

Solutions

Each answer shows the steps, then the **result**.

2.1

- for an object in equilibrium
- the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about the same point

2.2

- the resultant force is zero
- the resultant moment (torque) is zero ()

2.3

- principle of moments about the centre, $F_1 d_1 = F_2 d_2$: $0.40 \times d = 0.60 \times 0.20$

$$d = \frac{F_2 d_2}{F_1} = \frac{0.60 \text{ N} \times 0.20 \text{ m}}{0.40 \text{ N}} = \mathbf{0.30 \text{ m}}$$
 to the right of the centre

2.4

- moments about the left-hand end: $F_{N,\text{right}} \times 1.0 = 30 \times 0.50 + 50 \times 0.30 = 30 \text{ N m}$, so $F_{N,\text{right}} = \mathbf{30 \text{ N}}$
- resolving vertically, $F_{N,\text{left}} + F_{N,\text{right}} = F_{W,\text{shelf}} + F_{W,\text{box}}$: $F_{N,\text{left}} = 30 + 50 - 30 = \mathbf{50 \text{ N}}$

2.5

- resolving vertically: $2F_T \cos 30^\circ = F_W$

$$F_T = \frac{F_W}{2 \cos 30^\circ} = \frac{20 \text{ N}}{2 \cos 30^\circ} = \mathbf{12 \text{ N}}$$

3.1 B

- no resultant force means the three force vectors, added tip to tail, return to the start: a closed triangle
- **D** has the direction wrong: the resultant of two is equal and **opposite** to the third
D ,

3.2 B

- moments about the pivot: $F_T \times L = 12 \times \frac{L}{2}$, so $F_T = 6.0 \text{ N}$
- the pivot supplies the other 6.0 N; **C** forgets the pivot takes a share
6.0 N C

3.3

(a) take moments about A so that the force at A drops out A A

- principle of moments: $F_{N,B} \times 4.0 = 60 \times 2.0 + 40 \times 1.0 = 160 \text{ N m}$
B

$$F_{N,B} = \frac{M}{d} = \frac{160 \text{ N m}}{4.0 \text{ m}} = \mathbf{40 \text{ N}}$$

(b) resolving vertically, $F_{N,A} + F_{N,B} = F_{W,\text{beam}} + F_{W,\text{load}}$: $F_{N,A} = 60 + 40 - 40 = \mathbf{60 \text{ N}}$

3.4

(a) the resultant force on the sign is zero

- the resultant moment on the sign is zero

(b) horizontally: $F_{T,1} \sin 30^\circ = F_{T,2} \sin 60^\circ$, so $F_{T,1} = \sqrt{3} F_{T,2}$

- vertically: $F_{T,1} \cos 30^\circ + F_{T,2} \cos 60^\circ = 180$

- substituting: $\sqrt{3} F_{T,2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} F_{T,2} = 2 F_{T,2} = 180$, so $F_{T,2} = \mathbf{90 \text{ N}}$ 2

- $F_{T,1} = \sqrt{3} F_{T,2} = \sqrt{3} \times 90 = \mathbf{156 \text{ N}}$ 1

3.5

(a) for a body in **equilibrium**

- sum of **clockwise moments** about any point = sum of **anticlockwise moments** about that same point

(b)

- arrow **down** at the middle of the board (2.0 m from A) labelled *weight of board* / 800 N
- arrow **up** at P labelled *support force* P
- arrow **down** at B labelled *weight of swimmer* / 620 N B

(c) moments about A (hinge force has no moment) A

- clockwise $\sum M = F_{W,1} d_1 + F_{W,2} d_2 = (800 \text{ N})(2.0 \text{ m}) + (620 \text{ N})(4.0 \text{ m}) = 4080 \text{ N m}$
- anticlockwise = $F_P \times 1.2 \text{ m}$

$$F_P = \frac{\sum M}{d} = \frac{4080 \text{ N m}}{1.2 \text{ m}} = \mathbf{3400 \text{ N}}$$

(d) vertical equilibrium: up = down

- $\sum F_{\text{up}} = \sum F_{\text{down}}: 3400 \text{ N} + F_{\text{hinge}} = 800 \text{ N} + 620 \text{ N}$

- $F_{\text{hinge}} = -1980 \text{ N}$, so **1980 N downwards**

(e) the hinge force is unknown; moments **about** A give it zero moment, so it drops out
A

(f) no hinge: take moments about **P** P

- board's weight is $d = 2.0 \text{ m} - 1.2 \text{ m} = 0.80 \text{ m}$ to the right of P P

- clockwise: $M = Fd = (800 \text{ N})(0.80 \text{ m}) = 640 \text{ N m}$ P

- swimmer must be on the **other** side of P, a distance $(1.2 \text{ m} - x)$ from it
P

$$\sum M_{\text{anticlockwise}} = \sum M_{\text{clockwise}}: (620 \text{ N})(1.2 \text{ m} - x) = 640 \text{ N m} \Rightarrow x = \mathbf{0.17 \text{ m}}$$

from A

3.6

- moments about the hinge, where the hinge force has no moment: $F_T \sin 30^\circ \times 2.0 = 90 \times 1.0$

$$F_T = \frac{90 \text{ N} \times 1.0 \text{ m}}{2.0 \text{ m} \times \sin 30^\circ} = \mathbf{90 \text{ N}}$$

- horizontally, the hinge balances the pull of the wire towards the wall: $F_H = F_T \cos 30^\circ$

$$F_H = 90 \text{ N} \times \cos 30^\circ = \mathbf{78 \text{ N}} \text{ away from the wall}$$

- vertically, $F_V + F_T \sin 30^\circ = F_W$

$$F_V = 90 \text{ N} - 90 \text{ N} \times \sin 30^\circ = \mathbf{45 \text{ N}} \text{ upwards}$$