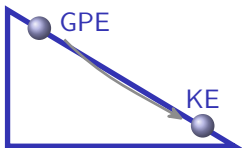


Work, Energy & Power

AS Physics ■ Topic 5

A-Level Physics



What we will cover

- 1 Work and energy 5.1
- 2 Efficiency and power 5.1.1
- 3 Potential and kinetic energy 5.2
- 4 Energy methods 5.2.1
- 5 Recap





Sheet 5.1 – what you need to know

You must be able to:

- define **work done** and use $W = Fs \cos \theta$
- say when a force does positive, negative or zero work
- state the principle of **conservation of energy**
- account for every joule, friction included, to find a speed or a height

What the exam asks

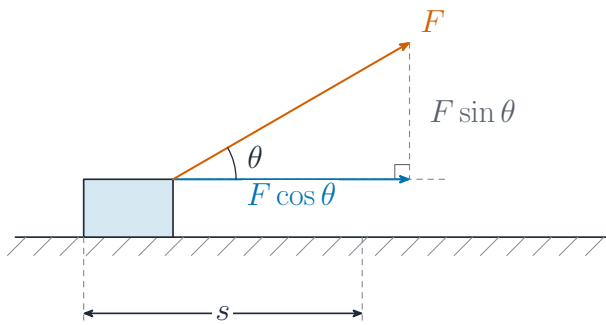
“State what is meant by the work done by a force.” [1]

“Calculate the work done by F for the movement of the trolley from X to Y.” [1]

“Use your answer and conservation of energy to determine the speed of the block at point A.” [3]

Practise on exercise sheet 5.1, then try the real exam questions on past-paper sheet 5.1.

Work done by a force



Work = force $\times$ distance **along the force**:

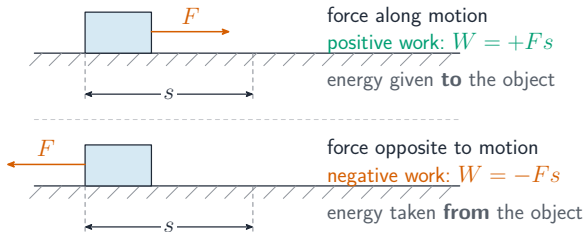
$$W = Fs \cos \theta, \quad [W] = \text{J} = \text{Nm}.$$

Work is a **scalar**.

Sledge: 20 N at 60° , 5.0 m

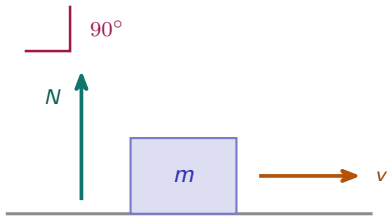
$$\begin{aligned} W &= Fs \cos \theta \\ &= (20 \text{ N})(5.0 \text{ m}) \cos 60^\circ \\ &= 50 \text{ J} \end{aligned}$$

Positive, negative and zero work



- $\theta = 0$: $W = +Fs$ – energy in
- $\theta = 90^\circ$: $W = 0$ – no work
(normal force on a flat road)
- $\theta = 180^\circ$: $W = -Fs$ – energy out (friction)

A force at 90° does no work

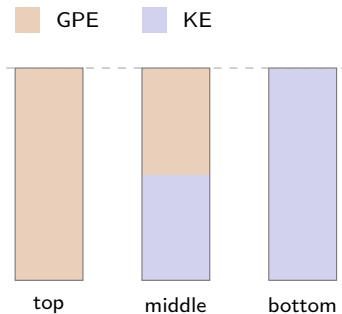


N is vertical. The motion is horizontal.

$$W = Fs \cos \theta$$

At 90° , $\cos \theta = 0$, so nothing is transferred. The **normal contact force** on a flat road does no work.

Conservation of energy



The total stays the same.

Energy is **never made or destroyed**. It only changes form.

In a **closed system** the total is constant.

On a frictionless **ramp** :

$$mgh = \frac{1}{2}mv^2, \quad v = \sqrt{2gh}.$$

With friction, some becomes **thermal energy** .

Pushing against friction

A box is pushed 3.0 m at a **constant speed** against a friction force of 40 N.

Constant speed: no resultant force, so the push equals the friction.

Where the work goes

work done by the push:

$$W = Fs = (40 \text{ N})(3.0 \text{ m}) = 120 \text{ J}$$

the kinetic energy does not change, so all 120 J becomes **thermal energy** of the box and the floor

friction does -120 J of work on the box

Down a slope with friction

A 2.0 kg block slides 4.0 m down a slope from rest, falling 1.5 m vertically, against a friction force of 3.0 N.

In and out

released:

$$\begin{aligned}\Delta E_P &= mg\Delta h = (2.0 \text{ kg})(9.81 \text{ m s}^{-2})(1.5 \text{ m}) \\ &= 29.4 \text{ J}\end{aligned}$$

against friction:

$$W = Fs = (3.0 \text{ N})(4.0 \text{ m}) = 12.0 \text{ J}$$

What is left, and the speed

$$E_K = \Delta E_P - W = 29.4 \text{ J} - 12.0 \text{ J} = 17.4 \text{ J}$$

$$v = \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{2 \times 17.4 \text{ J}}{2.0 \text{ kg}}} = 4.2 \text{ m s}^{-1}$$

Friction uses the distance **along the slope**; potential energy uses the **vertical** height.

A lift at constant speed

A 600 kg lift rises 15 m at a constant speed. Its motor transfers $1.20 \times 10^5 \text{ J}$.

Constant speed: the kinetic energy does not change, so the useful gain is potential energy only.

Account for every joule

useful gain:

$$\Delta E_P = mg\Delta h = (600 \text{ kg})(9.81 \text{ m s}^{-2})(15 \text{ m}) = 8.83 \times 10^4 \text{ J}$$

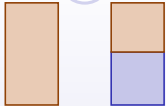
the rest, by conservation of energy:

$$\begin{aligned} E_{\text{wasted}} &= E_{\text{in}} - \Delta E_P \\ &= 1.20 \times 10^5 \text{ J} - 8.83 \times 10^4 \text{ J} = \\ &3.2 \times 10^4 \text{ J} \end{aligned}$$

as thermal energy in the motor and cables, and sound

Using energy methods

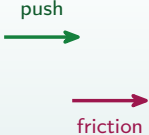
1



start end

Write the energy at the **start** and at the **end**.

2



A push puts energy in. Friction takes energy out.

3

$$E_{\text{start}} + W_{\text{in}} = E_{\text{end}} + W_{\text{lost}}$$



Sheet 5.1.1 – what you need to know

You must be able to:

- calculate an **efficiency** from energies or from powers
- define power and use $P = \frac{W}{t}$
- derive $P = Fv$ from the definitions of power and work
- use $P = Fv$ on the level, up a slope, and when drag grows with speed

What the exam asks

“Use the definition of power to show that the power P transferred by the force is given by $P = Fv$.”

[2]

“Calculate the percentage efficiency of the solar panel.” **[1]**

“Show that the resistive force opposing the forward motion of the lorry is 5200 N.” **[1]**

Practise on exercise sheet 5.1.1, then try the real exam questions on past-paper sheet 5.1.1.

Power – the rate of energy transfer

$$P = \frac{W}{t} = \frac{\Delta E}{\Delta t}, \quad [P] = W = \text{J s}^{-1}.$$

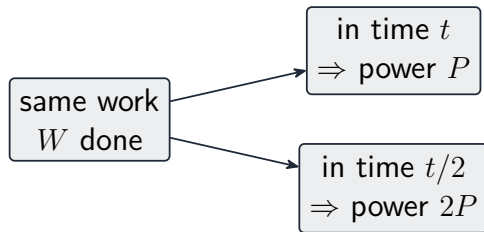
Power is a **scalar** .

same work
in less time



more power

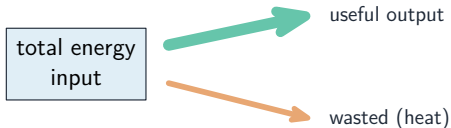
Same work, different power



$$P = \frac{W}{t} \text{ — less time, more power}$$

Power is **work per second**: climbing the stairs twice as fast does the *same* work against gravity, at **twice** the power.

Efficiency



$$\text{efficiency} = \frac{\text{useful output}}{\text{total input}} \times 100\%$$

$$\text{efficiency} = \frac{\text{useful output}}{\text{total input}}$$

A pump lifts 600 kg of water 12 m in 30 s, drawing 3.0 kW

useful power, the gain in potential energy per second:

$$P = \frac{mg\Delta h}{t} = \frac{(600 \text{ kg})(9.81 \text{ m s}^{-2})(12 \text{ m})}{30 \text{ s}} = 2350 \text{ W}$$

$$\text{efficiency} = \frac{2350 \text{ W}}{3000 \text{ W}} = 0.78, \text{ or } 78\%$$

The rest is wasted, usually as thermal energy. Real machines stay under 100%.

Power at a steady speed is force times velocity

$$P = Fv$$

In a time Δt the force moves through $v\Delta t$,
doing $Fv\Delta t$ of work. Divide by Δt .

Car at 25 m s^{-1} against 600 N

At constant speed the driving force
equals the resistive force, so

$$P = Fv = (600 \text{ N})(25 \text{ m s}^{-1}) = 15 \text{ kW}.$$

flat road

$$P = F_{\text{resist}} v$$

drag grows like v^2
double the speed, about
four times the power

steady lift

$$P = mgv$$

Up a slope at a steady speed

A 1200 kg car climbs a 5.0° slope at a steady 20 m s^{-1} against a resistive force of 600 N.

Along the slope the driving force balances the resistive force **and** the component of the weight.

Driving force, then power

$$mg \sin \theta = (1200 \text{ kg})(9.81 \text{ m s}^{-2}) \sin 5.0^\circ = 1030 \text{ N}$$

$$F = F_r + mg \sin \theta = 600 \text{ N} + 1030 \text{ N} = 1630 \text{ N}$$

$$P = Fv = (1630 \text{ N})(20 \text{ m s}^{-1}) = 3.3 \times 10^4 \text{ W}$$

Drag that grows with speed

The resistive force on a car is proportional to the **square** of its speed: 800 N at 20 m s^{-1} .

Double the speed: four times the force, **eight** times the power.

A steady 40 m s^{-1}

$$F \propto v^2: F_2 = F_1 \left(\frac{v_2}{v_1} \right)^2 =$$
$$800 \text{ N} \times \left(\frac{40 \text{ m s}^{-1}}{20 \text{ m s}^{-1}} \right)^2 = 3200 \text{ N}$$

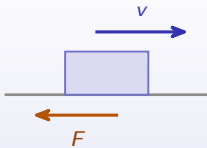
$$P_2 = F_2 v_2 = (3200 \text{ N})(40 \text{ m s}^{-1}) =$$
$$1.28 \times 10^5 \text{ W}$$

at 20 m s^{-1} :

$$P_1 = F_1 v_1 = (800 \text{ N})(20 \text{ m s}^{-1}) = 1.6 \times 10^4 \text{ W}$$

Reading $P = Fv$ three ways

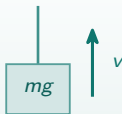
Force



$$F = P/v$$

a steady speed gives
the opposing force

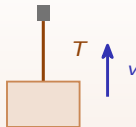
Lifting speed



$$\eta VI = mgv$$

voltage V , **current** I

Tension



$$T = P_{\text{useful}}/v$$

tension is useful
power over speed



Sheet 5.2 – what you need to know

You must be able to:

- derive $\Delta E_P = mg\Delta h$ from $W = Fs$
- derive $E_K = \frac{1}{2}mv^2$ from the equations of motion
- solve problems where potential and kinetic energy exchange
- follow the energy through a chain of transfers

What the exam asks

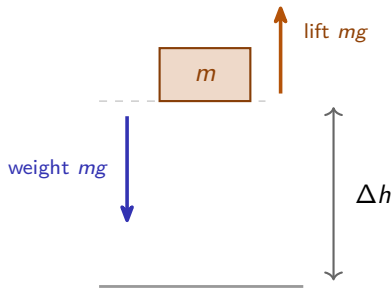
“Use the concept of work done to show that $\Delta E_P = mg\Delta h$.” [2]

“Calculate the kinetic energy of the block as it leaves the spring.” [2]

“Determine the gain in kinetic energy of the truck between A and B.” [3]

Practise on exercise sheet 5.2, then try the real exam questions on past-paper sheet 5.2.

Where $mg\Delta h$ comes from



Raising a **mass** m stores **gravitational potential energy** .

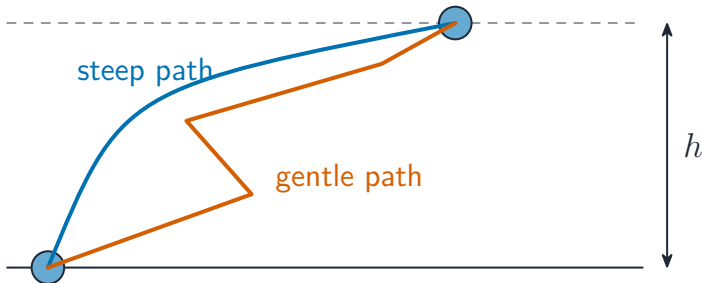
$$\Delta E_P = mg\Delta h$$

Lift it slowly, so no kinetic energy is gained. The upward force is mg , the work it does is $mg \cdot \Delta h$, and that work *is* the stored energy.

You need m , Δh and g – **never** speed or time.

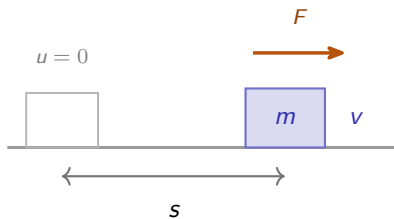
Only the height matters, not the path

same $\Delta E_P = mgh$ – only the height h matters



Steep or zig-zag, the gain is mgh either way: gravitational PE counts only the height risen.

Kinetic energy: where $\frac{1}{2}mv^2$ comes from



A push from rest, through a distance s , up to speed v .

$$v^2 = u^2 + 2as \text{ with } u = 0$$
$$\text{so } s = v^2 / (2a)$$

$$W = Fs = ma \cdot s$$

the a cancels

$$E_K = \frac{1}{2}mv^2$$

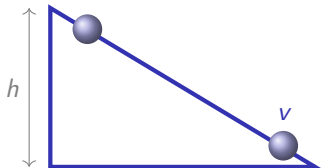
Worked example – speed down a ramp

From rest, smooth ramp, $h = 1.2 \text{ m}$

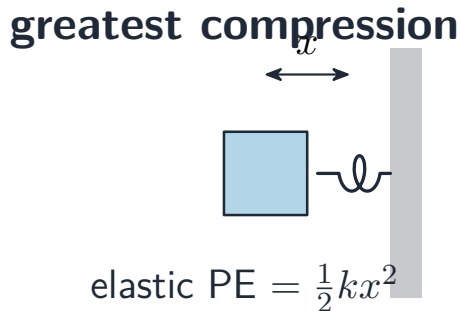
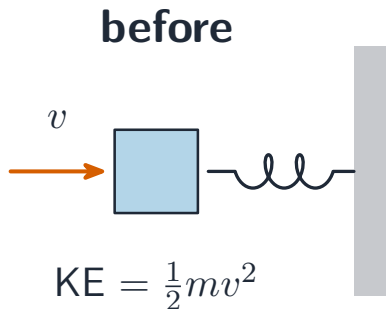
All the GPE becomes KE. Mass cancels.

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} = \sqrt{2(9.81 \text{ m s}^{-2})(1.2 \text{ m})} \approx 4.9 \text{ m s}^{-1}$$



Using energy methods, in detail



A block's kinetic energy becomes elastic potential energy as it compresses the spring



Sheet 5.2.1 – what you need to know

You must be able to:

- use the work done by the resultant force as the change in E_K
- compare kinetic energies, including with the same momentum
- find a speed from energy whatever the direction of motion
- find the work done against air resistance, and the average force

What the exam asks

“Calculate the maximum kinetic energy of the car.” [2]

“Determine the ratio of the kinetic energy of the child at Y with the resistive force to that without it.” [1]

“Explain why the work done by the upthrust is not equal to the gain in elastic potential energy of the spring.” [1]

Practise on exercise sheet 5.2.1, then try the real exam questions on past-paper sheet 5.2.1.

When you know the momentum, not the speed

$$p = mv \quad \text{and} \quad E_K = \frac{1}{2}mv^2$$


$$E_K = \frac{p^2}{2m}$$

A momentum change from p_1 to p_2 at constant mass changes the kinetic energy by $\frac{p_2^2 - p_1^2}{2m}$.

Reach for this whenever a question gives you **momentum** and asks for energy – there is no need to find v first.

Same momentum, different kinetic energy

$$E_K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

The same momentum with four times the mass: **one quarter** of the kinetic energy.

A 1000 kg car at 20 m s^{-1} ; a 4000 kg truck, same p

$$\text{car: } p = mv = (1000 \text{ kg})(20 \text{ m s}^{-1}) = 2.0 \times 10^4 \text{ kg m s}^{-1}$$

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}(1000 \text{ kg})(20 \text{ m s}^{-1})^2 = 2.0 \times 10^5 \text{ J}$$

$$\text{truck: } v = \frac{p}{m} = \frac{2.0 \times 10^4 \text{ kg m s}^{-1}}{4000 \text{ kg}} = 5.0 \text{ m s}^{-1}$$

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}(4000 \text{ kg})(5.0 \text{ m s}^{-1})^2 = 5.0 \times 10^4 \text{ J}$$

A speed from energy, whatever the direction

A stone is thrown horizontally at 8.0 m s^{-1} from a cliff 20 m high; no air resistance.

Energy gives the **speed**, not the direction or the time: use the equations of motion for those.

Conservation of energy; the mass cancels

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mg\Delta h$$

$$\begin{aligned} v &= \sqrt{u^2 + 2g\Delta h} = \\ &= \sqrt{(8.0 \text{ m s}^{-1})^2 + 2(9.81 \text{ m s}^{-2})(20 \text{ m})} \\ &= 21.4 \text{ m s}^{-1} \end{aligned}$$

Air resistance takes its share

A 0.20 kg ball is dropped from 5.0 m and lands with 8.0 J of kinetic energy.

With air resistance the total energy is **not** constant: the lower the ball, the less it has left.

The missing energy is the work done against air resistance

$$E_P = mgh = (0.20 \text{ kg})(9.81 \text{ m s}^{-2})(5.0 \text{ m}) = 9.81 \text{ J}$$

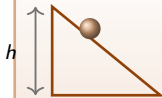
$$W = E_P - E_K = 9.81 \text{ J} - 8.0 \text{ J} = 1.8 \text{ J}$$

$$F = \frac{W}{s} = \frac{1.81 \text{ J}}{5.0 \text{ m}} = 0.36 \text{ N, on average}$$

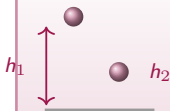
Four places to use it



Spring . At full **compression** ,
 $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$.



Constant speed.
The push pays for
the height and the
friction.



Bounce. The
fraction of the height
kept is h_2/h_1 .

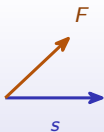
same speed



Same height, any
angle: it lands at the
same speed.



Exam tips

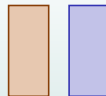


$$W = Fs \cos \theta$$

Use the distance
along the force.

GPE

KE



Energy stays

Loss in GPE = gain
in KE, plus work
against resistance.

 P

$$P = W/t = Fv$$

Efficiency is useful
output divided by
total input.



Vertical height

GPE uses the height
gained, not the
distance along the
slope.

Topic 5 – key takeaways

1

Work

$$W = Fs \cos \theta; \text{ energy in or out}$$

2

Energy

conserved; only changes form

3

Power

$$P = W/t = Fv; \text{ efficiency} < 100\%$$

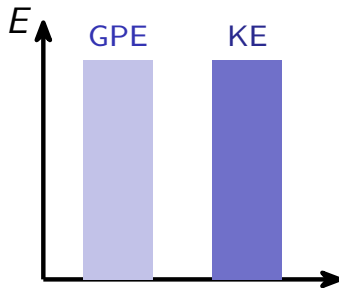
4

Stores

$$E_P = mg\Delta h, E_K = \frac{1}{2}mv^2$$

Check yourself

- 1 Work done at 90° to the motion?
- 2 Speed after a smooth drop of 5.0 m?
- 3 Power at 20 m s^{-1} against 400 N?
- 4 Why is efficiency never 100%?



Answers 1. zero 2. $v = \sqrt{2gh} \approx 9.9 \text{ m s}^{-1}$ 3. $P = Fv = 8.0 \text{ kW}$ 4. some energy always wasted as heat