Solutions

Each answer shows the steps, then the **result**.

2.1

- the total momentum of a system of objects is constant (total momentum before = total momentum after)
- provided no resultant external force acts on the system

2.2

(a) conservation of momentum, objects stick together: $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{3.0 \times 4.0 + 0}{3.0 + 1.0} = \mathbf{3.0}\ \frac{\text{m}}{\text{s}}$$

(b) $E_\text{k} = \frac{1}{2}mv^2$, before and after

$$E_\text{before} = \tfrac{1}{2}m_1u_1^2 = \tfrac{1}{2} \times 3.0 \times 4.0^2 = 24\ \text{J}$$

$$E_\text{after} = \tfrac{1}{2}(m_1 + m_2)v^2 = \tfrac{1}{2} \times 4.0 \times 3.0^2 = 18\ \text{J}$$

- the energy lost as a percentage of the energy before

$$\frac{E_\text{before} - E_\text{after}}{E_\text{before}} \times 100\% = \frac{24 - 18}{24} \times 100\% = \mathbf{25}\%$$

2.3

- conservation of momentum from rest: $0 = m_1v_1 + m_2v_2$

$$v_2 = -\frac{m_1v_1}{m_2} = -\frac{2.0 \times 6.0}{3.0} = -4.0\ \frac{\text{m}}{\text{s}}$$

- $\mathbf{4.0}\ \frac{\text{m}}{\text{s}}$, in the **opposite** direction to the 2.0 kg trolley

2.4

(a) ball 1 moves and ball 2 is at rest; total momentum $p = mv$ before and after, the same m for both

$$p_\text{before} = mu_1 + mu_2 = 0.50 \times 4.0 + 0 = 2.0\ \text{N s}$$

$$p_\text{after} = mv_1 + mv_2 = 0 + 0.50 \times 4.0 = 2.0\ \text{N s}$$

(b) total $E_\text{k} = \frac{1}{2}mv^2$, before and after

$$E_\text{before} = \tfrac{1}{2}mu_1^2 + 0 = \tfrac{1}{2} \times 0.50 \times 4.0^2 = 4.0 \text{ J}$$

$$E_\text{after} = 0 + \tfrac{1}{2}mv_2^2 = \tfrac{1}{2} \times 0.50 \times 4.0^2 = 4.0 \text{ J}$$

- the total kinetic energy is conserved, so the collision is **elastic**

(c) relative speed of approach $= u_1 - u_2 = 4.0 - 0 = 4.0 \ \dfrac{\text{m}}{\text{s}}$

- relative speed of separation $= v_2 - v_1 = 4.0 - 0 = 4.0 \ \dfrac{\text{m}}{\text{s}}$, the same

2.5

(a) the nucleus was at rest, so the total momentum before is **zero**

- momentum is conserved, so the two momenta after are equal in size and **opposite** in direction

(b) conservation of momentum, $m_\text{new}v_\text{new} = m_\alpha v_\alpha$, with the new mass $m_\text{new} = 212 - 4 = 208$ u

$$v_\text{new} = \frac{m_\alpha v_\alpha}{m_\text{new}} = \frac{4 \times 1.6 \times 10^7}{208} = \mathbf{3.1 \times 10^5} \ \frac{\text{m}}{\text{s}}$$

3.1 C

- the momenta are equal in size, so speed $\propto 1/\text{mass}$: $\dfrac{v_\alpha}{v_\text{new}} = \dfrac{m_\text{new}}{m_\alpha} = \dfrac{216}{4}$, with the new nucleus of mass $m_\text{new} = 220 - 4 = 216$ u
- **D** uses the mass of the nucleus before the decay; **A** and **B** are upside down D

3.2 C

- momentum: $mu = 3m\,v$, so $v = u/3$
- E_k after $= \frac{1}{2}(3m)(u/3)^2 = \frac{1}{3} \times \frac{1}{2}mu^2$, so one third remains and **two thirds** are transferred
- **A** is the fraction that remains A

3.3

(a) nucleon numbers and proton numbers both balance: ${}^{220}_{86}\text{Rn} \rightarrow {}^{216}_{84}\text{Po} + {}^{4}_{2}\alpha$

(b) the nucleus was at rest, so the total momentum stays **zero**

- conservation of momentum, $m_\alpha v_\alpha = m_{\text{Po}} v_{\text{Po}}$, masses in u
  u

$$v_\alpha = \frac{m_{\text{Po}} v_{\text{Po}}}{m_\alpha} = \frac{216 \times 2.2 \times 10^5}{4} = \mathbf{1.2 \times 10^7} \ \frac{\text{m}}{\text{s}}, \text{ opposite to the polonium}$$

(c) $E_{\text{k}} = \frac{1}{2} m v^2$, so $\dfrac{E_\alpha}{E_{\text{Po}}} = \dfrac{m_\alpha v_\alpha^2}{m_{\text{Po}} v_{\text{Po}}^2}$

- the momenta are equal in size, so $v_\alpha / v_{\text{Po}} = m_{\text{Po}} / m_\alpha$ and the ratio becomes $\dfrac{m_{\text{Po}}}{m_\alpha}$

$$\frac{E_\alpha}{E_{\text{Po}}} = \frac{m_{\text{Po}}}{m_\alpha} = \frac{216}{4} = \mathbf{54}$$

3.4

(a) acceleration is the gradient of the v–t line –

$$a = \frac{\Delta v}{\Delta t} = \frac{0.45 - 0.15}{0.30 - 0.10} = \mathbf{1.5} \text{ m s}^{\mathbf{-2}}$$

(b) conservation of momentum, bounce apart

$$m_{\text{P}} u_{\text{P}} + m_{\text{Q}} u_{\text{Q}} = m_{\text{P}} v_{\text{P}} + m_{\text{Q}} v_{\text{Q}} : \quad 0.40 \times 0.60 + 0.80 \times 0.15 = 0.40\, v_{\text{P}} + 0.80 \times 0.45$$

- so $0.36 = 0.40\, v_{\text{P}} + 0.36$ and $v_{\text{P}} = \mathbf{0}$: P stops P P

(c) a horizontal line at 0.60 $\dfrac{\text{m}}{\text{s}}$ from $t = 0$ to 0.10 s P

- a straight line from (0.10, 0.60) down to (0.30, 0): the change happens over the same contact time as for Q P Q
- a horizontal line along $v = 0$ from 0.30 to 0.50 s P

(d) add $E_{\text{k}} = \frac{1}{2} m v^2$ for the two trolleys, before and after

$$E_{\text{before}} = \tfrac{1}{2} m_{\text{P}} u_{\text{P}}^2 + \tfrac{1}{2} m_{\text{Q}} u_{\text{Q}}^2 = \tfrac{1}{2}(0.40)(0.60)^2 + \tfrac{1}{2}(0.80)(0.15)^2 = 0.081 \text{ J}$$

$$E_{\text{after}} = \tfrac{1}{2} m_{\text{P}} v_{\text{P}}^2 + \tfrac{1}{2} m_{\text{Q}} v_{\text{Q}}^2 = 0 + \tfrac{1}{2}(0.80)(0.45)^2 = 0.081 \text{ J}$$

- the total kinetic energy is conserved, so the collision is elastic