

## Solutions

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Each answer shows the steps, then the **result**.

### 2.1

- the total **kinetic energy** is conserved
- the **relative speed of approach** equals the **relative speed of separation**

### 2.2

- right positive; elastic, so  $u_A - u_B = v_B - v_A$ :  $3.0 - (-5.0) = 8.0 = 1.0 - v_A$
- $v_A = -7.0 \frac{\text{m}}{\text{s}}$ : A moves **left** at **7.0**  $\frac{\text{m}}{\text{s}}$                       A

### 2.3

- horizontal:  $p \cos \theta = 6.0 \cos 30^\circ = \mathbf{5.2} \text{ kg m s}^{-1}$
- vertical:  $p \sin \theta = 6.0 \sin 30^\circ = \mathbf{3.0} \text{ kg m s}^{-1}$

### 2.4

- momentum is conserved along each axis, so the lump has 4.0 N s east and 3.0 N s north
- Pythagoras:  $p = \sqrt{p_{\text{east}}^2 + p_{\text{north}}^2} = \sqrt{4.0^2 + 3.0^2} = \mathbf{5.0} \text{ N s}$
- $\theta = \tan^{-1} \frac{p_{\text{north}}}{p_{\text{east}}} = \tan^{-1} \frac{3.0}{4.0} = \mathbf{37^\circ}$  north of east

### 2.5

- Pythagoras:  $p = \sqrt{p_{\text{east}}^2 + p_{\text{north}}^2} = \sqrt{1.2^2 + 0.90^2} = 1.5 \text{ N s}$
- from  $p = mv$ :  $v = p/m = 1.5/0.50 = \mathbf{3.0} \frac{\text{m}}{\text{s}}$
- kinetic energy is a **scalar** that depends on the full speed,  $\frac{1}{2}mv^2$ ; a component gives a value that is too small

### 3.1 B

- equal masses in a head-on elastic collision **exchange velocities**: the first takes the second's velocity ( $v$ , backwards) and the second takes the first's ( $2v$ , backwards)
- check: approach  $2v + v = 3v$ , separation  $v + 2v = 3v$ ; **D** has the right directions but breaks momentum, **C** breaks the relative-speed rule and **A** is a sticking-type answer  
D                                              C                                              A

### 3.2 B

- from rest, the three momenta add to zero, so the third cancels 6.0 N s east and 8.0 N s north
- $p = \sqrt{p_{\text{east}}^2 + p_{\text{north}}^2} = \sqrt{6.0^2 + 8.0^2} = 10 \text{ N s}$ , directed south of west; **D** is the resultant of the other two, not its opposite, and **C** adds the magnitudes

D                      C

### 3.3

(a) the total kinetic energy is conserved

- the relative speed of approach equals the relative speed of separation

(b) conservation of momentum, Q at rest:  $m_P u_P = m_P v_P + m_Q v_Q$

P

$$0.40 \times 5.0 = 0.40 v_P + 0.60 v_Q, \quad \text{so} \quad 2.0 = 0.40 v_P + 0.60 v_Q$$

- elastic, so  $u_P - u_Q = v_Q - v_P$ :  $5.0 - 0 = v_Q - v_P$ , so  $v_Q = v_P + 5.0$
- $2.0 = 0.40 v_P + 0.60 v_P + 3.0$ , so  $v_P = -1.0 \frac{\text{m}}{\text{s}}$ : P moves **back** at **1.0**  $\frac{\text{m}}{\text{s}}$
- $v_Q = 4.0 \frac{\text{m}}{\text{s}}$  forwards Q

(c)  $E_k = \frac{1}{2}mv^2$  for P before and for Q after

P                      Q

$$E_P = \frac{1}{2}m_P u_P^2 = \frac{1}{2} \times 0.40 \times 5.0^2 = 5.0 \text{ J}$$

$$E_Q = \frac{1}{2}m_Q v_Q^2 = \frac{1}{2} \times 0.60 \times 4.0^2 = 4.8 \text{ J}$$

- the share of P's energy that Q receives
- Q                      P

$$\frac{E_Q}{E_P} \times 100\% = \frac{4.8}{5.0} \times 100\% = \mathbf{96\%}$$

### 3.4

(a) the total momentum of a system of objects is constant

- provided no resultant external force acts on the system

(b) momentum along east is conserved,  $m_A u_A = m_A v_A \cos 60^\circ + m_B v_x$

B

$$v_x = \frac{m_A u_A - m_A v_A \cos 60^\circ}{m_B} = \frac{0.25 \times 4.0 - 0.25 \times 2.0 \cos 60^\circ}{0.50} = 1.5 \frac{\text{m}}{\text{s}}$$

- along north the total before is zero,  $0 = m_A v_A \sin 60^\circ + m_B v_y$
- B

$$v_y = -\frac{m_A v_A \sin 60^\circ}{m_B} = -\frac{0.25 \times 2.0 \sin 60^\circ}{0.50} = -0.87 \frac{\text{m}}{\text{s}} \text{ (south)}$$

- Pythagoras:  $v = \sqrt{v_x^2 + v_y^2} = \sqrt{1.5^2 + 0.87^2} = \mathbf{1.7 \frac{m}{s}}$
- $\theta = \tan^{-1} \frac{|v_y|}{v_x} = \tan^{-1} \frac{0.87}{1.5} = \mathbf{30^\circ \text{ south of east}}$

(c) the total  $E_k = \frac{1}{2}mv^2$ , with the full speeds

$$E_{\text{before}} = \frac{1}{2}m_A u_A^2 = \frac{1}{2}(0.25)(4.0)^2 = 2.0 \text{ J}$$

$$E_{\text{after}} = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}(0.25)(2.0)^2 + \frac{1}{2}(0.50)(1.73)^2 = 1.25 \text{ J}$$

- the kinetic energy decreases, so the collision is **inelastic**

### 3.5

- momentum along east is conserved,  $Mu = m_1 v_1 \cos 53^\circ + m_2 v_x$

$$v_x = \frac{Mu - m_1 v_1 \cos 53^\circ}{m_2} = \frac{2.0 \times 6.0 - 1.0 \times 5.0 \cos 53^\circ}{1.0} = 9.0 \frac{\text{m}}{\text{s}}$$

- along north the total before is zero,  $0 = m_1 v_1 \sin 53^\circ + m_2 v_y$

$$v_y = -\frac{m_1 v_1 \sin 53^\circ}{m_2} = -\frac{1.0 \times 5.0 \sin 53^\circ}{1.0} = -4.0 \frac{\text{m}}{\text{s}} \text{ (south)}$$

- Pythagoras:  $v = \sqrt{v_x^2 + v_y^2} = \sqrt{9.0^2 + 4.0^2} = \mathbf{9.8 \frac{m}{s}}$
- $\theta = \tan^{-1} \frac{|v_y|}{v_x} = \tan^{-1} \frac{4.0}{9.0} = \mathbf{24^\circ \text{ south of east}}$