3.3.1 Elastic collisions and momentum in two dimensions

Name: _____ Class: _____ Date: _____ **Total: 37 marks**

KEY WORDS elastic collision • relative speed • component • conservation of momentum • kinetic energy

Objective

Answer the harder momentum questions: **elastic** collisions solved with the relative-speed rule, and conservation of momentum in **two dimensions** . Syllabus 3.3, learning outcomes 2 and 3.

You must be able to:

- recall the two facts about an elastic collision, and use the relative-speed rule with conservation of momentum to find two unknown velocities
- resolve a momentum into two **perpendicular components** , and apply conservation of momentum along each direction on its own
- combine two perpendicular momenta into a magnitude and a direction, and find the missing fragment of an explosion
- decide whether a collision in two dimensions is elastic, using the full speed of each object

1 Key knowledge and worked examples

■ Key knowledge: elastic collisions

Quantity or rule	Before collision	After collision
Total momentum	$m_1u_1 + m_2u_2$	$m_1v_1 + m_2v_2$
Total **kinetic energy**	$\frac{1}{2}(m_1u_1^2 + m_2u_2^2)$	$\frac{1}{2}(m_1v_1^2 + m_2v_2^2)$
Relative speed (head-on only)	Approach: $u_1 - u_2$	Separation: $v_2 - v_1$

In each row, **before = after** for an elastic collision. Choose one positive direction and use **signed velocities**: u before, v after.

With negligible external impulse, total momentum is conserved in **all** collisions. Total kinetic energy is conserved only in an **elastic** collision.

- relative speed: objects moving **towards** each other, **add** the speeds; moving the **same way**, **subtract** them
- to find two unknown velocities, use momentum **and** the relative-speed rule: two equations with no squares, quicker than the energy equation
- equal masses, one at rest, head-on and elastic: the two **exchange velocities**
- the relative-speed rule works only along the line of the collision; to test a collision in two dimensions, compare the total kinetic energy before and after

■ Key knowledge: momentum in two dimensions

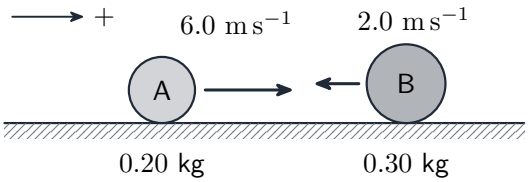
Momentum is a **vector**, so it is conserved along each of two perpendicular directions **separately**.

Step	Do this
1 choose axes	x along the incoming object, y across it: the total y-momentum before is then zero
2 resolve	a momentum p at angle θ to the x-axis has components $p\cos\theta$ along x and $p\sin\theta$ along y
3 two equations	total before = total after, once along x and once along y; label each equation with its axis
4 recombine	$p = \sqrt{p_x^2 + p_y^2}$ and $\theta = \tan^{-1}(p_y/p_x)$; give the direction with its sense, such as "36° south of east"

- an **explosion** from rest: the momenta of the fragments add to zero as vectors, so the last fragment cancels the resultant of all the others
- kinetic energy is a **scalar**: always use the full speed, never a component

■ Worked example: an elastic head-on collision

Ball A (0.20 kg) moves right at 6.0 $\frac{\text{m}}{\text{s}}$ and meets ball B (0.30 kg) moving left at 2.0 $\frac{\text{m}}{\text{s}}$. The collision is elastic. Find both velocities afterwards.



1. **Signs** – right positive: $u_\text{A} = +6.0\ \frac{\text{m}}{\text{s}}$, $u_\text{B} = -2.0\ \frac{\text{m}}{\text{s}}$.
2. **Momentum** – conservation of momentum, $m_\text{A}u_\text{A} + m_\text{B}u_\text{B} = m_\text{A}v_\text{A} + m_\text{B}v_\text{B}$:

   $$0.20 \times 6.0 + 0.30 \times (-2.0) = 0.20\,v_\text{A} + 0.30\,v_\text{B}, \quad \text{so} \quad 0.60 = 0.20\,v_\text{A} + 0.30\,v_\text{B}$$

3. **Relative speeds** – elastic, so approach = separation, $u_\text{A} - u_\text{B} = v_\text{B} - v_\text{A}$: $6.0 -$

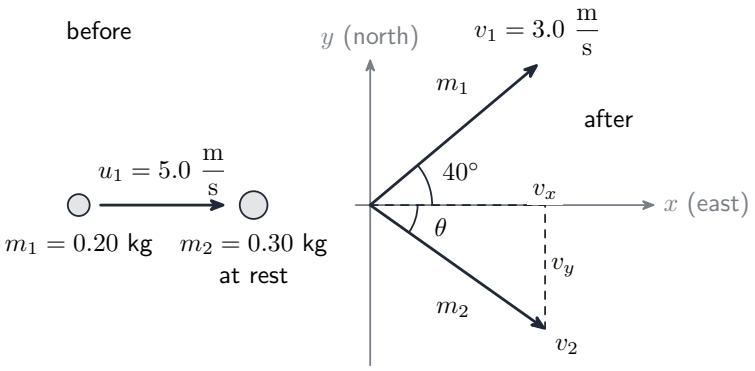
$(-2.0) = 8.0$, so $v_\mathrm{B} = v_\mathrm{A} + 8.0$.

4. **Solve** – substitute into step 2: $0.60 = 0.20\,v_\mathrm{A} + 0.30\,(v_\mathrm{A} + 8.0) = 0.50\,v_\mathrm{A} + 2.4$, so $v_\mathrm{A} = -3.6\ \dfrac{\mathrm{m}}{\mathrm{s}}$ (A moves **left** at 3.6 $\dfrac{\mathrm{m}}{\mathrm{s}}$) and $v_\mathrm{B} = +4.4\ \dfrac{\mathrm{m}}{\mathrm{s}}$ (B moves **right**).
5. **Check** – the total $E_\mathrm{k} = \frac{1}{2}mv^2$ must be the same before and after:

$$E_\mathrm{before} = \tfrac{1}{2}m_\mathrm{A}u_\mathrm{A}^2 + \tfrac{1}{2}m_\mathrm{B}u_\mathrm{B}^2 = \tfrac{1}{2}(0.20)(6.0)^2 + \tfrac{1}{2}(0.30)(2.0)^2 = 4.2\ \mathrm{J}$$

$$E_\mathrm{after} = \tfrac{1}{2}m_\mathrm{A}v_\mathrm{A}^2 + \tfrac{1}{2}m_\mathrm{B}v_\mathrm{B}^2 = \tfrac{1}{2}(0.20)(3.6)^2 + \tfrac{1}{2}(0.30)(4.4)^2 = 4.2\ \mathrm{J}$$

■ Worked example: a glancing collision



A 0.20 kg ball moving east at 5.0 $\dfrac{\mathrm{m}}{\mathrm{s}}$ strikes a stationary 0.30 kg ball. Afterwards the 0.20 kg ball moves at 3.0 $\dfrac{\mathrm{m}}{\mathrm{s}}$ at 40° north of east. Find the velocity of the 0.30 kg ball, and test the collision.

1. **Along x (east)** – momentum along x is conserved, $m_1u_1 = m_1v_1\cos 40° + m_2v_x$, with ball 1 the 0.20 kg ball:

$$v_x = \frac{m_1u_1 - m_1v_1\cos 40°}{m_2} = \frac{0.20 \times 5.0 - 0.20 \times 3.0\cos 40°}{0.30} = 1.80\ \frac{\mathrm{m}}{\mathrm{s}}$$

2. **Along y (north)** – the total before is **zero**, so $0 = m_1v_1\sin 40° + m_2v_y$:

$$v_y = -\frac{m_1v_1\sin 40°}{m_2} = -\frac{0.20 \times 3.0\sin 40°}{0.30} = -1.29\ \frac{\mathrm{m}}{\mathrm{s}}\ \text{(south)}$$

3. **Recombine** – Pythagoras for ball 2's speed v_2, $\tan^{-1}$ for the angle:

$$v_2 = \sqrt{v_x^2 + v_y^2} = \sqrt{1.80^2 + 1.29^2} = 2.2\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$\theta = \tan^{-1}\frac{|v_y|}{v_x} = \tan^{-1}\frac{1.29}{1.80} = 36°\ \text{south of east}$$

4. **Elastic?** – the total $E_\mathrm{k} = \frac{1}{2}mv^2$, with the full speeds:

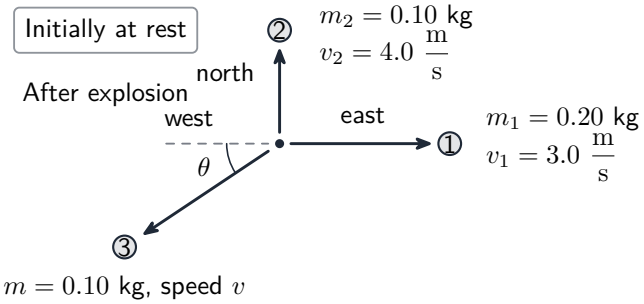
$$E_\mathrm{before} = \tfrac{1}{2}m_1u_1^2 = \tfrac{1}{2}(0.20)(5.0)^2 = 2.5\ \mathrm{J}$$

$$E_\mathrm{after} = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}(0.20)(3.0)^2 + \tfrac{1}{2}(0.30)(2.2)^2 = 1.6\ \mathrm{J}$$

Kinetic energy is lost, so the collision is **inelastic**.

Worked example: a firework in three pieces

A stationary firework explodes into three pieces that move in a horizontal plane: 0.20 kg at 3.0 $\frac{\text{m}}{\text{s}}$ east and 0.10 kg at 4.0 $\frac{\text{m}}{\text{s}}$ north. The third piece has a mass of 0.10 kg.



1. **The total stays zero** – conservation of momentum from rest, so the third piece cancels the momentum $p = mv$ of the other two along each axis:

$$p_{\text{east}} = m_1 v_1 = 0.20 \times 3.0 = 0.60 \text{ N s}$$

$$p_{\text{north}} = m_2 v_2 = 0.10 \times 4.0 = 0.40 \text{ N s}$$

2. **Magnitude** – the third piece's momentum has the size of their resultant (Pythagoras), and $p = mv$:

$$p = \sqrt{p_{\text{east}}^2 + p_{\text{north}}^2} = \sqrt{0.60^2 + 0.40^2} = 0.72 \text{ N s}$$

$$v = \frac{p}{m} = \frac{0.72 \text{ N s}}{0.10 \text{ kg}} = 7.2 \frac{\text{m}}{\text{s}}$$

3. **Direction** – opposite to the resultant of the other two: $\theta = \tan^{-1} \frac{p_{\text{north}}}{p_{\text{east}}} = \tan^{-1} \frac{0.40}{0.60} = 34°$ **south of west**.

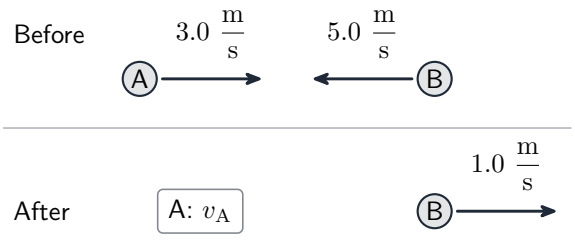
2 Practice

Now use the methods above.

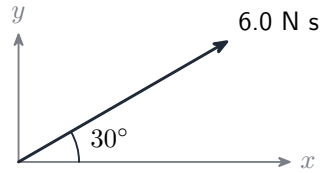
2.1 State **two** things that are true of an elastic collision, apart from the conservation of momentum. [2]

elastic collision

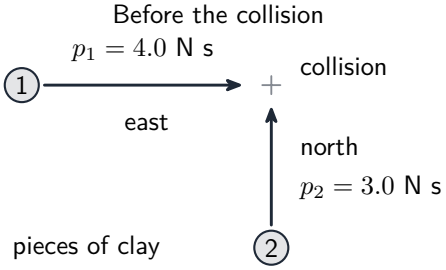
2.2 Ball A moves right at 3.0 $\frac{\text{m}}{\text{s}}$ and ball B moves left at 5.0 $\frac{\text{m}}{\text{s}}$. They collide head-on and elastically. Afterwards B moves right at 1.0 $\frac{\text{m}}{\text{s}}$. Use the relative-speed rule to find the velocity of A after the collision. [2]



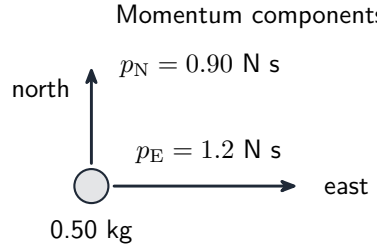
2.3 A momentum of 6.0 kg m s^{-1} is directed at 30° above the horizontal. Calculate its horizontal and vertical components. [2]



2.4 Two pieces of clay collide and stick together. Just before the collision one has momentum 4.0 N s due east and the other 3.0 N s due north. Calculate the magnitude and the direction of the momentum of the combined lump. [3]

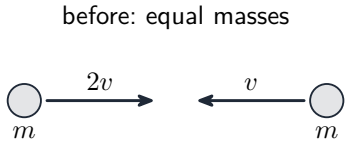


2.5 After a collision, a 0.50 kg ball has momentum components 1.2 N s east and 0.90 N s north. Calculate its speed, and state why its kinetic energy must be found from this speed and not from a component. [3]



3 Exam-style questions

3.1 Two balls of equal mass move towards each other along a straight line, one at speed $2v$ and the other at speed v. They collide head-on and elastically. What happens after the collision? [1]



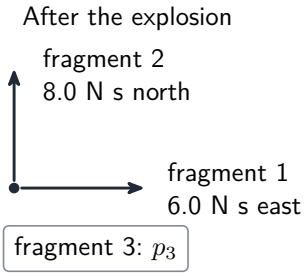
☐ **A** Both balls move at speed $1.5v$.

B Each ball moves back the way it came, the first at speed v and the second at speed $2v$.

C The first ball stops and the second moves back at speed $3v$.

D Each ball moves back the way it came, the first at speed $2v$ and the second at speed v.

3.2 A stationary object explodes into three fragments that move in a horizontal plane. One fragment has momentum 6.0 N s due east and another has momentum 8.0 N s due north. What is the momentum of the third fragment? [1]



☐ **A** 2.0 N s due south-west

B 10 N s, directed south of west

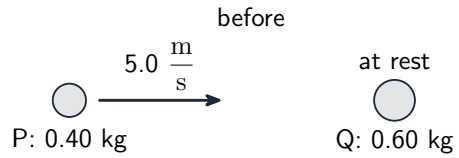
C 14 N s, directed south of west

D 10 N s, directed north of east

3.3 A ball P of mass 0.40 kg moving at 5.0 $\frac{\text{m}}{\text{s}}$ collides head-on and elastically with a stationary ball Q of mass 0.60 kg.

(a) State the **two** conditions, other than the conservation of momentum, that an elastic collision satisfies. [2]

(b) Use conservation of momentum and the relative-speed rule to **determine** the velocities of P and Q after the collision. [4]

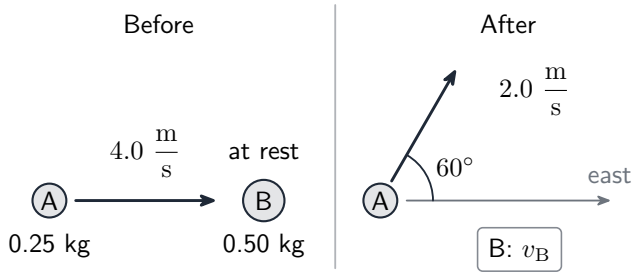


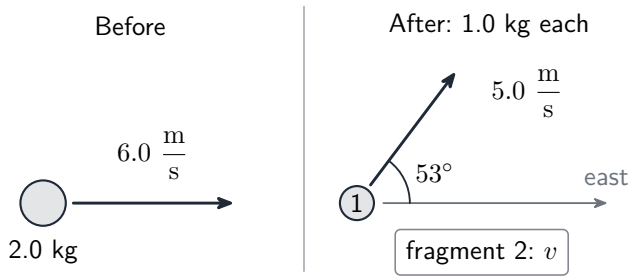
(c) Calculate the percentage of the initial kinetic energy of P that is transferred to Q. [2]

3.4 On a horizontal frictionless surface, a puck A of mass 0.25 kg moving due east at $4.0\ \frac{\text{m}}{\text{s}}$ strikes a stationary puck B of mass 0.50 kg. After the collision, A moves at $2.0\ \frac{\text{m}}{\text{s}}$ at 60° north of east.

(a) State the principle of conservation of momentum. [2]

(b) Calculate the magnitude and the direction of the velocity of B after the collision. [5]



(c) **Determine** whether the collision is elastic. Show your working. [3]

3.5 An object of mass 2.0 kg moving due east at $6.0\ \frac{\text{m}}{\text{s}}$ explodes into two fragments, each of mass 1.0 kg. One fragment moves at $5.0\ \frac{\text{m}}{\text{s}}$ at 53° north of east. Calculate the magnitude and the direction of the velocity of the other fragment. [5]