

Forces, Density & Pressure

AS Physics ■ Topic 4

A-Level Physics



What we will cover

1

Moments and couples

sheet 4.1: $M = Fd$, torque



2

Centre of gravity

sheet 4.1.1: angles, non-uniform



3

Equilibrium

sheet 4.2: moments, triangles



4

Density and pressure

sheet 4.3: $\Delta p = \rho g \Delta h$



5

Upthrust

sheet 4.3.1: floating, U-tubes



6

Putting it together

definitions, mistakes





Sheet 4.1 – what you need to know

You must be able to:

- define the **moment** of a force and the **torque** of a couple
- use the perpendicular distance to the line of action
- add moments with their sense, clockwise or anticlockwise
- say whether two forces form a **couple**

What the exam asks

“Define the moment of a force.”

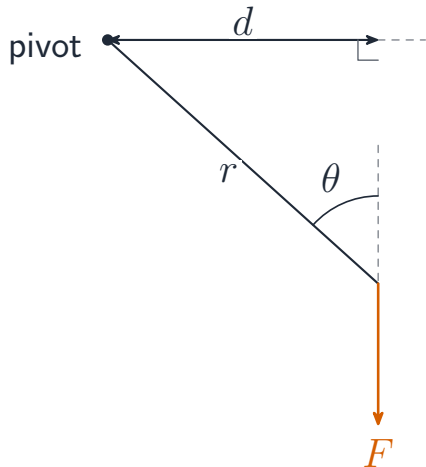
[1]

“State and explain whether the forces from A and C are a couple.” **[1]**

“Calculate the resultant moment about the centre of gravity.” **[2]**

Practise on exercise sheet 4.1, then try the real exam questions on past-paper sheet 4.1.

Moment of a force



The **moment** of a force about a point:

$$M = Fd \quad \text{unit: N m}$$

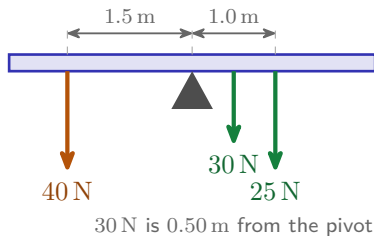
d is the **perpendicular distance** from the point to the **line of action** of the force.

If F acts at angle θ to a **lever arm** of length r from the **pivot** :

$$d = r \sin \theta \quad \Rightarrow \quad M = Fr \sin \theta$$

Only the part of F **perpendicular** to the lever arm turns it. Each moment is **clockwise** or **anticlockwise** .

Adding moments with their sense



Moments about the pivot, $M = Fd$

anticlockwise:

$$M = Fd = (40 \text{ N})(1.5 \text{ m}) = 60 \text{ N m}$$

clockwise: $M = F_1 d_1 + F_2 d_2$

$$= (25 \text{ N})(1.0 \text{ m}) + (30 \text{ N})(0.50 \text{ m}) = 40 \text{ N m}$$

resultant:

$$M = 60 \text{ N m} - 40 \text{ N m} = 20 \text{ N m}$$

anticlockwise

Give every moment its **sense**; the larger total wins.

Couples and torque

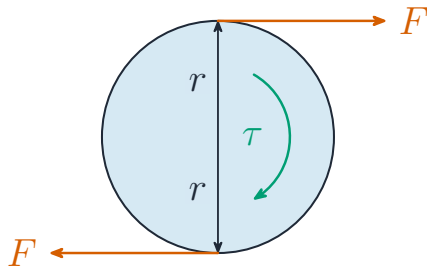
A **couple** is a pair of forces that are

- **equal** in size
- **opposite** in direction
- a perpendicular distance apart

The **torque** of a couple is its turning effect:

$$\tau = Fd \quad \text{unit: N m}$$

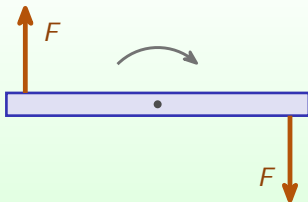
F is **one** of the forces; d is the perpendicular distance **between** their lines of action.



A couple **only turns**: its resultant force is zero. Its torque is the **same about any point**.

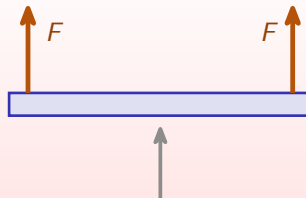
A couple, or not?

✓ a couple



resultant force 0: it only **turns**

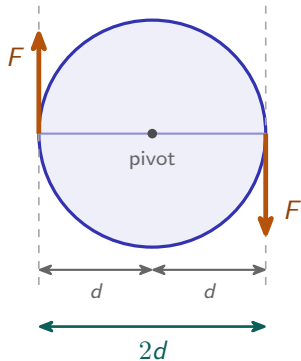
✗ not a couple



resultant $2F$: it **moves** (translation)

A multiple-choice trap: equal forces in the **same** direction are not a couple. A couple needs equal size **and** opposite directions.

Torque of a couple: watch the distance



Two forces F , each a distance d from a central pivot, on opposite sides:

between the lines of action: $\tau = F \times 2d = 2Fd$

moments about the pivot: $Fd + Fd = 2Fd$

The two-mark definition, one mark for each part: the torque of a couple is **the product of one of the forces** and **the perpendicular distance between the lines of action of the forces**.



Sheet 4.1.1 – what you need to know

You must be able to:

- define the **centre of gravity**, and find it by hanging
- take a moment when the force is at an angle
- find the moment of the weight of a tilted object
- find the weight or the centre of gravity of a non-uniform object

What the exam asks

“State what is meant by the centre of gravity of an object.” [1]

“Calculate the component of the weight that is perpendicular to the trapdoor.” [1]

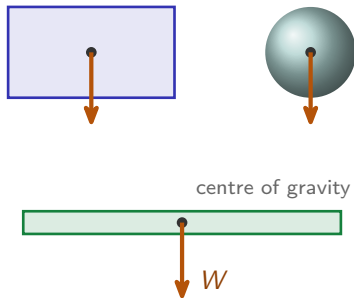
“Calculate the magnitude of the force F .” [2]

Practise on exercise sheet 4.1.1, then try the real exam questions on past-paper sheet 4.1.1.

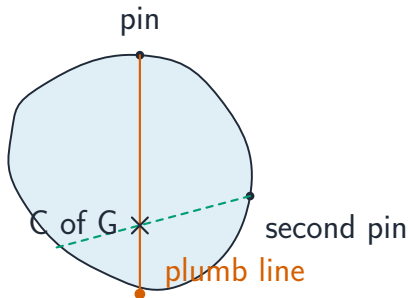
Centre of gravity

The **weight** of a large object can be treated as acting at **one point**: its **centre of gravity**.

- in a uniform gravitational field, it is the **centre of mass**
- a uniform, regular shape: at the **middle**
- on a **free-body diagram**, draw the weight **from there**



Finding the centre of gravity of an irregular shape



hanging freely, the sheet settles
with its centre of gravity vertically
below the pin
hang it from a second point:
the two plumb lines cross at
the centre of gravity

A **plumb line** hung from the pin marks the vertical. “Explain why it hangs like this”:
the centre of gravity is **directly below the support**, so the weight acts through the
pivot and has **no moment** about it.

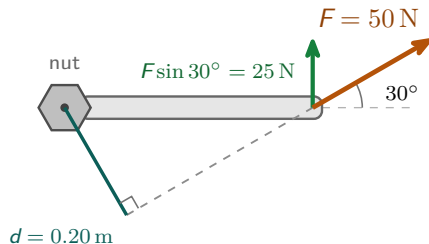
Worked example – turning a nut with a spanner

$F = 50 \text{ N}$ on a 0.40 m spanner, at 30° to it

View 1. Only the perpendicular part turns the nut: $F_\perp = F \sin 30^\circ = 25 \text{ N}$, so $M = F_\perp r = (25 \text{ N})(0.40 \text{ m})$.

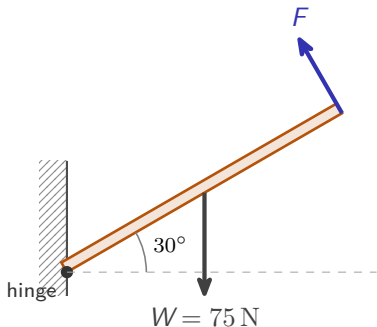
View 2. The perpendicular distance to the line of action is $d = r \sin 30^\circ = 0.20 \text{ m}$, so $M = Fd = (50 \text{ N})(0.20 \text{ m})$.

$$M = Fr \sin \theta = 10 \text{ N m}$$



Both views give the same moment. A push

The weight of a tilted object



A uniform trapdoor, 0.80 m long

perpendicular to the door:

$$W_{\perp} = W \cos 30^{\circ} = (75 \text{ N}) \cos 30^{\circ} = 65 \text{ N},$$

acting 0.40 m out

moment of the weight:

$$M = W_{\perp} d = (65 \text{ N})(0.40 \text{ m}) = 26 \text{ N m}$$

moments balance:

$$F = \frac{M}{L} = \frac{26 \text{ N m}}{0.80 \text{ m}} = 32 \text{ N}$$

The weight is always **vertical**: use its component perpendicular to the object.

A non-uniform bar

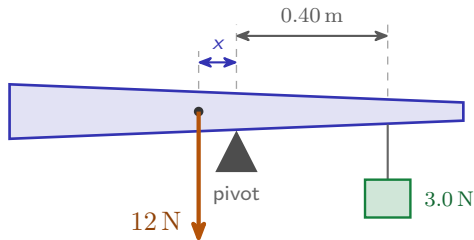
A **non-uniform** bar has its centre of gravity **off the middle**. Find it by

- balancing it on a pivot: the centre of gravity is then **above the pivot**
- taking moments against a **known** weight

A 12 N bar pivoted at its middle

It balances with 3.0 N hung 0.40 m out on the thin side, so $Wx = Fd$:

$$x = \frac{Fd}{W} = \frac{(3.0 \text{ N})(0.40 \text{ m})}{12 \text{ N}} = 0.10 \text{ m} \text{ from the pivot, on the thick side.}$$





Sheet 4.2 – what you need to know

You must be able to:

- state the **principle of moments** and the two conditions for equilibrium
- take moments about the point where an unknown force acts
- find support and hinge forces, with their direction
- solve three forces with a closed **vector triangle**

What the exam asks

“State the principle of moments.”

[2]

“By taking moments about the pivot, calculate the tension T .”

[3]

“State and explain whether the block is in equilibrium.” **[2]**

Practise on exercise sheet 4.2, then try the real exam questions on past-paper sheet 4.2.

Two conditions for equilibrium

A body is in **equilibrium** only when **both** hold:

$$\sum F = 0$$

the **resultant force** is zero:
no straight-line acceleration



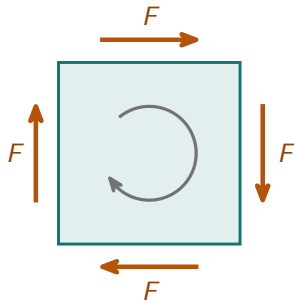
$$\sum M = 0$$

the resultant moment
about **any** point is zero:
no **angular acceleration**



Both are needed: a body with no resultant force can still be **turning**, and a body with no resultant moment can still be **moving** in a straight line.

Zero resultant force is not enough



Four equal forces: two pairs, opposite in direction but **not** along the same lines.

$\sum F = 0$, so the centre of mass moves at **constant velocity**

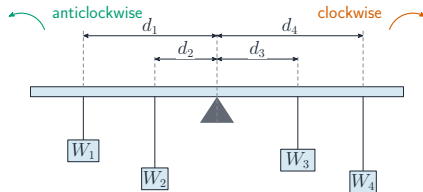
but the two couples give a **resultant torque**, so it spins faster and faster

So this body is **not** in equilibrium. It is in equilibrium only when both conditions hold.

The principle of moments

Principle of moments : learn it word for word

For a body in equilibrium, **the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about that same point.**



Left of the pivot: **anticlockwise**; right of it: **clockwise**. Balanced: the two totals are equal.

How to solve a balance problem

1 Choose a pivot

usually where an **unknown** force acts: its distance is zero, so it drops out

2 List the forces

every force, with its **perpendicular** distance from that pivot

3 Balance moments

$\sum M_{\text{clockwise}} = \sum M_{\text{anticlockwise}}$ about that pivot

4 Use $\sum F = 0$

when you need a second equation, often for the force at the pivot

A heavy uniform rod: include its **own weight**, acting at its centre. A ruler balanced on a pivot with masses on each side is solved exactly this way.

Worked example – balancing a beam

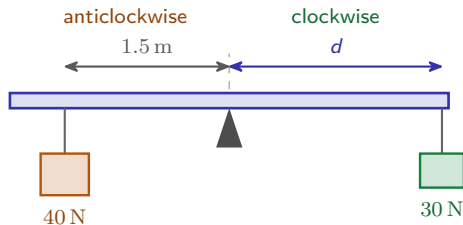
Where must the 30 N weight hang?

A uniform beam, 4.0 m long, is pivoted at its centre. A 40 N weight hangs 1.5 m from the pivot on one side.

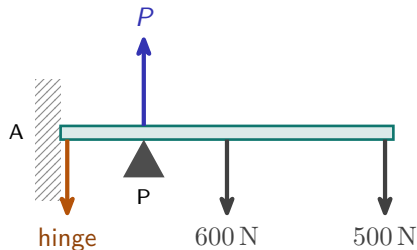
The beam's own weight acts **at the pivot**, so it has no moment. Clockwise = anticlockwise:

$$F_1 d_1 = F_2 d_2$$

$$d_2 = \frac{F_1 d_1}{F_2} = \frac{(40 \text{ N})(1.5 \text{ m})}{30 \text{ N}} = 2.0 \text{ m}$$



A hinge force can point down



Board 4.0 m, 600 N; **P** at 1.0 m; 500 N at the end

moments about A (the hinge force drops out):

$$Pd_P = W_b d_b + W_s d_s$$

$$P(1.0 \text{ m}) = (600 \text{ N})(2.0 \text{ m}) + (500 \text{ N})(4.0 \text{ m}), \text{ so}$$

$$P = 3200 \text{ N}$$

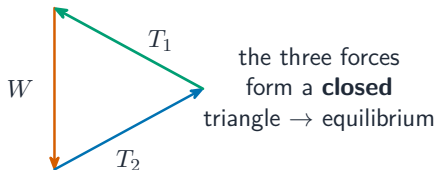
vertically: $P + F = W_b + W_s$

$$F = W_b + W_s - P = 600 \text{ N} + 500 \text{ N} - 3200 \text{ N} = -2100 \text{ N}$$

2100 N downwards

Support between the hinge and the load:
the hinge pulls **down**.

Three forces: the closed triangle



Three **coplanar** forces in equilibrium, drawn tip to tail, form a **closed triangle**. Or resolve:

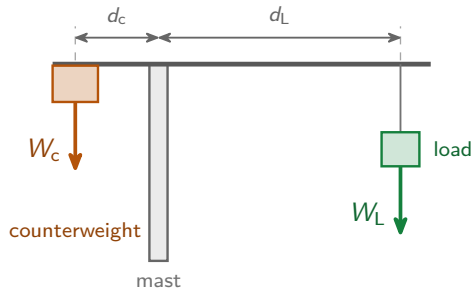
A 250 N sign: one wire horizontal, one at 40° to the vertical

vertical first (one unknown):

$$T \cos 40^\circ = 250, \quad T = 326 \text{ N}$$

$$\text{horizontal: } F = T \sin 40^\circ = 210 \text{ N}$$

Moments at work: a tower crane



The **counterweight's** moment about the mast balances the **load's**:
 $W_c d_c = W_L d_L$.



Sheet 4.3 – what you need to know

You must be able to:

- define **density** and **pressure**, with their units
- derive $\Delta p = \rho g \Delta h$ from the two definitions
- find the pressure at a depth, with and without the atmosphere
- find a density from measurements, with its uncertainty

What the exam asks

“Define pressure.” [1]

“Show that the pressure due to a liquid is $\rho g \Delta h$.” [3]

“Calculate the density of the air surrounding the balloon.” [2]

Practise on exercise sheet 4.3, then try the real exam questions on past-paper sheet 4.3.

Density and pressure

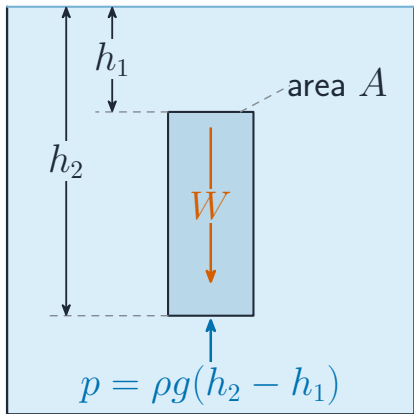
- **density** : mass per unit volume,
 $\rho = \frac{m}{V}$, in kg m^{-3}
- **pressure** : **normal** force per unit area,
 $p = \frac{F}{A}$, in $\text{Pa} = \text{N m}^{-2}$
- in base units: $\text{Pa} = \text{kg m}^{-1} \text{s}^{-2}$
- $1 \text{ g cm}^{-3} = 1000 \text{ kg m}^{-3}$;
 $1 \text{ cm}^2 = 10^{-4} \text{ m}^2$

A 12 kg crate on a $0.20 \text{ m} \times 0.15 \text{ m}$ base

force: $W = mg =$
 $(12 \text{ kg})(9.81 \text{ m s}^{-2}) = 118 \text{ N}$

$$p = \frac{F}{A} = \frac{118 \text{ N}}{(0.20 \text{ m})(0.15 \text{ m})} = 3.9 \times 10^3 \text{ Pa}$$

The same weight on a **smaller** area gives a **larger** pressure.

Deriving $\Delta p = \rho g \Delta h$ 

$$\text{volume } V = A \Delta h$$

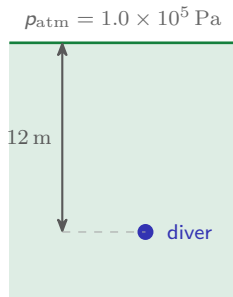
$$\text{mass } m = \rho V = \rho A \Delta h$$

$$\text{weight } W = mg = \rho A \Delta h g$$

$$\Delta p = \frac{W}{A} = \rho g \Delta h$$

The area cancels: the pressure depends only on the **depth** and the **density**.

Pressure at a depth



Sea water, $\rho = 1030 \text{ kg m}^{-3}$

due to the water: $\Delta p = \rho g \Delta h =$
 $(1030 \text{ kg m}^{-3})(9.81 \text{ m s}^{-2})(12 \text{ m}) = 1.21 \times 10^5 \text{ Pa}$

total: $p = p_{\text{atm}} + \Delta p =$
 $1.0 \times 10^5 \text{ Pa} + 1.21 \times 10^5 \text{ Pa} = 2.2 \times 10^5 \text{ Pa}$

from 12 m to 12.5 m: $\Delta p = \rho g \Delta h =$
 $(1030 \text{ kg m}^{-3})(9.81 \text{ m s}^{-2})(0.50 \text{ m}) = 5.1 \times 10^3 \text{ Pa}$

Read which pressure the question wants: the pressure **due to the liquid**, or the **total**.

A density with its uncertainty

$m = (0.243 \pm 0.001) \text{ kg}$; **sides** 5.41, 11.09, 1.62 cm, **each** $\pm 0.01 \text{ cm}$

$$V = xyz =$$

$$(5.41 \text{ cm})(11.09 \text{ cm})(1.62 \text{ cm}) =$$

$$97.2 \text{ cm}^3 = 9.72 \times 10^{-5} \text{ m}^3$$

$$\rho = \frac{m}{V} = \frac{0.243 \text{ kg}}{9.72 \times 10^{-5} \text{ m}^3} = 2500 \text{ kg m}^{-3}$$

Multiplied and divided, so **add the percentages**:

$$\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

$$\frac{\Delta \rho}{\rho} = \frac{0.001}{0.243} + \frac{0.01}{5.41} + \frac{0.01}{11.09} + \frac{0.01}{1.62}$$

$$\frac{\Delta \rho}{\rho} = 0.41\% + 0.18\% + 0.09\% + 0.62\% = 1.3\%$$

The **smallest** side has the largest percentage uncertainty: that is the measurement to improve.



Sheet 4.3.1 – what you need to know

You must be able to:

- explain **upthrust** from the pressure difference
- use $F = \rho g V$ with the density of the **fluid**
- balance the forces when floating, weighed in a liquid, or sinking
- use “same level, same pressure” in a U-tube

What the exam asks

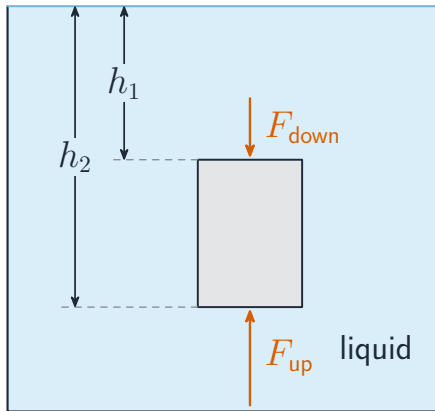
“Explain how hydrostatic pressure results in an upthrust force.” [2]

“Show that the upthrust acting on the ball is $2.8 \times 10^{-3} \text{ N}$.” [1]

“Calculate the density of the sphere.” [2]

Practise on exercise sheet 4.3.1, then try the real exam questions on past-paper sheet 4.3.1.

Where upthrust comes from



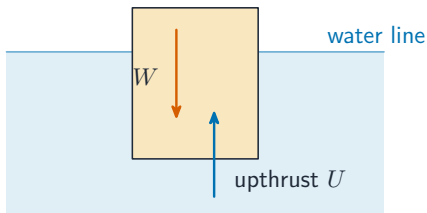
Pressure grows with depth, so the fluid pushes **up** on the bottom harder than **down** on the top:

$$F_2 - F_1 = (p_2 - p_1)A = \rho g \Delta h A = \rho g V$$

Archimedes' principle : **upthrust** = the weight of the fluid displaced, $F = \rho_{\text{fluid}} g V$. It does not depend on the depth, or on what the object is made of.

Floating

floats when $U = W$



A floating object sinks until the **upthrust equals its weight**:

$$\rho_{\text{fluid}} V_{\text{below}} g = \rho_{\text{object}} V g \Rightarrow \frac{V_{\text{below}}}{V} = \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}}$$

A foam raft, 40 kg m^{-3} , 0.60 m^3

weight: $W = \rho_f V g =$

$$(40 \text{ kg m}^{-3})(0.60 \text{ m}^3)(9.81 \text{ m s}^{-2}) = 235 \text{ N}$$

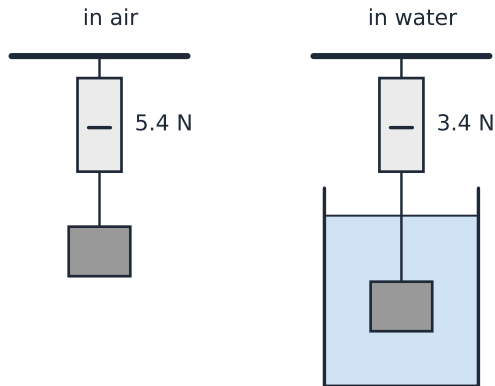
largest upthrust: $U = \rho_w V g =$

$$(1000 \text{ kg m}^{-3})(0.60 \text{ m}^3)(9.81 \text{ m s}^{-2}) = 5890 \text{ N}$$

largest load:

$$L = U - W = 5890 \text{ N} - 235 \text{ N} = \mathbf{5650 \text{ N}}$$

Weighing an object in a liquid



5.4 N in air, 3.4 N in water

equilibrium, $T + U = W$:

$$U = W - T = 5.4 \text{ N} - 3.4 \text{ N} = 2.0 \text{ N}$$

$$V = \frac{U}{\rho g} =$$

$$\frac{2.0 \text{ N}}{(1000 \text{ kg m}^{-3})(9.81 \text{ m s}^{-2})} =$$

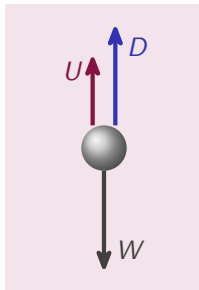
$$2.04 \times 10^{-4} \text{ m}^3$$

$$m = \frac{W}{g} = \frac{5.4 \text{ N}}{9.81 \text{ m s}^{-2}} = 0.550 \text{ kg}$$

$$\rho = \frac{m}{V} = \frac{0.550 \text{ kg}}{2.04 \times 10^{-4} \text{ m}^3} =$$

$$2700 \text{ kg m}^{-3}$$

Falling through a liquid



$$m = 1.6 \times 10^{-3} \text{ kg}, \quad V = 2.0 \times 10^{-7} \text{ m}^3, \quad \text{oil} \\ 900 \text{ kg m}^{-3}$$

terminal velocity, no resultant force: $W = U + D$, with $D = kv$

$$W = mg = (1.6 \times 10^{-3} \text{ kg})(9.81 \text{ m s}^{-2}) = 1.57 \times 10^{-2} \text{ N}$$

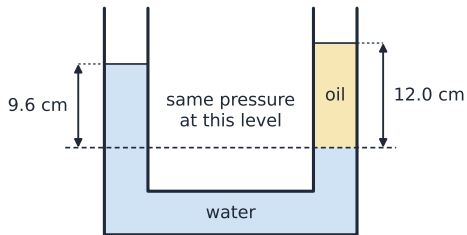
$$U = \rho g V = (900 \text{ kg m}^{-3})(9.81 \text{ m s}^{-2})(2.0 \times 10^{-7} \text{ m}^3) = 1.77 \times 10^{-3} \text{ N}$$

$$D = W - U = 1.57 \times 10^{-2} \text{ N} - 0.18 \times 10^{-2} \text{ N} = 1.39 \times 10^{-2} \text{ N}$$

$$v = \frac{D}{k} = \frac{1.39 \times 10^{-2} \text{ N}}{1.4 \times 10^{-2} \text{ N s m}^{-1}} = 0.99 \text{ m s}^{-1}$$

Write $W = U + D$ first: it is the method mark.

Two liquids in a U-tube



At the level of the boundary both arms are joined by water: **same level, same pressure.**

$$\rho_{\text{water}} g h_{\text{water}} = \rho_{\text{oil}} g h_{\text{oil}}$$

$$\rho_{\text{oil}} = \frac{\rho_{\text{water}} h_{\text{water}}}{h_{\text{oil}}} = \frac{(1000 \text{ kg m}^{-3})(9.6 \text{ cm})}{12.0 \text{ cm}} = 800 \text{ kg m}^{-3}$$

g and the air pressure cancel; the heights may stay in cm.



Definitions the examiner accepts

moment of a force

the force $\times$ the perpendicular distance from the point to the line of action of the force

couple

a pair of equal and opposite forces whose lines of action are parallel but separated

torque of a couple

one force $\times$ the perpendicular distance between the lines of action of the forces

centre of gravity

the point at which the whole weight of an object may be taken to act

principle of moments

in equilibrium, the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about that point

equilibrium

no resultant force and no resultant moment

density

mass per unit volume

pressure

normal force per unit area

upthrust

the upward force of a fluid on an object, equal to the weight of the fluid displaced

Common mistakes

The distance along the bar

A moment needs the **perpendicular** distance to the line of action, $d = L \sin \theta$.

Both forces of a couple

Torque = **one** force $\times$ the separation: the diameter, not the radius.

The beam's own weight

It acts at the centre of gravity and has a moment: include it.

The object's density in $\rho g V$

Upthrust uses the density of the **fluid** and the volume displaced.



Topic 4 – key takeaways

1

Moments

$M = Fd$, d perpendicular to the line of action; couple: $\tau = Fd$

2

Equilibrium

no resultant force and no resultant moment

3

Pressure

$p = F/A$; $\Delta p = \rho g \Delta h$, depth and density only

4

Upthrust

$F = \rho g V$ of the fluid; floating: upthrust = weight

Check yourself

1

The unit of the torque of a couple?

2

What is the resultant force of a couple?

3

Where do you take moments to remove an unknown force?

4

Does the upthrust on a fully submerged object change with depth?

Try all four first – the answers appear on the next slide.

Check yourself

1

The unit of the torque of a couple?

⇒ N m: a force times a distance, not a joule

2

What is the resultant force of a couple?

⇒ **zero**: it turns the object but does not accelerate it

3

Where do you take moments to remove an unknown force?

⇒ about the point where that force **acts**

4

Does the upthrust on a fully submerged object change with depth?

⇒ **no**: it is $\rho g V$, the weight of the fluid displaced