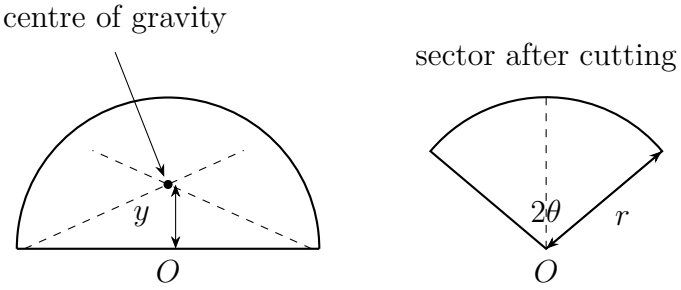
Centre of gravity of circular sectors

Name: _____ Class: _____ Date: _____

Aim Locate the centre of gravity of card sectors and use a graph to estimate their radius. Allow 60–75 minutes.

Key ideas A freely suspended object settles with its **centre of gravity** vertically below the support. Two plumb lines locate it. For a uniform circular sector of total angle 2θ, use the supplied model $y = (120r/\pi)(\sin\theta/\theta)$, with θ entered in degrees. No derivation is required.

Apparatus Uniform semicircular card of radius about 145 mm, scissors, millimetre ruler, protractor, clamped pin and plumb line with a small mass. Keep spare card available.



Left: each dashed line is the vertical through a different suspension hole. Right: cut symmetrically about the same centre line; retain O and the original radius.

Safety Keep the pin clamped with its sharp end away from faces. Handle scissors carefully and keep the plumb mass clear of fingers and the bench edge.

1 Plan before cutting

1. Measure r. Mark O and the symmetry axis. Plan six total sector angles, for example $180°, 160°, 140°, 120°, 100°, 80°$.
2. Suspend the semicircle from a small hole near one corner. Hang the plumb line from the same support; let both settle and mark the vertical on the card.
3. Repeat from a second, separated hole. Their intersection locates the centre of gravity. Measure its distance y from O along the symmetry axis.
4. Cut down to the next smaller angle, symmetrically. Repeat the suspension measurements and label the new lines by angle so you cannot confuse them.
5. Continue through your planned angles. Work largest to smallest: a removed piece cannot be restored. Use a third suspension line at each angle to judge how tightly the lines meet.

Processing check (synthetic data) A fitted gradient is 5.54 ± 0.20 m degree. Calculate r_{graph} and Δr_{graph}. Is this consistent with $r = 0.145 \pm 0.002$ m? Compare intervals.

2 Measurements and processing

Measured $r =$ _____ m Estimated $\Delta r =$ _____ m.

Set the calculator to degrees. θ is *half* the measured sector angle. Divide the numerical value of $\sin\theta$ by the numerical degree value of θ. This degree-based horizontal variable has units degree^{-1}.

2θ/degree	θ/degree	$\sin\theta$	X/degree^{-1}	y/m

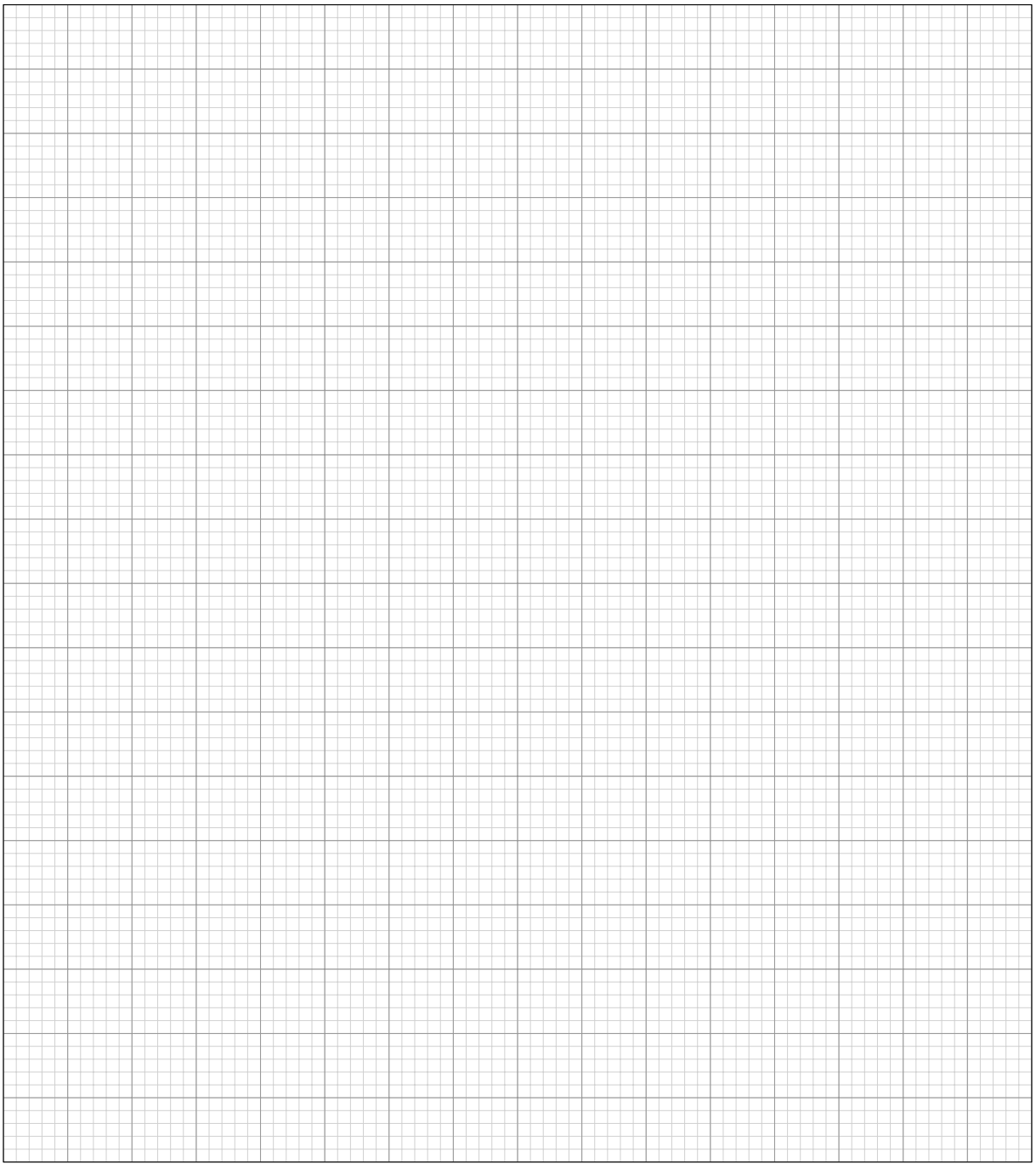
Check one processed value For $2\theta = 180°$, $\theta = 90°$ and $X = 1/90 = 0.01111$ degree^{-1}. Show the processing for another row.

Range Explain why your angle range produces a useful spread in X, even though no point will be near $X = 0$.

Observations Was the centre of gravity on the symmetry axis? How well did the third plumb line agree? Estimate Δy from the intersection region and $\Delta\theta$ from angle marking.

3 Graph

Plot distance y against $X = \sin\theta/\theta$ calculated in degrees. Draw a best-fit line; do not force it through an off-page origin. Extension beyond Paper 3: add justified uncertainty bars in both variables and a defensible extreme line; use the gradient difference as an estimate of Δs. If no straight line fits the bars, discuss the model or underestimated errors.



Choose scales and label both axes with quantities and units. Mark a large gradient triangle on the best-fit line where appropriate.

4 Test the sector model

$$y = sX, \qquad s = \frac{120r}{\pi}, \qquad r_{\text{graph}} = \frac{\pi s}{120}.$$

Here the numerical factor 120 includes the degree-to-radian conversion. Do not use the radian formula with degree data.

1. Use a large triangle on the fitted line to determine s. State its degree-based unit. Calculate r_{graph} in metres.

2. Compare r_{graph} with your directly measured r. Calculate the percentage difference. Judge consistency using Δr and a justified range of graph gradients.

3. What intercept would the ideal model predict? If the origin is off the grid, calculate $b = y - sX$ using a fitted-line point. Discuss what a non-zero b might indicate.

4. Identify two specific limitations, such as thick plumb lines or non-uniform card. Match each to an improvement.

5. Why must you not cut through the centre mark or reuse a line from a different sector angle?

Conclusion State your radius estimate and whether the linear trend supports the supplied model.

Before you leave Return the apparatus, clear the bench and submit your recorded data.