6.1 Hooke's law and the spring constant

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS Hooke's law • spring constant • extension • limit of proportionality • load

Objective

Answer exam questions on **Hooke's law** and the **spring constant**: the words for deformation, reading a force–extension graph, and springs joined in series and in parallel. Syllabus 6.1, learning outcomes 1 to 4.

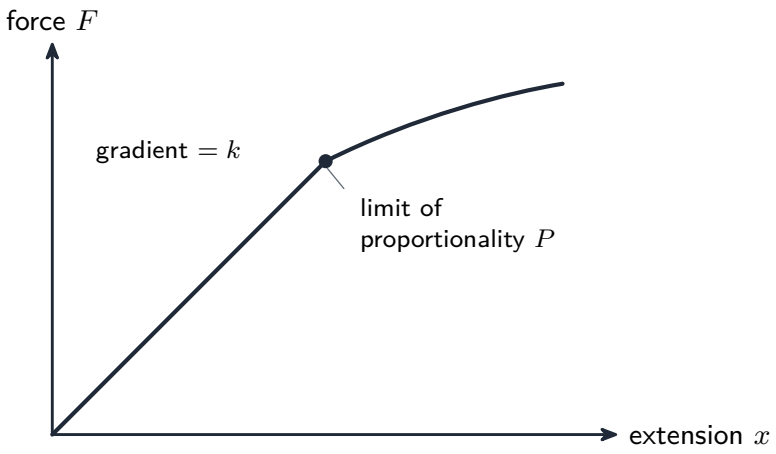
You must be able to:

- use the terms tensile, compressive, load, extension, compression and limit of proportionality
- state Hooke's law, and use the spring constant $k = F/x$
- read a force–extension graph: its gradient, and where it stops being straight
- find the extension and the spring constant of springs joined in series or in parallel

1 Key knowledge and worked examples

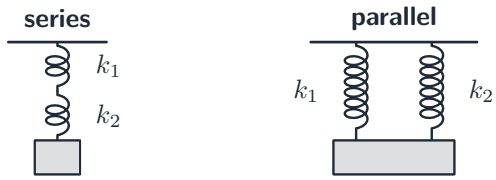
Key knowledge: the words and the law

Term	Meaning
tensile / **compressive** forces	a pair of forces that stretch / squash an object (one force alone would accelerate it)
load	the force applied, often the weight of hung masses, $F_W = mg$
extension / **compression**	the increase / decrease in length from the original length
Hooke's law (2 marks)	the extension is proportional to the load, provided the **limit of proportionality** is not exceeded
limit of proportionality	the point beyond which the force–extension graph is no longer a straight line
spring constant	force per unit extension, $k = \frac{F}{x}$, in N m^{-1}: the **gradient** of the straight part of a force–extension graph



- a stiffer spring has a larger k and a steeper force–extension line
- extension is measured from the **original length**: new length − original length

Key knowledge: springs joined together



$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \qquad k = k_1 + k_2$$

Joined	Load on each spring	Extension of the pair	Pair constant (two identical springs)
series	full load F	extensions add	$k/2$
parallel	half the load $F/2$	same as each spring	$2k$

Worked example: a spring constant

A spring extends by 2.4 cm when a load of 6.0 N hangs from it.

1. **Spring constant** – $k = \dfrac{F}{x} = \dfrac{6.0\ \text{N}}{0.024\ \text{m}} = 250\ \text{N m}^{-1}$.
2. **A new load** – a mass of 0.40 kg: $F = mg = 3.9$ N, so from Hooke's law $x = \dfrac{F}{k} = \dfrac{3.9}{250} = 0.016$ m, provided the limit of proportionality is not passed.
3. **New length** – if the unstretched spring is 15.0 cm long, it becomes $L = L_0 + x = 15.0 + 1.6 = 16.6$ cm.

■ Worked example: reading a force–extension graph

A spring's force–extension graph is a straight line from the origin to (4.0 cm, 12 N), then it curves.

1. **Spring constant** – the gradient of the straight part: $k = \dfrac{\Delta F}{\Delta x} = \dfrac{12\ \text{N}}{0.040\ \text{m}} = 300\ \text{N m}^{-1}$.
2. **Limit of proportionality** – where the line stops being straight: at a load of 12 N. Beyond it, $F = kx$ no longer holds.

■ Worked example: two springs, two ways

Two identical springs each have $k = 200\ \text{N m}^{-1}$. They carry a load of 12 N.

1. **In series** – each spring carries the whole 12 N, so each extends by $x = \dfrac{F}{k}$, and the two extensions add:

$$x = \frac{F}{k} = \frac{12\ \text{N}}{200\ \text{N m}^{-1}} = 0.060\ \text{m}, \qquad x_{\text{pair}} = 2x = 0.12\ \text{m}$$

$$k_{\text{pair}} = \frac{F}{x_{\text{pair}}} = \frac{12\ \text{N}}{0.12\ \text{m}} = 100\ \text{N m}^{-1}$$

2. **In parallel** – each spring carries half the load, 6.0 N, and the pair extends as far as one spring:

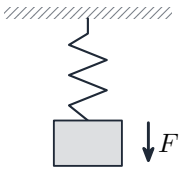
$$x = \frac{F}{k} = \frac{6.0\ \text{N}}{200\ \text{N m}^{-1}} = 0.030\ \text{m}$$

$$k_{\text{pair}} = \frac{F}{x} = \frac{12\ \text{N}}{0.030\ \text{m}} = 400\ \text{N m}^{-1}$$

2 Practice

Now apply the methods above.

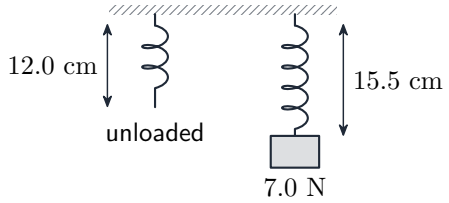
2.1 State **Hooke's law**. [2]



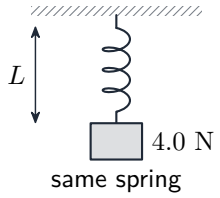
2.2 Define the **spring constant** of a spring. [1]

spring constant

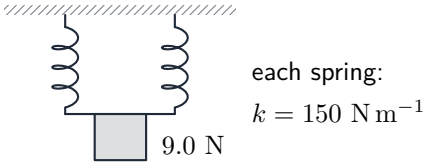
2.3 A spring of original length 12.0 cm is 15.5 cm long when it supports a load of 7.0 N. Calculate its spring constant. [2]



2.4 The spring in **2.3** now supports a load of 4.0 N. Calculate its length. [2]



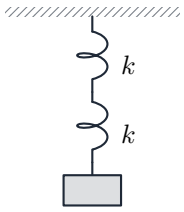
2.5 Two identical springs, each of spring constant 150 N m^{-1}, hang side by side and together support a load of 9.0 N. Calculate the extension of the springs. [2]



3 Exam-style questions

3.1 Two identical springs, each of spring constant k, are joined end to end in series. What is the spring constant of the combination? [1]

- [ ] **A** $\frac{k}{2}$
- [ ] **B** k
- [ ] **C** $2k$
- [ ] **D** $4k$



3.2 What is meant by the **limit of proportionality** of a spring? [1]

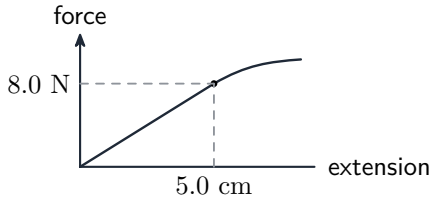
- [ ] **A** the largest load for which the spring returns to its original length
- [ ] **B** the point beyond which the extension is no longer proportional to the load
- [ ] **C** the load at which the spring breaks
- [ ] **D** the extension at which the spring constant is greatest

limit of proportionality

3.3 A student hangs loads from a spring and measures the extension. The force–extension graph is a straight line through the origin up to a load of 8.0 N and an extension of 5.0 cm, and then curves.

(a) State **Hooke's law**. [2]

(b) **Determine** the spring constant of the spring. [2]



(c) A mass of 0.45 kg hangs from the spring. Calculate the extension. [2]

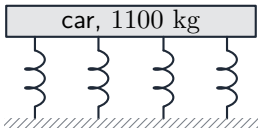
(d) The 0.45 kg mass is now hung from two of these springs joined in series. Calculate the total extension. [1]

(e) State why the answer to (c) could not be found with the same method for a load of 10 N. [1]

3.4 A spring in a car suspension has a spring constant of 3.2×10^4 N m^{-1}. Four such springs share the weight of a car of mass 1100 kg equally.

(a) State whether the springs are in tension or in compression. [1]

(b) Calculate the compression of each spring. [2]



four springs

each $k = 3.2 \times 10^4$ N m^{-1}

(c) Four passengers with a total mass of 280 kg get in. Calculate how far the car sinks. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the extension of a spring is proportional to the load (force) on it
- provided the limit of proportionality is not exceeded

2.2

- the force per unit extension, $k = F/x$

2.3

- extension $x = L - L_0 = 15.5 - 12.0 = 3.5 \text{ cm} = 0.035 \text{ m}$

$$k = \frac{F}{x} = \frac{7.0 \text{ N}}{0.035 \text{ m}} = \mathbf{200} \text{ N m}^{\mathbf{-1}}$$

2.4

- Hooke's law, $x = \dfrac{F}{k} = \dfrac{4.0}{200} = 0.020 \text{ m} = 2.0 \text{ cm}$
- length $L = L_0 + x = 12.0 + 2.0 = \mathbf{14.0}$ cm

2.5

- in parallel each spring carries 4.5 N

$$x = \frac{F}{k} = \frac{4.5 \text{ N}}{150 \text{ N m}^{-1}} = \mathbf{0.030} \text{ m}$$

3.1 A

- each spring carries the whole load, so the extensions add: twice the extension for the same force means half the spring constant
- **C** is two springs in parallel

3.2 B

- beyond the limit of proportionality the force–extension graph is no longer straight
- **A** describes the elastic limit, a different point

3.3

(a) the extension is proportional to the load

- provided the limit of proportionality is not exceeded

(b) the gradient of the straight part, $k = \dfrac{F}{x}$

$$k = \frac{F}{x} = \frac{8.0 \text{ N}}{0.050 \text{ m}} = \mathbf{160} \text{ N m}^{\mathbf{-1}}$$

(c) $F = mg = 0.45 \times 9.81 = 4.4$ N, within the straight part

$$x = \frac{F}{k} = \frac{4.4 \text{ N}}{160 \text{ N m}^{-1}} = \mathbf{0.028} \text{ m}$$

(d) in series each spring carries the whole load, so the extension doubles: **0.055** m

(e) 10 N is beyond the limit of proportionality, so the extension is no longer F/k

3.4

(a) **compression**

(b) each spring carries a quarter of the weight: $F = \dfrac{mg}{4} = \dfrac{1100 \times 9.81}{4} = 2700$ N

$$x = \frac{F}{k} = \frac{2700 \text{ N}}{3.2 \times 10^4 \text{ N m}^{-1}} = \mathbf{0.084} \text{ m}$$

(c) extra force on each spring, a quarter of the extra weight: $\Delta F = \dfrac{\Delta m\, g}{4} = \dfrac{280 \times 9.81}{4} =$ 687 N

$$\Delta x = \frac{\Delta F}{k} = \frac{687 \text{ N}}{3.2 \times 10^4 \text{ N m}^{-1}} = \mathbf{0.021} \text{ m}$$