

Physical Quantities & Units

AS Physics ■ Topic 1

A-Level Physics

the length of an object



1.5 m

What we will cover

1

Physical quantities

a number and a unit; estimates

1.5 m



2

SI units

base and derived units, prefixes

kg

m

s

A

3

Errors & uncertainties

precision, accuracy, uncertainty



4

Scalars and vectors

adding and resolving vectors



5

Putting it together

definitions, exam tips, mistakes





A number *and* a unit

A **physical quantity** has two parts:

- a **magnitude** – the number
- a **unit** – what the number means

The number alone tells you **nothing**.

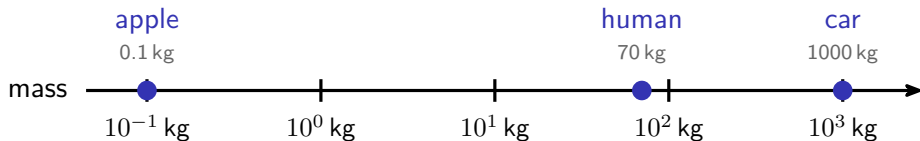
1.5	m
	
magnitude	unit

"1.5" ❌

"1.5 m" ✅

Estimate the everyday

A good estimate has the right **order of magnitude** – the right power of ten. Paper 1 usually opens with one.



Each step along the line is a **factor of ten**. Pick the answer on the right step: a human is about 10^2 kg, not 10^1 or 10^3 .

Estimates to know by heart

human mass

70 kg

human weight

700 N

human height

1.7 m

an apple

0.1 kg

speed of sound in air

340 m s^{-1}

speed of light
in a **vacuum**

$3.0 \times 10^8 \text{ m s}^{-1}$

free fall

$g \approx 9.81 \text{ m s}^{-2}$

An apple weighs about 1 N. That one number lets you check every other force estimate.

More values to know

Sizes

- a car: $\sim 1000 \text{ kg}$
- height of a room: $\sim 3 \text{ m}$
- visible light wavelength:
 $\sim 5 \times 10^{-7} \text{ m}$
- **diameter** of an atom:
 $\sim 10^{-10} \text{ m}$; a nucleus:
 $\sim 10^{-15} \text{ m}$

Speeds and power

- walking: $\sim 1.5 \text{ m s}^{-1}$
- a car on a fast road:
 $\sim 30 \text{ m s}^{-1}$
- a kettle: $\sim 2 \text{ kW}$
- a person climbing
stairs: $\sim 300 \text{ W}$

Materials and the air

- **density** : water
 1000 kg m^{-3} , air
 $\sim 1.2 \text{ kg m}^{-3}$, steel
 $\sim 8000 \text{ kg m}^{-3}$
- **atmospheric pressure** :
 $\sim 1.0 \times 10^5 \text{ Pa}$
- room temperature:
 $\sim 20^\circ\text{C} \approx 293 \text{ K}$

The right power of ten matters most: for a human, 70 kg is a good estimate; 7 kg is not.

Building an estimate

Not on the list? **Build it** from values that are.

Kinetic energy of a car

$m \approx 1000 \text{ kg}$, $v \approx 30 \text{ m s}^{-1}$:

$$\frac{1}{2}mv^2 \approx \frac{1}{2} \times 1000 \times 30^2 \approx 5 \times 10^5 \text{ J}$$

Pressure under a standing person

weight $\approx 700 \text{ N}$ on about 0.02 m^2 :

$$p \approx \frac{700}{0.02} \approx 4 \times 10^4 \text{ Pa}$$

Always check that the **power of ten** looks sensible before you move on.



SI base quantities

The SI system has **seven base quantities** ; these **five** are used here:

kg
mass

m
length

s
time

A
current

K
temperature

Combine them with $\times$ and $\div$ to get **every other unit** – N, J, W, Pa, ...

Build any derived unit

Each **derived unit** is just base units, from its defining equation:

N

newton
force

 kg m s^{-2} **J**

joule
energy

 $\text{kg m}^2 \text{s}^{-2}$ **W**

watt
power

 $\text{kg m}^2 \text{s}^{-3}$ **Pa**

pascal
pressure

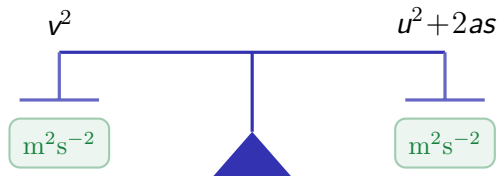
 $\text{kg m}^{-1} \text{s}^{-2}$

Read the unit straight off the defining equation: **speed** = m s^{-1} , **acceleration** = m s^{-2} , and **stress** is force per area, so it takes the pascal too.

Do the units balance?

Homogeneous = both sides have the **same base units**. Test $v^2 = u^2 + 2as$:

- LHS = m^2s^{-2}
- $u^2 = \text{m}^2\text{s}^{-2}$; $2as = \text{m}^2\text{s}^{-2}$



Matching units don't **prove** an equation right – but **mismatched** units prove it wrong.

Worked example – the units of a constant

Drag force $F = kv^2$: base units of k ?

Rearrange: $k = F/v^2$. In base units,

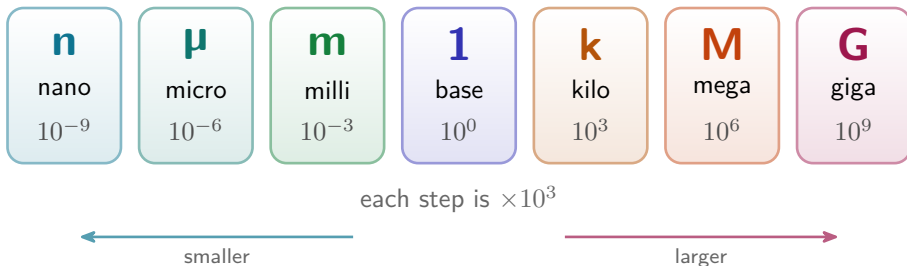
$$k: \frac{\text{kg m s}^{-2}}{\text{m}^2 \text{s}^{-2}} = \text{kg m}^{-1}$$

“Which equation could be correct?”

- write each option in **base units**
- only a **homogeneous** equation *can* be correct
- a pure number (2 , π , $\frac{1}{2}$) has **no unit**, so it never changes the check

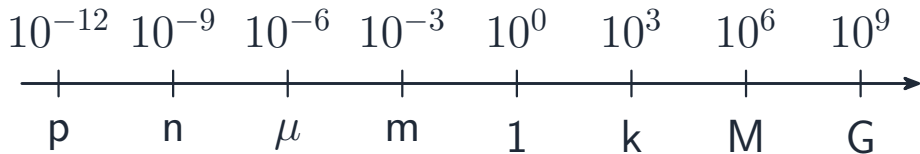
Prefixes – powers of ten

A **prefix** scales a unit by a power of ten.



Also learn: **T** tera 10^{12} **d** deci 10^{-1} **c** centi 10^{-2} **p** pico 10^{-12}

Prefixes on the page



each step is $\times 10^3$

Each step is a factor of 1000, so converting is a shift of three places – never a new calculation.

Square the prefix too: $1 \text{ mm}^2 = (10^{-3} \text{ m})^2 = 10^{-6} \text{ m}^2$, not 10^{-3} .

Worked example – converting prefixes

$$0.25 \text{ kN mm}^{-2} \rightarrow \text{N m}^{-2}$$

Replace each prefix – and **square** the mm:

$$0.25 \frac{10^3}{(10^{-3})^2} = 2.5 \times 10^8 \text{ N m}^{-2}.$$

$$0.25 \text{ kN mm}^{-2}$$



replace prefixes
 $\times 10^3 / 10^{-6}$



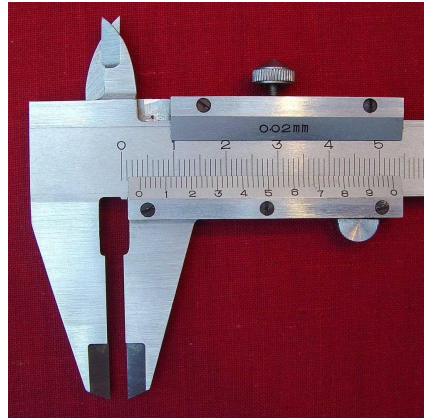
$$2.5 \times 10^8 \text{ N m}^{-2}$$



Every measurement is uncertain

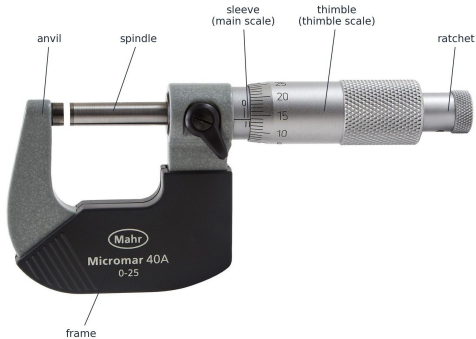
Every measurement carries some **uncertainty** . A good experimenter:

- knows where it comes from
- estimates it fairly
- carries it to the answer

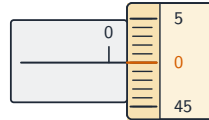


Calipers measure with a small uncertainty

The micrometer – to 0.01 mm

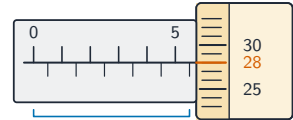


(a) zero check



reads 0.00 mm

(b) a measurement

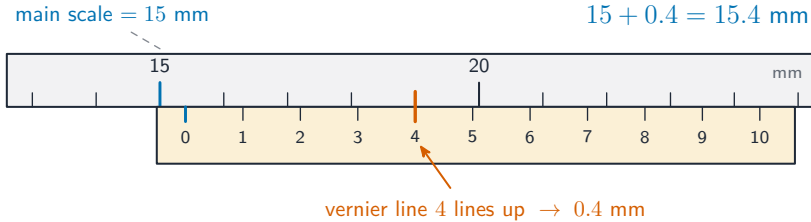


5.5 mm

$5.5 + 0.28 = 5.78 \text{ mm}$

A **micrometer** reads to 0.01 mm: reading = **sleeve** (main scale) + **thimble** scale.

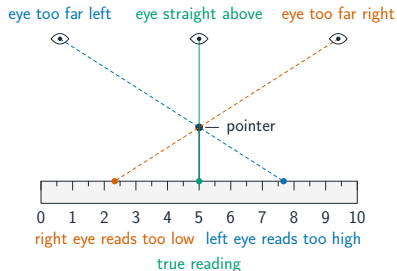
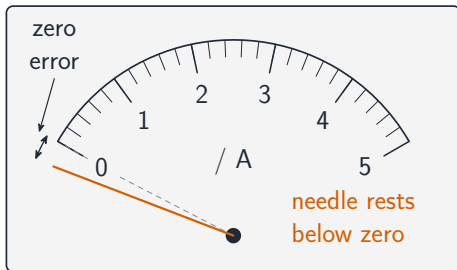
The vernier caliper – to 0.1 mm



Whole mm from the **main scale**, then tenths from the **vernier** mark that lines up.

Systematic errors

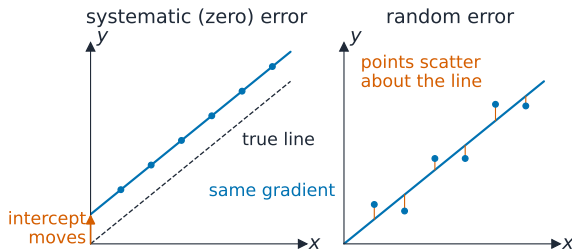
A **systematic error** shifts **every** reading by the same amount, in the same direction. Causes: a **zero error**, a **calibration** error, **parallax**.



It spoils **accuracy**, not precision, and repeating cannot reveal it. **Reduce it:** subtract the zero error; calibrate against a known standard; read the scale from directly in front.

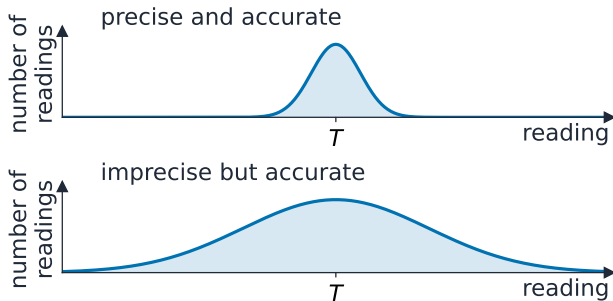
Random errors

A **random error** scatters readings **above and below** the true value at random. On a graph it scatters the points; a systematic error moves the **intercept**.



It spoils the **precision**. **Causes**: how you read the scale, changing conditions, the smallest step the **instrument** shows. **Reduce it**: take the **mean** of many repeats; use smaller divisions; time many oscillations, not one.

Precision : how narrow is the spread?

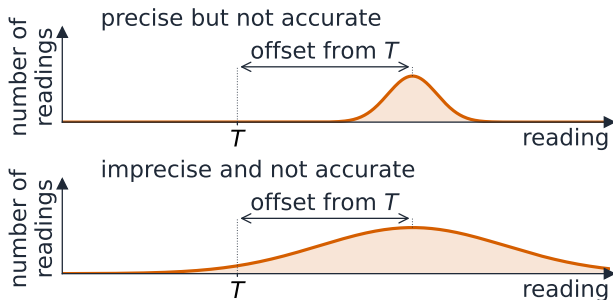


precision: how closely repeated readings agree
(random error / spread)

precision : how close **repeated readings** are to each other.

- precise readings are **grouped** close together
- the narrower the curve, the more precise
- **random** errors spoil it

Accuracy : is the peak on the true value?



accuracy: how close readings are to the true value
(systematic error reduces accuracy)

accuracy : how close a reading – or the **mean** of several – is to the **true value**.

- a peak away from T is **not accurate**
- **systematic** errors spoil it
- with enough repeats, the mean can still be accurate despite random errors

Precise, accurate, both or neither

precise and accurate

close together, near the true value

precise, not accurate

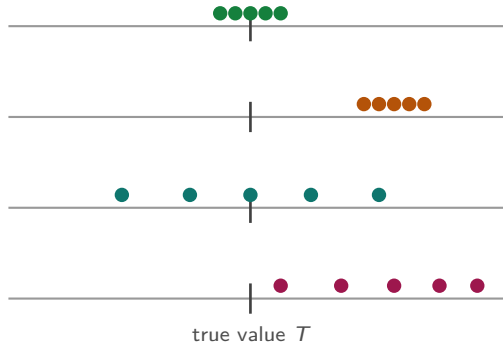
close together, away from it: systematic error

accurate, not precise

spread out, but the mean is near it

neither

spread out, and away from the true value



In a table of repeated readings, judge the **spread** (precision) and the **mean** (accuracy) separately.

The uncertainty in one reading

analogue: $\pm$ half a division

A metre rule reads to 1 mm, so one reading is ± 0.5 mm. A length needs **two** readings, so ± 1 mm.

digital: ± 1 in the last digit

A stopwatch showing 2.47 s is ± 0.01 s.

timing by hand

Your **reaction time** matters more than the display: allow ± 0.1 to 0.2 s, and time **many** oscillations.

repeats: $\pm$ half the range

2.42, 2.48, 2.45 s: mean 2.45 s, range 0.06 s, so $t = (2.45 \pm 0.03)$ s.

Be honest: a shadow edge that is hard to see is uncertain by several mm, however fine the scale. “The smallest **division** ” is not accepted for a difficult reading.

Combining uncertainties

A **derived quantity** is one you *calculate* from measured values. Each is $x \pm \Delta x$; its **percentage uncertainty** $= \frac{\Delta x}{|x|} \times 100\%$.

- **add / subtract** $\Rightarrow$ add **absolute uncertainty**
- **multiply / divide** $\Rightarrow$ add the %
- **power** $a^n \Rightarrow$ multiply % by $|n|$

$$y = \frac{ab}{c}$$



$$\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$$

Adding or subtracting – add the *absolute* uncertainties

$$\Delta y = \Delta a + \Delta b$$

Both $y = a + b$ and $y = a - b$ add them.

A *difference* keeps the whole Δ but has a far smaller value, so its % is the worst of the three.

$$a = (50.0 \pm 0.5) \text{ mm} \quad \text{┃───●───┃} \quad 1.0\%$$

$$b = (20.0 \pm 0.2) \text{ mm} \quad \text{┃───●───┃} \quad 1.0\%$$

$$a - b = (30.0 \pm 0.7) \text{ mm} \quad \text{┃───●───┃} \quad 2.3\%$$

one scale for all three – the *difference* is the widest

Multiplying or dividing – add the *percentage* uncertainties

$$\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$$

Density $\rho = \frac{m}{V}$ from

$m = (250 \pm 1) \text{ g}$ and

$V = (100 \pm 2) \text{ cm}^3$.

The **bigger** % wins: improve V , not m .

$$\frac{\Delta m}{m} = \frac{1}{250} \quad \text{0.4\%}$$

$$\frac{\Delta V}{V} = \frac{2}{100} \quad \text{2.0\%}$$

$$\frac{\Delta \rho}{\rho} \quad \text{2.4\%}$$

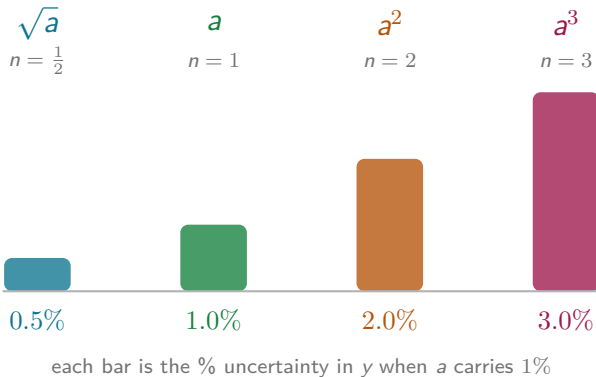
$$\rho = (2.50 \pm 0.06) \text{ g cm}^{-3}$$

Powers – multiply the percentage by the index

$$\frac{\Delta y}{|y|} = |n| \frac{\Delta a}{|a|}$$

A 1% error in a length becomes
3% in a volume – three edges
each carry it.

Next slide: d carries 0.38%,
so $V \propto d^3$ carries 1.14%.



Worked example – uncertainty in a volume



$$d = (5.26 \pm 0.02) \text{ cm}$$

$$V = \frac{\pi}{6} d^3 = 76.2 \text{ cm}^3$$

1**% uncertainty in d**

$$\frac{0.02}{5.26} \times 100\% \\ = 0.38\%$$

2 **$V \propto d^3$: triple it**

$$3 \times 0.38\% \\ = 1.14\%$$

3**back to absolute**

$$1.14\% \times 76.2 \\ \Delta V = 0.9 \text{ cm}^3$$

$$V = (76.2 \pm 0.9) \text{ cm}^3$$

Do two values agree?

Do not just say “they are close”. Work out the **percentage difference** :

$$\frac{|k_1 - k_2|}{\text{mean of } k_1 \text{ and } k_2} \times 100\%$$

Smaller than the percentage uncertainty in k (or the criterion given, often 10%): the values **agree** and the relationship is supported. **Larger**: they do not.

Test against a 10% criterion

$k_1 = 2.46$, $k_2 = 2.61$: difference 0.15, mean 2.535.

$$\frac{0.15}{2.535} \times 100\% = 5.9\%$$

$5.9\% < 10\%$, so the values **agree**.

Significant figures

Give the answer the **same number of significant figures** as the least precise measurement you used – usually 2 or 3.

- too many looks too exact; too few loses information
- **justify** it by naming the raw readings: “*a* has 2 s.f. because *L* and *t* were measured to 2 s.f.”
- “because of the raw data” alone earns **no mark**; in a table, judge each row from its own least precise reading

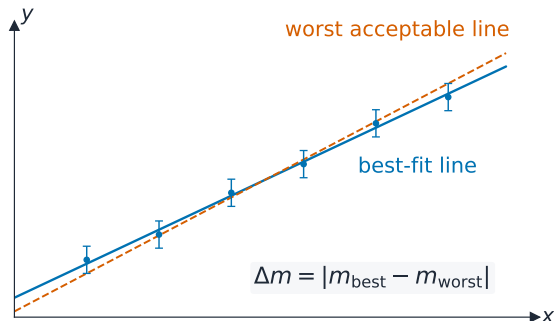
76.2 34...

drop extra

76.2 cm³ (3 s.f.)

Uncertainties on a graph

- an **error bar** on each point: its length is the absolute uncertainty
- draw the **line of best fit**, then the **worst acceptable line**: the steepest (or shallowest) line through **every** bar
- $\Delta m = |m_{\text{best}} - m_{\text{worst}}|$, the intercept the same way; its % flows into anything calculated from m



Plotting a logarithm? The uncertainty in $\ln x$ is $\Delta x/x$ ($0.434 \Delta x/x$ in $\lg x$): work it out for the largest and smallest x .



Size only, or size *and* direction?

Scalar

5 kg

magnitude only

mass · time · temperature · energy
work · power · speed · distance
pressure · density
electric charge

Vector



magnitude + direction

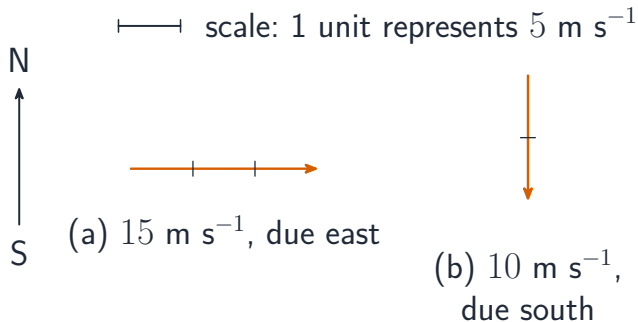
displacement · velocity
acceleration · momentum
force (including weight)

Quick test – can you ask “in which direction?” If yes, it is a **vector**.

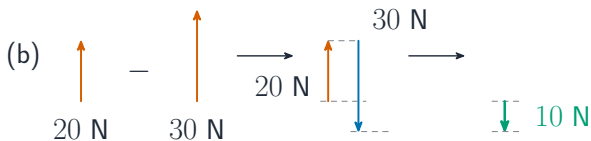
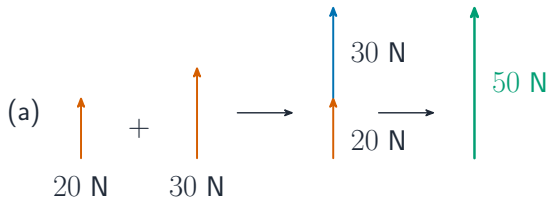
Drawing vectors to scale

A vector is an arrow:

- **direction** of arrow = direction
- **length** (to scale) = magnitude

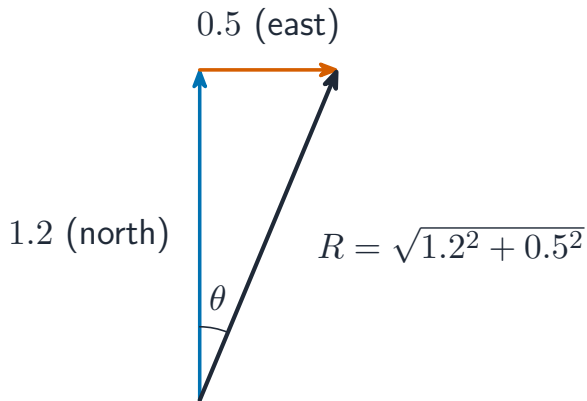


Adding tip to tail



- add **coplanar** vectors **tip to tail**
- the **resultant** runs first tail → last tip
- to subtract $\vec{X} - \vec{Y}$, add the reverse of $\vec{Y}$

Vectors at right angles



For **perpendicular** vectors:

$$|\vec{R}| = \sqrt{X^2 + Y^2}, \quad \tan \theta = \frac{Y}{X}.$$

Swimmer

1.2 m s⁻¹ north, current

0.5 m s⁻¹ east:

$|\vec{R}| = 1.3 \text{ m s}^{-1}$, $\theta \approx 23^\circ$ east of north.

Two equal forces at an angle

Same size F , with 2α between them:

$$R = 2F \cos \alpha$$

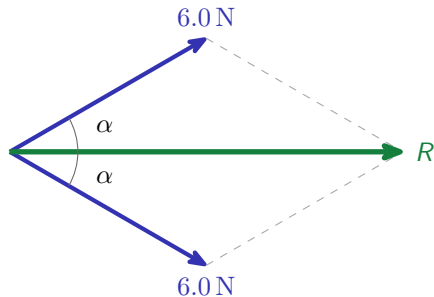
along the line that **cuts the angle in half**.

Two 6.0 N forces, 60° apart

$\alpha = 30^\circ$, so

$$R = 2 \times 6.0 \times \cos 30^\circ = 10.4 \text{ N}$$

Any other angle: a **scale drawing** of the vector triangle, or resolve both into components and recombine with $\sqrt{X^2 + Y^2}$.



Worked example – a change in velocity

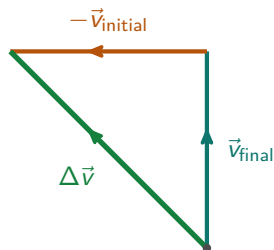
East at 5.0 m s^{-1} , then north at 5.0 m s^{-1}

A change in velocity is a **vector subtraction**:

$$\Delta \vec{v} = \vec{v}_{\text{final}} - \vec{v}_{\text{initial}}$$

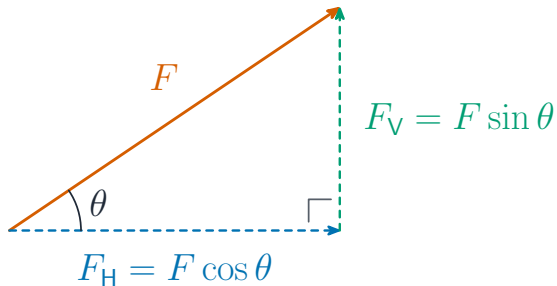
Draw 5.0 north, then add the **reverse** of the initial velocity, 5.0 west:

$$|\Delta \vec{v}| = \sqrt{5.0^2 + 5.0^2} = 7.1 \text{ m s}^{-1} \text{ north-west}$$



The **speed** did not change, but the **velocity** did – so a resultant force must have acted (topic 3).

Resolving into components

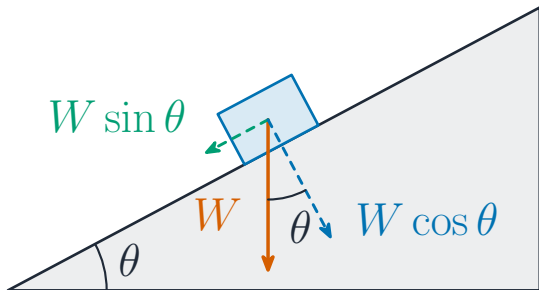


Split into two **perpendicular components** – usually **horizontal** and **vertical**, or along and across a surface:

$$F_H = F \cos \theta, \quad F_V = F \sin \theta.$$

Resolve when you need how much of a vector acts in **one direction**: a force along a slope, or a ball's horizontal and vertical velocity.

Weight on a slope



On an **inclined plane**, split the weight along the slope and at right angles to it:

$$W_{\parallel} = W \sin \theta, \quad W_{\perp} = W \cos \theta$$

where θ is the **angle of the slope** to the horizontal.

The angle between W and the perpendicular to the slope is **the slope angle** θ too.



Definitions the examiner accepts

A definition is marked against **fixed wording**: learn these exactly, and give one answer only.

physical quantity

a numerical magnitude together with a unit

homogeneous

an equation in which every term has the same

equation

base units

scalar

a quantity with magnitude only

vector

a quantity with magnitude and direction

precision

how close repeated readings are to each other

accuracy

how close a reading, or the mean of several readings, is to the true value

Definitions: errors and uncertainties

Five more, marked the same way:

systematic error

an error that shifts every reading in the same direction by the same amount; not reduced by repeating

random error

an error that scatters readings above and below the true value; reduced by repeating and averaging

zero error

the reading an instrument shows when the true value is zero

absolute uncertainty

the range within which the true value is expected to lie, in the units of the quantity

percentage uncertainty

the absolute uncertainty divided by the value, multiplied by 100%

Exam tips

- Give every answer a **unit**, and check **homogeneity** – both sides of an equation must have the same base units.
- Distinguish **random error** (reduce by repeating and averaging) from **systematic error** (a zero or calibration error that repeats do not remove).
- Combine uncertainties: **add absolute** uncertainties when adding/subtracting, **add percentage** uncertainties when multiplying/dividing (and multiply the % by any power).
- Distinguish **precision** (small spread) from **accuracy** (close to the true value).
- Resolve a vector into perpendicular components ($F \cos \theta$, $F \sin \theta$); add vectors tip-to-tail or by components.

Common mistakes

✗ The mistake

Substituting 5 cm as 5, or 20 g as 20.

The same % uncertainty for T^2 as for T .

Treating the % uncertainty and the absolute uncertainty as the same number.

“The smallest division” as the uncertainty of a hard reading.

Two answers to a definition.

✓ What is right

Convert the prefix first: 0.05 m, 0.020 kg.

Squaring doubles the % uncertainty; a square root halves it.

Absolute is in the **units** of the quantity; percentage is $\Delta x/x \times 100\%$.

Estimate **honestly**: a difficult edge can be uncertain by several mm.

Commit to one.



Topic 1 – key takeaways

1

Magnitude + unit

every quantity is a number *and* a unit

2

Derived units

built from 5 base units; check **homogeneity**

3

Precision vs accuracy

spread of repeats vs the true value

4

Vectors

add tip-to-tail; resolve into components

Check yourself

1

SI base units of the **pascal**?

2

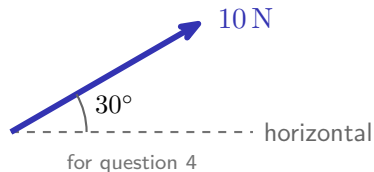
Readings close together, far from the true value: precise or accurate?

3

$y = ab^2$: how do the % uncertainties combine?

4

10 N at 30° : horizontal component?



Try all four first – the answers appear on the next slide.

Check yourself

1

SI base units of the **pascal**?

$$\Rightarrow \text{kg m}^{-1} \text{s}^{-2}$$

2

Readings close together, far from the true value: precise or accurate?

$\Rightarrow$ **precise**, not accurate

3

$y = ab^2$: how do the % uncertainties combine?

$$\Rightarrow \frac{\Delta y}{y} = \frac{\Delta a}{a} + 2 \frac{\Delta b}{b}$$

4

10 N at 30° : horizontal component?

$$\Rightarrow 10 \cos 30^\circ = \mathbf{8.7 \text{ N}}$$

