

10.2.1 Solving circuits with Kirchhoff's laws

Name: _____ Class: _____ Date: _____

Total: **37 marks**

KEY WORDS junction • parallel • series • electromotive force
• power

Objective

Answer exam questions that **solve circuits** with Kirchhoff's laws: reducing a network of resistors one step at a time, finding every current and p.d., comparing the power in each component, and a loop that contains two cells. Syllabus 10.2, learning outcome 7.

You must be able to:

- reduce a series-parallel network to a single resistance, one step at a time
- find the current in, and the p.d. across, every resistor of a network
- say which component dissipates the most power, in series and in parallel
- apply the second law round a loop with two cells, including cells that oppose

1 Key knowledge and worked examples

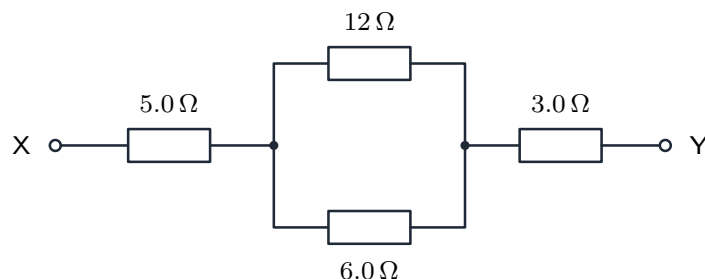
■ Key knowledge: solving a circuit

Step	Do this
1	combine the innermost parallel group into one resistance; then add series resistances; repeat
2	find the total current from the supply: $I = \frac{E}{R_{\text{total}}}$
3	work back out: the p.d. across each group is IR ; in a parallel group the p.d. is shared by every branch
4	the current in each branch is $\frac{V}{R}$; check that the branch currents add up (first law)

- in **series** the same current: the **larger** resistance takes more power, $P = I^2 R$; in **parallel** the same p.d.: the **smaller** resistance takes more, $P = \frac{V^2}{R}$
- two cells round one loop: add their e.m.f.s if they push the same way, subtract if

they oppose

■ **Worked example: reducing a network one step at a time**



Never try to see the whole network at once. Replace one group, redraw, repeat.

1. **The parallel pair** – for resistors in parallel the reciprocals add:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{12\ \Omega} + \frac{1}{6.0\ \Omega} = \frac{3}{12\ \Omega} \Rightarrow R_p = 4.0\ \Omega$$

2. **The series chain** – what is left is three resistances in series, which add:

$$R_{XY} = R_a + R_p + R_b = 5.0\ \Omega + 4.0\ \Omega + 3.0\ \Omega = \mathbf{12\ \Omega}$$

- For **two** resistors only, $R_p = \frac{R_1 R_2}{R_1 + R_2}$ is quicker. It is wrong for three.
- A parallel combination is always **smaller** than its smallest resistor; a series one is always larger than its largest. Use that to check every answer.

■ **Worked example: which component dissipates the most power**

Two rules settle this with no arithmetic at all, and which one applies depends on what the components share.

- **Same current** (in series): $P = I^2 R$, so the **larger** resistance dissipates more.
- **Same p.d.** (in parallel): $P = \frac{V^2}{R}$, so the **smaller** resistance dissipates more.

A $4.0\ \Omega$ and an $8.0\ \Omega$ resistor are joined to a $12\ \text{V}$ supply of negligible internal resistance.

1. **In series** – the resistances add, $R = R_4 + R_8 = 12\ \Omega$, so the same current flows in both:

$$I = \frac{V}{R} = \frac{12\ \text{V}}{12\ \Omega} = 1.0\ \text{A}$$

$$P_4 = I^2 R_4 = (1.0\ \text{A})^2 (4.0\ \Omega) = 4.0\ \text{W}$$

$$P_8 = I^2 R_8 = (1.0\ \text{A})^2 (8.0\ \Omega) = 8.0\ \text{W}$$

The larger resistor wins.

2. **In parallel** – both have the full 12 V across them:

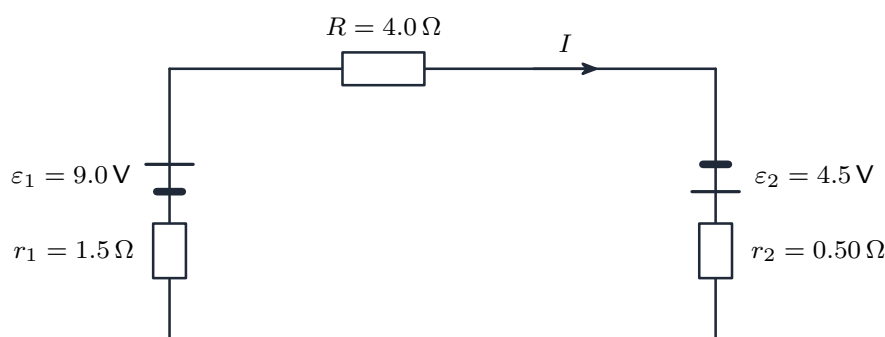
$$P_4 = \frac{V^2}{R_4} = \frac{(12 \text{ V})^2}{4.0 \, \Omega} = 36 \text{ W}$$

$$P_8 = \frac{V^2}{R_8} = \frac{(12 \text{ V})^2}{8.0 \, \Omega} = 18 \text{ W}$$

Now the smaller resistor wins.

- In a mixed circuit, the resistor carrying the **whole** current usually dissipates the most, because every parallel branch carries only a share of it.

■ **Worked example: a loop with two cells that oppose**



The second law handles this with no extra ideas: go once round the loop and add the e.m.f.s with their signs.

1. **Net e.m.f.** – the cells push opposite ways round the loop, so their e.m.f.s subtract:

$$\varepsilon_{\text{net}} = \varepsilon_1 - \varepsilon_2 = 9.0 \text{ V} - 4.5 \text{ V} = 4.5 \text{ V}$$

2. **Total resistance** – the two **internal resistances** are in series with the resistor:

$$R_{\text{total}} = r_1 + r_2 + R = 1.5 \, \Omega + 0.50 \, \Omega + 4.0 \, \Omega = 6.0 \, \Omega$$

3. **Current** – driven the way the larger e.m.f. pushes:

$$I = \frac{\varepsilon_{\text{net}}}{R_{\text{total}}} = \frac{4.5 \text{ V}}{6.0 \, \Omega} = \mathbf{0.75 \text{ A}}$$

4. **Terminal p.d. of the 9.0 V cell** – its e.m.f. less the p.d. across its own internal resistance:

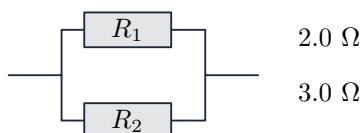
$$V_1 = \varepsilon_1 - Ir_1 = 9.0 \text{ V} - (0.75 \text{ A})(1.5 \, \Omega) = \mathbf{7.9 \text{ V}}$$

- The smaller cell is being driven backwards, so it is charging: its terminal p.d. $V_2 = \varepsilon_2 + Ir_2 = 4.9 \text{ V}$ is *larger* than its e.m.f.

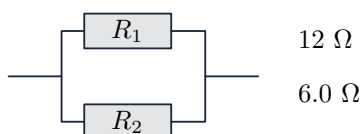
2 Practice

Now use the methods above.

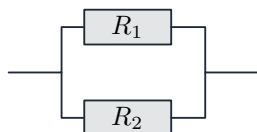
2.1 Calculate the combined resistance of $2.0\ \Omega$, $3.0\ \Omega$ and $6.0\ \Omega$ connected in parallel.[2]



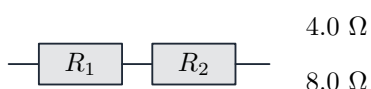
2.2 For the network between X and Y in the worked example above, the $12\ \Omega$ resistor is replaced by a second $6.0\ \Omega$ resistor. Calculate the new resistance between X and Y. [3]



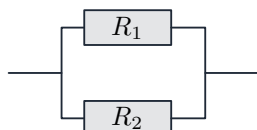
2.3 Explain why connecting a second resistor in parallel with the first always **decreases** the resistance between the same two points. [2]



2.4 A $4.0\ \Omega$ and an $8.0\ \Omega$ resistor are connected in **series** across a supply. State which dissipates the greater power, and by what factor. [2]



2.5 The same two resistors are now connected in **parallel** across the same supply. State which dissipates the greater power, and explain why the answer has changed. [2]



3 Exam-style questions

3.1 A $6.0\ \Omega$ resistor and a $3.0\ \Omega$ resistor are connected in parallel. This combination is in series with a $4.0\ \Omega$ resistor across a supply of e.m.f. $12\ \text{V}$ and negligible internal resistance. What is the current from the supply? [1]

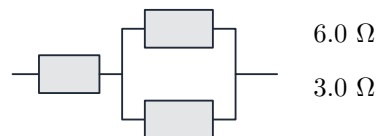
☐

A $0.50\ \text{A}$

B $1.0\ \text{A}$

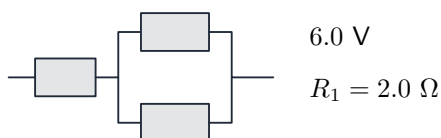
C $2.0\ \text{A}$

D $3.0\ \text{A}$



3.2 A battery of e.m.f. $6.0\ \text{V}$ and negligible internal resistance is connected to $R_1 = 2.0\ \Omega$ in series with two resistors $R_2 = 4.0\ \Omega$ and $R_3 = 4.0\ \Omega$ in **parallel**.

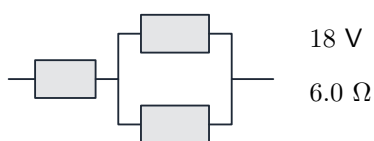
(a) Calculate the total resistance and the current from the battery. [3]



(b) Calculate the current in **each** of R_2 and R_3 . [3]

3.3 A battery of e.m.f. 18 V and negligible internal resistance is connected to a $6.0\ \Omega$ resistor in series with a parallel combination of a $9.0\ \Omega$ resistor and an $18\ \Omega$ resistor.

(a) Show that the resistance of the parallel combination is $6.0\ \Omega$. [2]

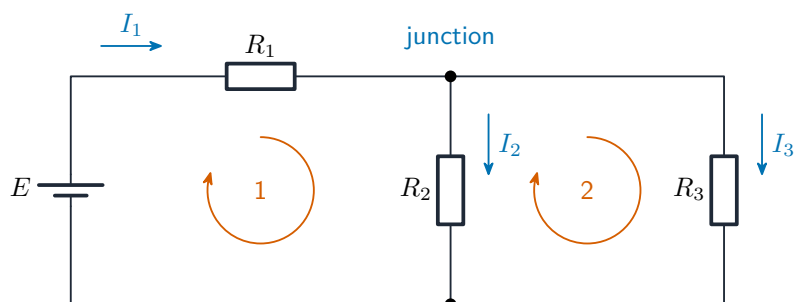


(b) Calculate the current in the battery. [2]

(c) Calculate the p.d. across the parallel combination and the current in each of its branches. [3]

(d) State, with a reason, which of the three resistors dissipates the greatest power. [2]

3.4 A cell of e.m.f. 12 V and negligible internal resistance is connected to $R_1 = 3.0\ \Omega$ in series with a parallel combination of $R_2 = 12\ \Omega$ and $R_3 = 4.0\ \Omega$, as shown.



(a) Write down the equation that Kirchhoff's first law gives for the junction. [1]

(b) Calculate the current I_1 in the cell and the currents I_2 and I_3 in the two branches. [4]

(c) Calculate the p.d. across R_1 . [2]

(d) Show that the total power dissipated in the three resistors equals the power supplied by the cell, and state which conservation law this illustrates. [3]