5.1.1 Efficiency, power and P = Fv

Name: _____ Class: _____ Date: _____ **Total: 28 marks**

KEY WORDS efficiency • power • work done • resistive force • useful energy

Objective

Answer exam questions on **efficiency** and **power**: the efficiency of a machine, power as the rate of doing work, and the derivation and use of $P = Fv$ for vehicles on the level and on slopes. Syllabus 5.1, learning outcomes 3 to 7.

You must be able to:

- calculate an efficiency from energies or from powers, including over several stages
- define power, and use $P = W/t$
- derive $P = Fv$ from the definitions of power and work
- use $P = Fv$ for a vehicle at a steady speed, on the level or up a slope, and when the drag grows with speed

1 Key knowledge and worked examples

Key knowledge: efficiency and power

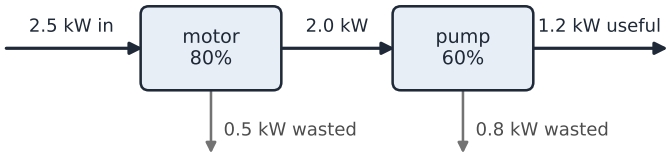
Quantity	Definition	Equation	Unit
efficiency	useful energy output ÷ total energy input	$\frac{\text{useful output}}{\text{total input}}$, as energies or as powers	none; never above 1
power	work done per unit time (rate of transfer of energy)	$P = \frac{W}{t}$	W = J s^{-1}
power of a force	the force × the velocity along it	$P = Fv$	W

Deriving $P = Fv$ (2 marks): $P = \frac{W}{t}$ and $W = Fs$, so $P = \frac{Fs}{t}$; and $\frac{s}{t} = v$, so $P = Fv$.

$P = Fv$

Situation	Force to use in $P = Fv$
steady speed on a level road	driving force = total resistive force
steady speed up a slope at θ	driving force = resistive force $+mg\sin\theta$
lifting at a steady speed	force = weight mg, so $P = mgv$
drag $\propto v^2$	doubling the speed makes the drag 4× and the power 8×

- efficiency over stages: multiply them, $\eta = \eta_1\eta_2$, e.g. $\eta = 0.80 \times 0.90 = 0.72$



- wasted energy = input − useful output, usually thermal energy and sound

Worked example: the efficiency of a pump

A pump raises 600 kg of water through 12 m in 30 s. It draws 3.0 kW.

1. **Useful power** – power is work done per unit time, the work being the gain in potential energy: $P = \frac{mg\Delta h}{t} = \frac{600 \times 9.81 \times 12}{30} = 2350$ W.
2. **Efficiency** – efficiency $= \frac{\text{useful output}}{\text{total input}} = \frac{2350\text{ W}}{3000\text{ W}} = 0.78$.

■ **Worked example: a car at a steady speed**

A car moves at a steady 35 $\frac{\text{m}}{\text{s}}$ along a level road against a total resistive force of 1750 N.

1. **Driving force** – steady speed, so no resultant force: $F = 1750$ N.
2. **Power** – $P = Fv = 1750 \times 35 = 6.1 \times 10^4$ W.

■ **Worked example: up a slope**

A car of mass 1200 kg climbs a slope at 5.0° to the horizontal at a steady 20 $\frac{\text{m}}{\text{s}}$. The resistive force is 600 N.

1. **Driving force** – along the slope it balances the resistive force and the component of the weight: $F = 600 + mg\sin\theta = 600 + 1200 \times 9.81 \times \sin 5.0° = 600 + 1030 = 1630$ N.
2. **Power** – $P = Fv = 1630 \times 20 = 3.3 \times 10^4$ W.

■ **Worked example: drag that grows with speed**

The resistive force on a car is proportional to the square of its speed: 800 N at 20 $\frac{\text{m}}{\text{s}}$.

1. **Force at** 40 $\frac{\text{m}}{\text{s}}$ – $F \propto v^2$ and the speed doubles: $F = 800 \times \left(\frac{40}{20}\right)^2 = 3200$ N.
2. **Power at a steady** 40 $\frac{\text{m}}{\text{s}}$ – $P = Fv = 3200 \times 40 = 1.28 \times 10^5$ W, eight times the 1.6×10^4 W needed at 20 $\frac{\text{m}}{\text{s}}$.

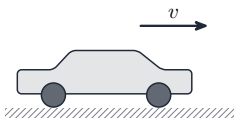
2 Practice

Now apply the methods above.

2.1 Define **power**. [1]

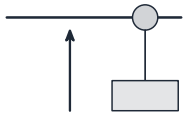
power

2.2 Show that the power of a force F moving an object at a constant velocity v in the direction of the force is $P = Fv$. [2]



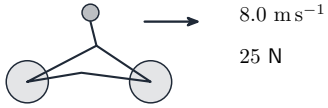
2.3 A motor lifts a load of mass 50 kg through 8.0 m in 10 s.

(a) Calculate the useful power output of the motor. [2]



(b) The input power is 500 W. Calculate the efficiency. [1]

2.4 A cyclist rides at a steady 8.0 $\frac{\text{m}}{\text{s}}$ along a level road against a total resistive force of 25 N. Calculate the useful power output of the cyclist. [2]



2.5 An electric motor is 80% efficient and drives a gearbox that is 90% efficient. The motor takes 5.0 kW. Calculate the useful power from the gearbox. [2]

80%

90%

3 Exam-style questions

3.1 What is the watt expressed in SI base units? [1]

watt

☐ **A** kg m^2 s^{-2}
B kg m^2 s^{-3}
C kg m s^{-3}
D kg m^2 s^{-1}

3.2 The resistive force on a boat is proportional to the square of its speed. At 15 $\frac{\text{m}}{\text{s}}$ the resistive force is 600 N. What power is needed to keep the boat moving at a steady 30 $\frac{\text{m}}{\text{s}}$? [1]

☐ **A** 18 kW
B 36 kW
C 72 kW
D 144 kW

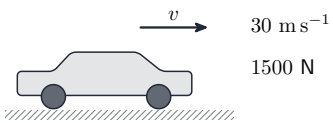
15 m s^{-1}

600 N

3.3 An electric car travels at a constant speed of 30 $\frac{\text{m}}{\text{s}}$ along a straight level road. The total resistive force on the car is 1500 N.

(a) **Define** power. [1]

(b) Calculate the power delivered to the wheels. [2]

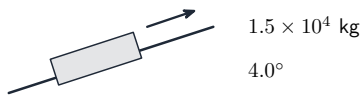


(c) Calculate the useful work done by the motor as the car travels 12 km. [2]

(d) The motor is 85% efficient. Calculate the electrical power input to the motor. [2]

3.4 A lorry of mass 1.5×10^4 kg climbs a straight road inclined at 4.0° to the horizontal, at a steady speed of 12 $\frac{\text{m}}{\text{s}}$. The total resistive force on the lorry is 3.0×10^3 N.

(a) **Show that** the driving force on the lorry is about 1.3×10^4 N. [2]



(b) Calculate the useful output power of the engine. [2]

(c) The engine's efficiency is 35%. Calculate the rate at which the engine uses energy from its fuel. [2]

(d) At the top of the hill the road becomes level. The engine keeps the same output power and the resistive force stays the same. **Explain** why the lorry speeds up, and calculate its greatest possible speed on the level. [3]