8.4 The diffraction grating

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS diffraction grating • wavelength • diffraction • normal • order

Objective

Build the skills to answer exam questions on the **diffraction grating** .

You must be able to:

- use $d \sin\theta = n\lambda$ in **both** directions (find an angle; find the grating)
- find the slit spacing from **lines per mm** and the **highest order**
- describe using a grating to measure **wavelength**

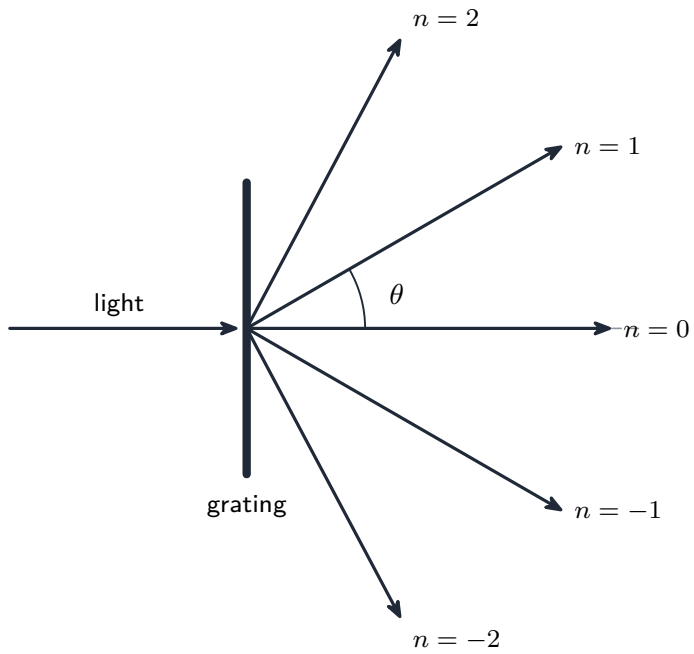
1 Key knowledge and worked examples

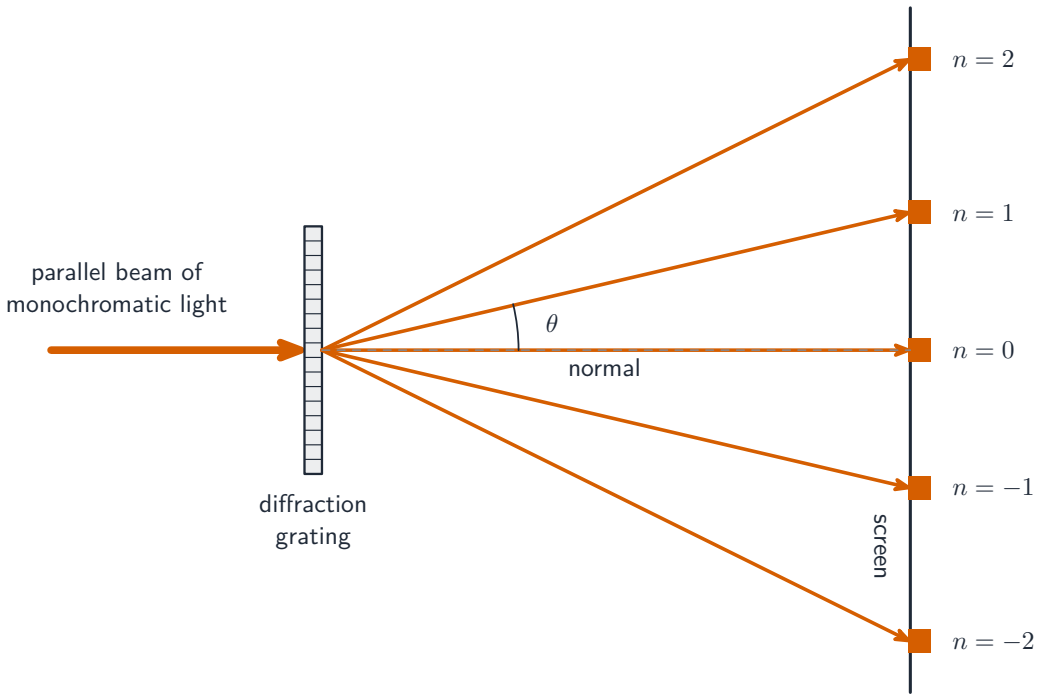
Study these first. Each one shows the method for a question type used later – follow the steps and you can do the Practice and Exam-style questions yourself.

■ The grating equation

A grating has many equally spaced slits, slit spacing d. Bright maxima (orders n) appear where

$$d \sin\theta = n\lambda.$$





A diffraction grating spreads the orders

Worked example (find the angle). 500 lines/mm, $\lambda = 600$ nm, first order:

- line spacing: $d = \dfrac{1}{N} = \dfrac{1}{500}$ mm $= 2.0 \times 10^{-6}$ m
- angle: $\sin\theta = \dfrac{n\lambda}{d} = \dfrac{600 \times 10^{-9}}{2.0 \times 10^{-6}} = 0.30$, so $\theta \approx 17°$

Working backwards – find the grating

Exam questions often **give** an order's angle and ask for the grating. Rearrange to $d = \dfrac{n\lambda}{\sin\theta}$, then **lines per mm** $= \dfrac{1}{d\ (\text{in mm})}$.

Worked example. The **second-order** ($n = 2$) maximum for $\lambda = 730$ nm is at $\theta = 35°$.

- spacing: $d = \dfrac{n\lambda}{\sin\theta} = \dfrac{2 \times 730 \times 10^{-9}}{\sin 35°} = 2.55 \times 10^{-6}\ \text{m} = 2.55 \times 10^{-3}\ \text{mm}$
- **lines per mm**: $N = \dfrac{1}{d} = \dfrac{1}{2.55 \times 10^{-3}\ \text{mm}} \approx 390$

Slit spacing and highest order

N lines per mm $\rightarrow d = 10^{-3}/N$ m. Since $\sin\theta \leq 1$, the **highest order** is $n_{\max} = \lfloor d/\lambda \rfloor$.

Finding wavelength

Shine parallel light normally on the grating, measure θ_1 for the first order, then $\lambda = d\sin\theta_1$. A grating gives **sharper** maxima than a double slit (many slits add).

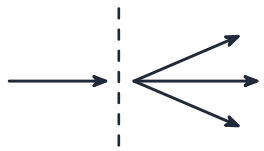
Key knowledge: paired orders, overlap and graphs

- Angle between the $+n$ and $-n$ beams is $2\theta_n$. Halve it before using $d\sin\theta_n = n\lambda$.
- Orders of different wavelengths overlap when $n_1\lambda_1 = n_2\lambda_2$.
- A graph of $\sin\theta$ against n has gradient $G = \lambda/d$, so $d = \lambda/G$. Convert spacing to metres before finding lines/mm: $N = 10^{-3}/d$.
- For a screen perpendicular to the central ray at distance L, an order at displacement y has $\tan\theta = y/L$; do not use the double-slit fringe-spacing formula for a grating.

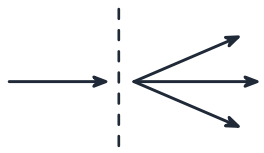
2 Practice

Now apply the methods above.

2.1 State what d represents in $d\sin\theta = n\lambda$. [1]

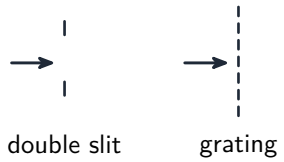


2.2 A grating has 500 lines per mm. Find the slit spacing d in metres. [2]

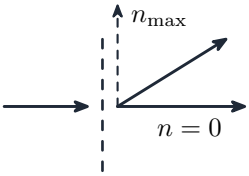


500 lines mm^{-1}

2.3 State **one** reason a grating gives **sharper** maxima than a double slit. [1]

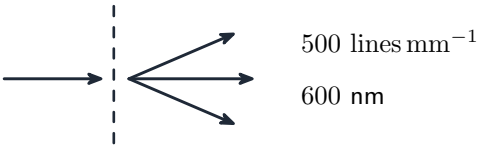


2.4 Write the formula for the **highest order** observable. [1]

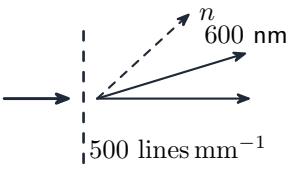


3 Exam-style questions

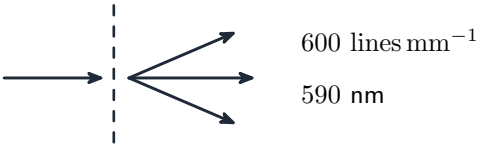
3.1 A grating with 500 lines per mm is illuminated normally by light of wavelength 600 nm. Calculate the angle of the **first-order** maximum. [3]



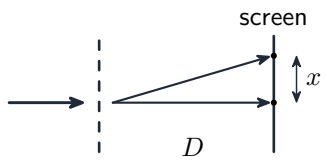
3.2 For the same grating and wavelength, calculate the **highest order** that can be seen. [2]



3.3 A grating has 600 lines per mm. Light of wavelength 590 nm is incident normally on it. **Determine** the highest order of maximum that can be observed, and **explain** what limits it. [4]

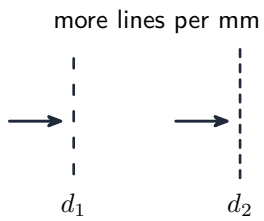


3.4 Describe how a diffraction grating can be used to **measure the wavelength** of light. [3]

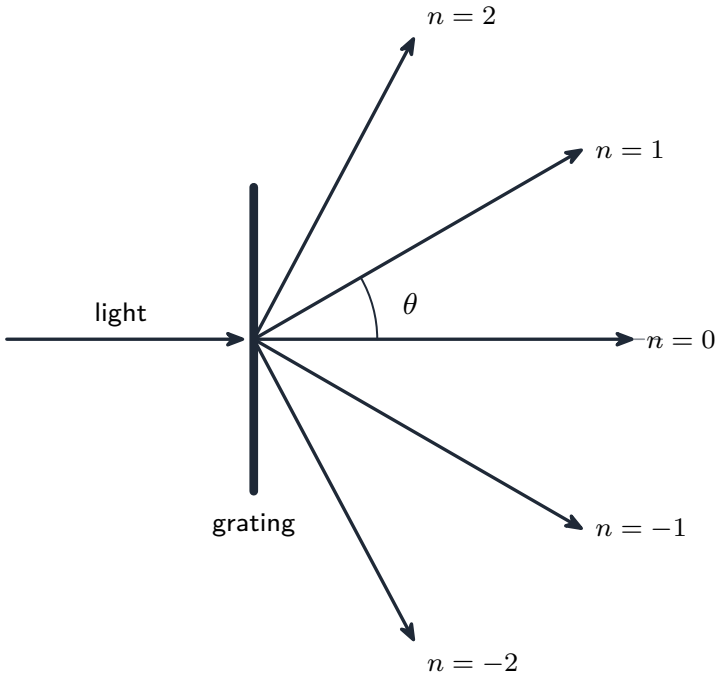


3.5 A grating with **more** lines per mm (smaller d) is used. The diffraction angles become: [1]

- [ ] **A** smaller
- [ ] **B** larger
- [ ] **C** unchanged
- [ ] **D** zero



3.6 A grating gives second-order beams separated by 60° for 600 nm light. Find the slit spacing and lines per mm. At the same angle, what order is produced by 400 nm light?[5]



Solutions

Each answer shows the steps, then the **result**.

2.1

- the **slit spacing** – the centre-to-centre distance between adjacent slits

2.2

- $d = 1/N$ per millimetre

$$d = \frac{1}{N} = \frac{1.0 \times 10^{-3}\ \text{m}}{500} = \mathbf{2.0 \times 10^{-6}}\ \text{m}$$

2.3

- many slits contribute, so the maxima are **sharper / brighter** and other directions are more completely cancelled

2.4

- $n_{\text{max}} = \lfloor d/\lambda \rfloor$ – the largest integer with $\sin\theta \leq 1$

3.1

- $d = 2.0 \times 10^{-6}$ m

$$\sin\theta = \frac{n\lambda}{d} = \frac{(1)(600 \times 10^{-9}\ \text{m})}{2.0 \times 10^{-6}\ \text{m}} = 0.30$$

- $\theta = \mathbf{17}^\circ$

3.2

$$\frac{d}{\lambda} = \frac{2.0 \times 10^{-6}\ \text{m}}{600 \times 10^{-9}\ \text{m}} = 3.33$$

- highest order $n = \mathbf{3}$ ($n = 4$ needs $\sin\theta > 1$)

3.3

$$d = \frac{1}{N} = \frac{1.0 \times 10^{-3}\ \text{m}}{600} = 1.67 \times 10^{-6}\ \text{m}$$

- largest angle is 90°, where $\sin\theta = 1$

$$n_{\text{max}} = \frac{d}{\lambda} = \frac{1.67 \times 10^{-6}\ \text{m}}{590 \times 10^{-9}\ \text{m}} = 2.8$$

- an order must be a whole number and $\sin\theta$ cannot exceed 1, so the highest observed is the **2nd order**

3.4

- shine **parallel monochromatic** light normally on the grating
- measure the **angle** θ of a known order n from the straight-through beam
- calculate $\lambda = d\sin\theta/n$

3.5 B

- more lines per mm $\Rightarrow$ smaller $d \Rightarrow$ larger $\sin\theta$ for each order $\Rightarrow$ **larger** angle

3.6

- $\theta = 60°/2 = 30°$.
- $d = n\lambda/\sin\theta = 2 \times 600 \times 10^{-9}/\sin 30° = 2.40 \times 10^{-6}$ m.
- $N = 10^{-3}/d = 10^{-3}/(2.40 \times 10^{-6}) = 417$ lines/mm.
- $n_2 = n_1\lambda_1/\lambda_2 = 2 \times 600/400 = 3$: third order.