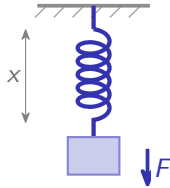


Deformation of Solids

AS Physics ■ Topic 6

A-Level Physics



What we will cover

- 1 Hooke's law 6.1
- 2 The Young modulus 6.1.1
- 3 Elastic and plastic 6.2
- 4 Definitions and tips





Sheet 6.1 – what you need to know

You must be able to:

- use the words **tensile**, compressive, load, extension and compression
- state **Hooke's law** and use $k = \frac{F}{x}$
- read a force–extension graph: its gradient, and where it stops being straight
- find the spring constant of springs in series or in parallel

What the exam asks

“State Hooke's law.” [1]

“Determine the spring constant k of the rope.” [2]

“State and explain qualitatively how the spring constant of the second sample compares with that of the original sample.” [2]

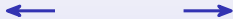
Practise on exercise sheet 6.1, then try the real exam questions on past-paper sheet 6.1.

Forces that change shape

A force along the length changes the shape. That change is a **deformation** . Both cases below are measured from the natural length.

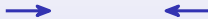
Tensile

stretches the solid
extension x



Compressive

squeezes the solid
compression , a negative x



Tensile testing machine

The applied force is the **load** . A spring's own mass is usually **negligible** , so the forces on it are the load and the tension it provides.

Hooke's law

For small extensions the extension is proportional to the load. This is **Hooke's law**.

$$F = kx \quad k = \frac{F}{x} \quad [k] = \text{N m}^{-1}$$

The **spring constant** k is the **force per unit extension**.

Not “extension per unit force” (that is $1/k$), and not $\frac{1}{2}Fx$ (that is the energy stored).

2.0 N stretches a spring 4.0 cm

$$4.0 \text{ cm} = 0.040 \text{ m}$$

$$k = \frac{F}{x} = \frac{2.0 \text{ N}}{0.040 \text{ m}} = 50 \text{ N m}^{-1}$$

Two lengths give k and L_0

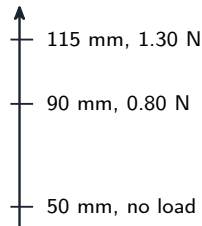
A spring is 90 mm long under 0.80 N, and 115 mm under 1.30 N.

The extra 0.50 N produces an extra
25 mm = 0.025 m:

$$k = \frac{\Delta F}{\Delta x} = \frac{0.50 \text{ N}}{0.025 \text{ m}} = 20 \text{ N m}^{-1}$$

At 0.80 N the extension is

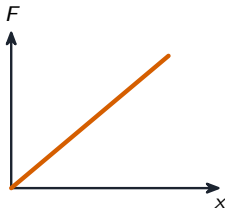
$$x = \frac{F}{k} = \frac{0.80 \text{ N}}{20 \text{ N m}^{-1}} = 0.040 \text{ m} = 40 \text{ mm}.$$



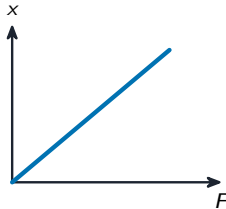
Unstretched length $L_0 = L - x = 90 \text{ mm} - 40 \text{ mm} = 50 \text{ mm}.$

Which gradient is k ?

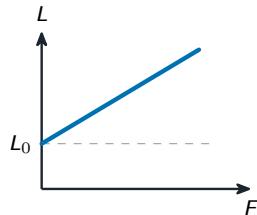
Read the axes before you quote a gradient.



F against x
gradient = k



x against F
gradient = $1/k$

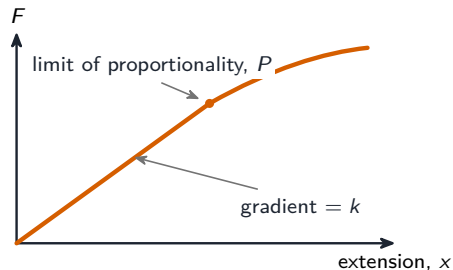


L against F
gradient = $1/k$

If force is vertical and the horizontal axis is length L rather than extension x , the gradient is still k , because $L = L_0 + x$.

The limit of proportionality

- Hooke's law holds only up to the **limit of proportionality** .
- Up to P the F - x line is straight and its gradient is k .
- Past P the line curves. The material may still be **elastic** a little further, then it turns **plastic** .



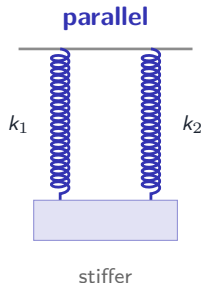
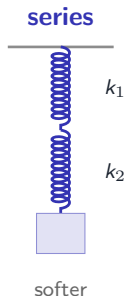
Springs in series and parallel

- **Series** : the **same load** passes through both springs and the extensions **add**.

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \text{ (softer).}$$

- **Parallel** : identical springs **share the load** and the extensions are **equal**.

$$k = k_1 + k_2 \text{ (stiffer).}$$



A series calculation

Springs 6.0 and 4.0 N cm^{-1} , with a load of 80 N , joined end to end

Each spring carries the full 80 N :

$$x_1 = \frac{F}{k_1} = \frac{80 \text{ N}}{6.0 \text{ N cm}^{-1}} = 13.3 \text{ cm}$$

$$x_2 = \frac{F}{k_2} = \frac{80 \text{ N}}{4.0 \text{ N cm}^{-1}} = 20 \text{ cm}$$

$$x = x_1 + x_2 = 13.3 \text{ cm} + 20 \text{ cm} = 33 \text{ cm}$$

The same result from $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$, so $k = 2.4 \text{ N cm}^{-1}$ and $x = \frac{F}{k} = \frac{80 \text{ N}}{2.4 \text{ N cm}^{-1}} = 33 \text{ cm}$.

Sharing a load, and the unit of k

- Two **identical** springs in parallel: each carries **half** the force.
- Energy stored goes as F^2 . Each spring stores a **quarter** of the energy one spring would store under the whole load. The pair together stores **half**.
- Three identical springs: the largest combined constant is all three in parallel ($3k$); the smallest is all three in series ($k/3$).
- $50 \text{ N m}^{-1} = 0.50 \text{ N cm}^{-1}$. A value in N cm^{-1} is **100 times smaller** than the same stiffness in N m^{-1} . Give the unit the question asks for.



Sheet 6.1.1 – what you need to know

You must be able to:

- define **stress**, **strain** and the **Young modulus**, with units
- calculate E for a wire from its load, extension, length and diameter
- find E from the gradient of a stress–strain or load–extension graph
- describe the wire experiment, and which measurement limits its accuracy

What the exam asks

“Define the Young modulus of a material.” [1]

“Show that the spring constant k_0 of the wire is given by $k_0 = \frac{EA}{L}$.” [2]

“State and explain the effect, if any, that this has on the strain in the wire.” [2]

Practise on exercise sheet 6.1.1, then try the real exam questions on past-paper sheet 6.1.1.

Stress and strain

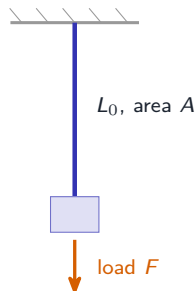
- **Stress** is the force per unit cross-sectional area .

$$\sigma = \frac{F}{A} \quad (\text{Pa} = \text{N m}^{-2})$$

In base units, $\text{Pa} = \text{kg m}^{-1} \text{s}^{-2}$.

- **Strain** is the extension per unit original length.

$$\varepsilon = \frac{x}{L_0} \quad (\text{no unit})$$



Area, and where the stress is larger

- A round wire: $A = \pi d^2/4 = \pi r^2$. Using the diameter **as the radius** makes the area four times too big. Halve d first.
- $1 \text{ mm}^2 = 10^{-6} \text{ m}^2$.
- 18 N on 3.2 mm^2 : $\sigma = \frac{F}{A} = \frac{18 \text{ N}}{3.2 \times 10^{-6} \text{ m}^2} = 5.6 \times 10^6 \text{ Pa}$.
- The same force acts all along the wire. A bolt of diameter $2d$ at one end and d at the other has **four times** the stress at the narrow end ($\sigma \propto 1/d^2$).
- Three times the radius means **one ninth** of the stress.

The Young modulus

The **Young modulus** is stress divided by strain, in the Hooke's-law region:

$$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{x/L_0} = \frac{FL_0}{Ax}.$$

- Unit: Pa. Metals are about 10^{11} Pa. Steel is 2.0×10^{11} Pa.
- E belongs to the **material**. A thicker wire of the same metal has the same E .
- The spring constant also depends on size:
 $k = EA/L_0$.

It stretches less because the area is larger, not because the metal is stiffer.

A steel wire

$L_0 = 2.0 \text{ m}$, $A = 1.5 \times 10^{-7} \text{ m}^2$, $x = 1.0 \text{ mm}$ **under** 15 N

The extension must be in metres: $1.0 \text{ mm} = 1.0 \times 10^{-3} \text{ m}$.

$$E = \frac{FL_0}{Ax} = \frac{(15 \text{ N})(2.0 \text{ m})}{(1.5 \times 10^{-7} \text{ m}^2)(1.0 \times 10^{-3} \text{ m})} = 2.0 \times 10^{11} \text{ Pa.}$$

A thicker steel wire has this same Young modulus.

Size changes k , not E

A “show that $k = EA/L_0$ ” answer is two lines:

$$k = \frac{F}{x}, \quad E = \frac{FL_0}{Ax}, \quad \text{so} \quad E = \frac{kL_0}{A} \quad \text{and} \quad k = \frac{EA}{L_0}.$$

- While Hooke's law holds, k stays **constant**, whatever the force.
- Same metal, **twice the diameter**: four times the area, so four times k . The Young modulus is unchanged.

A copper wire's spring constant

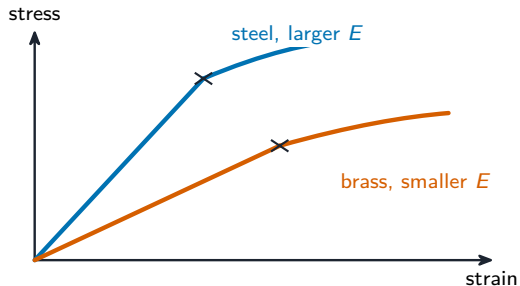
$$L_0 = 1.7 \text{ m, diameter } 0.64 \text{ mm, } E = 1.2 \times 10^{11} \text{ Pa}$$

Halve the diameter: $r = 0.32 \times 10^{-3} \text{ m}$.

$$A = \pi r^2 = \pi(0.32 \times 10^{-3} \text{ m})^2 = 3.2 \times 10^{-7} \text{ m}^2$$

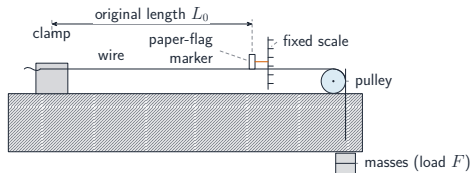
$$k = \frac{EA}{L_0} = \frac{(1.2 \times 10^{11} \text{ Pa})(3.2 \times 10^{-7} \text{ m}^2)}{1.7 \text{ m}} = 2.3 \times 10^4 \text{ N m}^{-1}.$$

Reading E off a graph



- On a stress–strain graph the gradient of the straight part is E .
- The steeper line is the stiffer material.
- Each cross is that metal's limit of proportionality.
- On a force–extension graph the gradient is EA/L_0 , so $E = \text{gradient} \times L_0/A$.

Measuring E for a wire



- 1 Long thin wire over a **pulley** .
- 2 L_0 with a metre rule, from the clamp to a marker.
- 3 **Diameter** d with a **micrometer** , at several places. $A = \pi d^2/4$.
- 4 Add loads. Read the extension from the marker against a fixed scale.
- 5 Plot F against x . In the straight region, $E = \text{gradient} \times L_0/A$.

The uncertainty in E

$d = (0.50 \pm 0.01) \text{ mm}$, $x = (2.5 \pm 0.1) \text{ mm}$; F
and L together $\pm 1\%$

diameter: $\frac{\Delta d}{d} = \frac{0.01 \text{ mm}}{0.50 \text{ mm}} = 2\%$, doubled

because $A \propto d^2$: 4%

extension: $\frac{\Delta x}{x} = \frac{0.1 \text{ mm}}{2.5 \text{ mm}} = 4\%$

$E = \frac{FL}{Ax}$ multiplies and divides, so the percentages
add:

$$\frac{\Delta E}{E} = 2 \frac{\Delta d}{d} + \frac{\Delta x}{x} + 1\% = 4\% + 4\% + 1\% = 9\%$$

$$E = (2.0 \pm 0.2) \times 10^{11} \text{ Pa}$$

The diameter is squared, so its percentage uncertainty counts **twice**: measure it with a micrometer, in several places.

A long wire gives a larger extension, and so a smaller percentage uncertainty in x .

What earns the experiment marks

Measure

L_0 with a metre rule.

d with a micrometer,
in several places.

Load from the masses.

Extension from a marker
on a fixed scale.

Graph

Plot F against x .

The gradient is EA/L_0 .

Or plot stress against
strain. That gradient
is E itself.

Precautions

Safety goggles: the wire
can snap.

A small load first, to
take out kinks.

Read on unloading too,
to check it stayed elastic.

Long and thin, so the extension is large enough to measure. Repeating d cuts **random error** and checks the wire is uniform.



Sheet 6.2 – what you need to know

You must be able to:

- use elastic deformation, plastic deformation and elastic limit
- find the work done from the area under a force–extension graph
- use $E_P = \frac{1}{2}Fx = \frac{1}{2}kx^2$
- explain why energy is not all recovered after plastic deformation

What the exam asks

“Show that the elastic potential energy in the rope is 75 kJ.” [2]

“Calculate the elastic potential energy E_P of the compressed spring.” [2]

“Explain why the work done in stretching the wire to an extension of 12 mm is not equal to the energy recovered when the tensile force is removed.” [2]

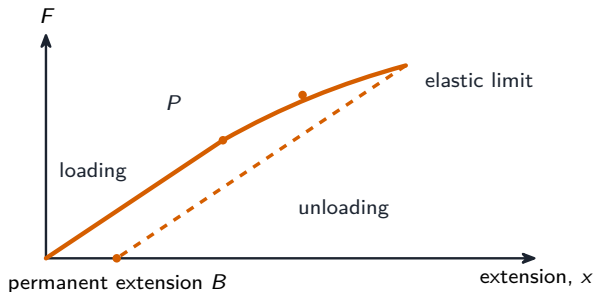
Practise on exercise sheet 6.2, then try the real exam questions on past-paper sheet 6.2.

Elastic, then plastic

- 1 **Straight and elastic**, up to the limit of proportionality. Hooke's law holds. Remove the load and the object returns to its first length.
- 2 **Curved but still elastic**, on to the **elastic limit** . Extension is no longer proportional to load, but unloading still returns the original length.
- 3 **Plastic**, past the elastic limit. A permanent extension stays.

On a loading curve the end of the straight part is the limit of proportionality. The elastic limit lies a little beyond it, and a loading line alone cannot show which later point it is.

After the elastic limit

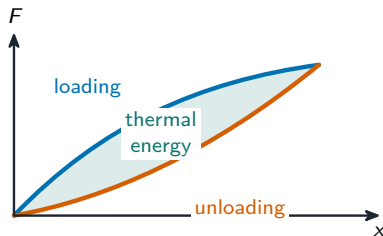


The unloading line is parallel to the first Hooke line, shifted right.

Where it meets the axis, a permanent extension B is left.

The area between the two lines is energy turned into **thermal energy**.

A rubber band is elastic



A **rubber band** returns to its original length, so the deformation is elastic.

The two curves are different. The area between them is **elastic hysteresis** : energy dissipated as thermal energy.

Energy recovered on unloading is the area under the **lower** curve.

A wire taken past its elastic limit does not do this. Part of the deformation is plastic, a permanent extension remains, and part of the work done became thermal energy in the wire.

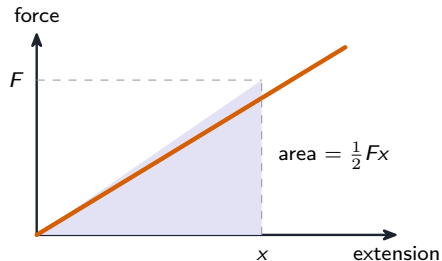
Energy stored is an area

Work done stretching a material is the **area** under the force–extension graph.

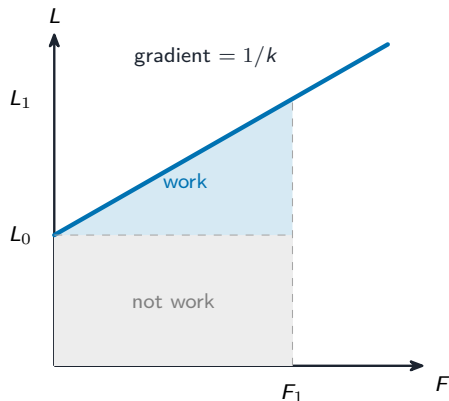
The force grows as the material stretches, so the work is the sum of $F\Delta x$. Within the limit of proportionality the average force is $\frac{1}{2}F$:

$$E_P = \frac{1}{2}Fx = \frac{1}{2}kx^2 = \frac{F^2}{2k}.$$

E_P here is energy, not the Young modulus.



Work on a length–force graph



- The intercept on the length axis is L_0 .
- The gradient is $1/k$, not k .
- Work done is the **triangle** between the line and the level $L = L_0$.
- The rectangle below L_0 is the unstretched length times the force. It is not work.

One joule in a spring

$k = 50 \text{ N m}^{-1}$, stretched by 0.20 m , within the limit of proportionality

$$E_P = \frac{1}{2} kx^2 = \frac{1}{2} (50 \text{ N m}^{-1}) (0.20 \text{ m})^2 = 1.0 \text{ J}.$$

This **elastic potential energy** is stored because the spring was stretched or compressed inside its limit of proportionality.

Between two extensions

A wire with $k = 2.0 \times 10^4 \text{ N m}^{-1}$ is already extended by 2.0 mm. The work to reach 3.0 mm is the **difference** of two stored energies:

$$\begin{aligned} W &= \frac{1}{2}k(x_2^2 - x_1^2) \\ &= \frac{1}{2}(2.0 \times 10^4 \text{ N m}^{-1})((3.0 \times 10^{-3} \text{ m})^2 - (2.0 \times 10^{-3} \text{ m})^2) \\ &= \mathbf{0.050 \text{ J}} \end{aligned}$$

Not $\frac{1}{2}k(\Delta x)^2$. That would be a new stretch starting from zero.

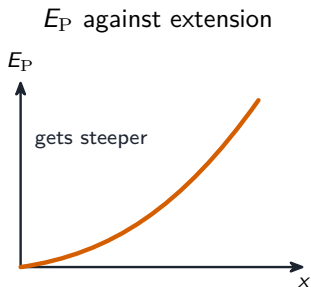
- $E_P \propto x^2$. Double the extension and the energy is **four times** larger.
- For $k = 400 \text{ N m}^{-1}$ and 0.32 J :

$$x = \sqrt{\frac{2E_P}{k}} = \sqrt{\frac{2(0.32 \text{ J})}{400 \text{ N m}^{-1}}} = 0.040 \text{ m}.$$
- Two wires end to end feel the **same** tension. Add $\frac{1}{2}Fx$ for each wire's own extension.

When the line is not straight

Past the limit of proportionality, or for a rubber band, the graph is not a triangle.

- Find the area by counting grid squares (a part square counts as a half) or by **trapezia** .
- One square is worth the force step times the extension step.
- The area under the whole curve, up to breaking, is still the work done.



A curve through the origin, not a straight line.

Comparing stored energy

Using $E_P = \frac{1}{2}Fx$:

- Same force, **softer** spring (smaller k): larger extension, **more** energy.
- Same extension, **stiffer** spring (larger k): larger force, **more** energy.
- Two wires, same length, same area, same load: the one with the **smaller** Young modulus extends more and stores more energy.

Released onto a mass, the elastic potential energy becomes **kinetic energy** :

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2. \text{ Add } mgh \text{ if the mass also rises.}$$



Definitions to memorise

One answer only, in this wording.

Hooke's law	the extension is directly proportional to the applied force, provided the limit of proportionality is not exceeded
spring constant	the force per unit extension
limit of proportionality	the point beyond which the extension is no longer proportional to the applied force
elastic limit	the maximum force (or extension) for which the material returns to its original length when the force is removed
elastic deformation	the material returns to its original length when the force is removed

Five more definitions

plastic deformation

the material does not return to its original length when the force is removed; a permanent extension remains

stress

the force per unit cross-sectional area

strain

the extension per unit original length

Young modulus

the ratio of stress to strain, within the limit of proportionality

elastic potential energy

the energy stored in an object because it has been stretched or compressed

Exam tips

- **Hooke's law** ($F = kx$) holds only up to the limit of proportionality. That condition is part of the statement.
- **Stress** = F/A , **strain** = x/L_0 , **Young modulus** = **stress/strain** (the gradient of the straight part of a stress–strain graph). The unit is the pascal.
- Energy stored is the **area under the force–extension graph**, and $\frac{1}{2}Fx$ only while the line is straight.
- **Elastic** means it returns to its original length. **Plastic** means a permanent extension remains.

Mistakes that cost the mark

- Using the diameter as the radius in $A = \pi r^2$, or leaving an area in mm^2 .
- Calling the whole area down to the axis “work” on a length–force graph. Only the triangle above L_0 is work.
- Treating stored energy as proportional to extension. It goes as x^2 .
- Stating Hooke’s law and leaving out the limit of proportionality.
- Swapping the limit of proportionality (end of the straight line) and the elastic limit.
- Saying a thicker wire has a larger Young modulus. The modulus is the material’s; the thicker wire has a larger spring constant.

Topic 6 key takeaways

1

Hooke's law

$F = kx$ only up to the limit of proportionality

2

Young modulus

$E = \sigma/\epsilon$, a property of the material

3

Elastic / plastic

plastic leaves a permanent extension

4

Stored energy

area under $F-x$; $\frac{1}{2}kx^2$ while straight

Check yourself

- 1 What is the unit of the spring constant k ?
- 2 A graph plots length against force. What is its gradient?
- 3 A wire's diameter doubles. What happens to E , and what happens to k ?
- 4 How much energy is stored when $k = 200 \text{ N m}^{-1}$ and $x = 0.10 \text{ m}$?

Answers

1. N m^{-1}

2. $1/k$

3. E unchanged, k four times larger

4. 1.0 J