02 \text{ V} \times \frac{75.0 \text{ cm}}{51.0 \text{ cm}} = 1.50 \text{ V}$$

At balance **no current** flows in the cell, so there is no Ir : the balance measures the **e.m.f.**, which no real voltmeter can.

The driver cell needs a **larger** e.m.f. than the cells it measures, or there is no balance point.



Exam tips

- Apply **Kirchhoff's laws**: current into a junction = current out (charge conserved); $\sum \text{e.m.f.} = \sum \text{p.d. round a loop}$ (energy conserved).
- Combine resistors: **series** $R = R_1 + R_2$; parallel $1/R = 1/R_1 + 1/R_2$.
- A **potential divider** splits voltage in the ratio of the resistances.
- Include **internal resistance**: $\text{e.m.f.} = I(R + r)$ – the “lost volts” are Ir .



Topic 10 – key takeaways

1 e.m.f. and r

$E = I(R + r)$ and $V = E - Ir$: the lost volts Ir heat the cell

2 Kirchhoff

junction: charge is conserved;
loop: energy is conserved

3 Dividers

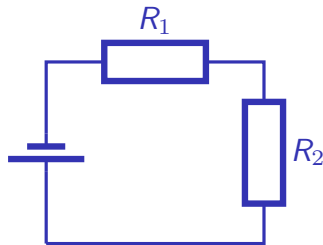
the p.d. splits in proportion to R ;
a sensor changes its share

4 Potentiometer

balance length $\propto$ e.m.f.; no
current flows at balance

Check yourself

- 1 Terminal p.d. of a cell on open circuit?
- 2 Two $6\ \Omega$ resistors in parallel – combined R ?
- 3 Which law comes from conservation of charge?
- 4 Why use the null method to compare e.m.f.s?



Answers 1. equals the e.m.f. ($I = 0$) 2. $3\ \Omega$ 3. the junction (first) law 4. internal resistance has no effect at balance