4.3 Density, pressure and hydrostatic pressure

Name: _____ Class: _____ Date: _____ **Total: 36 marks**

KEY WORDS density • pressure • hydrostatic pressure • atmospheric pressure • percentage uncertainty

Objective

Answer exam questions on **density** and **pressure**, including the **hydrostatic pressure** equation $\Delta p = \rho g \Delta h$ and its derivation. Syllabus 4.3, learning outcomes 1 to 4.

You must be able to:

- define density and pressure, with their units
- calculate the pressure under a solid object, and a density from measurements with its percentage uncertainty
- derive $\Delta p = \rho g \Delta h$ from the definitions of pressure and density
- use it for the pressure at a depth, the total pressure with the atmosphere, and the difference between two depths

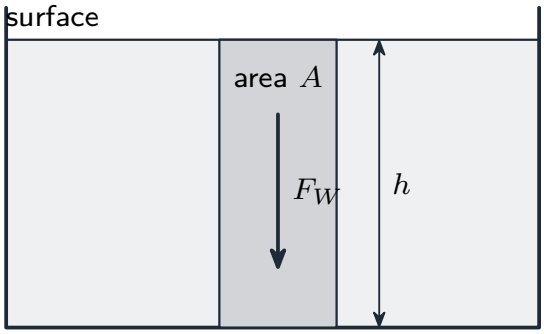
1 Key knowledge and worked examples

Key knowledge: the definitions

Quantity	Definition	Equation	Unit
density	mass per unit volume	$\rho = \frac{m}{V}$	kg m^{-3}
pressure	normal force per unit area	$p = \frac{F}{A}$	Pa = N m^{-2}
hydrostatic pressure	the pressure due to a depth Δh of a fluid	$\Delta p = \rho g \Delta h$	Pa

- conversions: 1 g cm^{-3} = 1000 kg m^{-3} (water); 1 cm^2 = 10^{-4} m^2; 1 cm^3 = 10^{-6} m^3
- under a solid: $p = \dfrac{mg}{A}$; the same weight on a smaller area gives a larger pressure
- in SI base units a pascal is kg m^{-1} s^{-2}
- a density from measurements: add the **percentage** uncertainties of the mass and of every length multiplied together

■ **Key knowledge: deriving $\Delta p = \rho g \Delta h$**



$\Delta p = \rho g h$

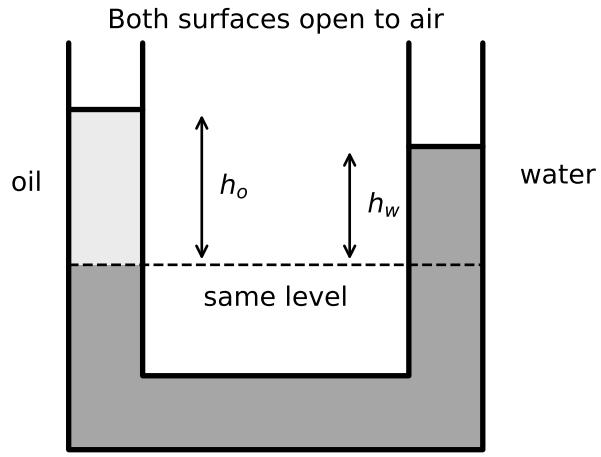
The four lines the mark scheme looks for, for a column of liquid of cross-sectional area A and height Δh:

1. volume $V = A\,\Delta h$
2. mass $m = \rho V = \rho A\,\Delta h$
3. weight $F_W = mg = \rho A\,\Delta h\,g$
4. pressure $= \dfrac{\text{force}}{\text{area}} = \dfrac{\rho A\,\Delta h\,g}{A}$, so $\Delta p = \rho g \Delta h$

Fact	**Consequence**
Δp depends only on the depth and the density	not on the shape of the container or the area
total pressure $= p_{\text{atm}} + \rho g h$, with $p_{\text{atm}} \approx 1.0 \times 10^5$ Pa	"the pressure at a depth" may mean either: read what the question asks
same depth in the same liquid at rest	the same pressure, whatever lies between
a U-tube with two liquids	at the level of their boundary the pressures are equal: $\rho_1 h_1 = \rho_2 h_2$

■ Key knowledge: connected liquids

In a stationary connected liquid, pressures at the **same horizontal level in the same liquid** are equal. Start at a free surface, add $\rho g h$ when moving down, and subtract it when moving up. Both open surfaces have atmospheric pressure, which cancels.



Worked example. Oil of density 800 kg/m^3 has a 0.10 m column above the oil–water boundary in an open U-tube. The water surface is h_w above that level. $p_{\text{atm}} + \rho_o g h_o = p_{\text{atm}} + \rho_w g h_w$, hence $h_w = \rho_o h_o / \rho_w = 800 \times 0.10/1000 = 0.080$ m. Compare heights from the same level, not from the tube's bottom.

■ Worked example: a density with its uncertainty

A block has $m = (0.300 \pm 0.001)$ kg and sides $x = (5.00 \pm 0.01)$ cm, $y = (10.00 \pm 0.01)$ cm and $z = (2.00 \pm 0.01)$ cm.

1. $V = xyz = 5.00 \times 10.00 \times 2.00 = 100\ \text{cm}^3 = 1.00 \times 10^{-4}\ \text{m}^3$.
2. $\rho = m/V = 0.300/(1.00 \times 10^{-4}) = 3.00 \times 10^3\ \text{kg m}^{-3}$.
3. For products/quotients add fractional uncertainties:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

$$\frac{\Delta\rho}{\rho} = \frac{0.001}{0.300} + \frac{0.01}{5.00} + \frac{0.01}{10.00} + \frac{0.01}{2.00} = 0.0113$$

4. The percentage uncertainty is 1.1%; the smallest side has the largest fractional uncertainty, so improving its measurement has the largest effect.

■ Worked example: pressure under a solid

A crate of mass 12 kg has a base 0.20 m by 0.15 m.

1. **Force** – the weight, $F_W = mg = 12 \times 9.81 = 118$ N.
2. **Pressure** – from the definition, $p = \dfrac{F}{A} = \dfrac{118 \text{ N}}{0.20 \text{ m} \times 0.15 \text{ m}} = 3.9 \times 10^3$ Pa.

■ Worked example: pressure at a depth

A diver is 12 m below the surface of sea water of density 1030 kg m^{-3}. Atmospheric pressure is 1.0×10^5 Pa.

1. **Pressure due to the water** – hydrostatic pressure, $\Delta p = \rho g \Delta h = 1030 \times 9.81 \times 12 = 1.21 \times 10^5$ Pa.
2. **Total pressure** – add the atmosphere: $p = p_{\text{atm}} + \rho g h = 1.0 \times 10^5 + 1.21 \times 10^5 = 2.2 \times 10^5$ Pa.
3. **Difference between two depths** – only Δh matters: between 12 m and 12.5 m, $\Delta p = \rho g \Delta h = 1030 \times 9.81 \times 0.50 = 5.1 \times 10^3$ Pa. This difference, over the area of an object, is where upthrust comes from (sheet 4.3.1).

■ Worked example: a jet of water on a wall

A horizontal jet of water, density 1000 kg m^{-3} and cross-sectional area 2.0×10^{-4} m^2, hits a wall at 6.0 $\dfrac{\text{m}}{\text{s}}$ and runs down it.

1. **Mass per second** – density $\times$ the volume arriving each second: $\dfrac{m}{t} = \rho A v = 1000 \times 2.0 \times 10^{-4} \times 6.0 = 1.2$ kg s^{-1}.
2. **Force** – rate of change of momentum, the water losing all its horizontal speed: $F = \dfrac{m}{t} v = 1.2 \times 6.0 = 7.2$ N.
3. **Pressure** – force per unit area: $p = \dfrac{F}{A} = \dfrac{7.2}{2.0 \times 10^{-4}} = 3.6 \times 10^4$ Pa.

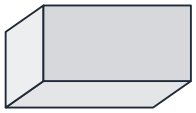
2 Practice

Now apply the methods above.

2.1 Define **pressure**. [1]

pressure

2.2 A block has mass 2400 kg and volume 0.30 m^3. Calculate its **density**. [2]



2400 kg

0.30 m^3

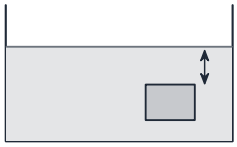
2.3 A person of mass 60 kg stands on both feet. Each foot has a contact area of 0.020 m^2. Calculate the pressure on the floor. [2]

60 kg

0.020 m^2

2.4 A swimming pool is 2.5 m deep. Atmospheric pressure is 1.0×10^5 Pa. Calculate, at the bottom of the pool:

(a) the pressure due to the water [2]

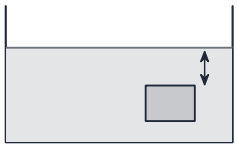


2.5 m

1.0×10^5 Pa

(b) the total pressure. [1]

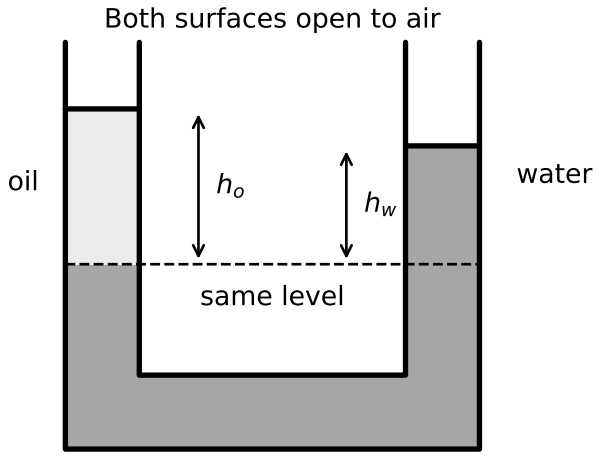
2.5 Two points in an oil of density 850 kg m^{-3} are 0.40 m apart vertically. Calculate the difference in pressure between them. [2]



850 kg m^{-3}

0.40 m

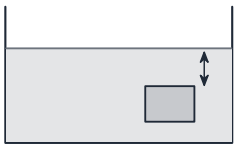
2.6 Both arms of a U-tube are open to air. A 0.15 m column of oil of density 820 kg/m^3 sits above water. The water density is 1000 kg/m^3. Find the water-surface height above the oil–water boundary and the difference between the free-surface heights. [4]



3 Exam-style questions

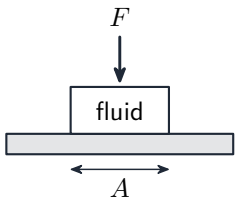
3.1 The pressure due to a liquid at the bottom of a container depends on [1]

- ☐ **A** the cross-sectional area of the container only
- **B** the depth and the density of the liquid
- **C** the shape of the container
- **D** the total volume of liquid



3.2 What is the pascal expressed in SI base units? [1]

- [ ] **A** kg m s^{-2}
B kg m^{-1} s^{-2}
C kg m^{-2} s^{-2}
D kg m^2 s^{-2}



3.3 A tank contains water of density 1000 kg m^{-3} to a depth of 3.2 m. A square hatch of side 0.40 m is set in the flat bottom of the tank.

(a) **Show that** the pressure due to a depth Δh of a liquid of density ρ is $\Delta p = \rho g \Delta h$. [3]

1000 kg m^{-3}
3.2 m
0.40 m

(b) Calculate the pressure due to the water at the bottom of the tank. [2]

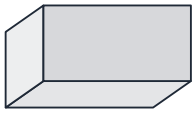
(c) Calculate the force that the water exerts on the hatch. [2]

(d) The tank is replaced by a narrower one holding water to the same depth. State and explain what happens to the pressure on the hatch. [2]

3.4 A student measures a rectangular metal block to find the density of the metal.

quantity	measurement
mass m	(0.512 ± 0.001) kg
length x	(8.02 ± 0.01) cm
width y	(4.15 ± 0.01) cm
height z	(2.05 ± 0.01) cm

(a) **Define** density. [1]



(0.512 ± 0.001) kg

(8.02 ± 0.01) cm

(b) **Determine** the density of the metal. [3]

(c) **Determine** the percentage uncertainty in the density. [3]

(d) **State**, with a reason, which single measurement the student should improve first. [2]

(e) The block rests on the bench on its largest face. Calculate the pressure it exerts on the bench. [2]