

5.1.1 Efficiency, power and $P = Fv$

Name: _____ Class: _____ Date: _____

Total: 28 marks

KEY WORDS efficiency • power • work done • resistive force
• useful energy

Objective

Answer exam questions on **efficiency** and **power** : the efficiency of a machine, power as the rate of doing work, and the derivation and use of $P = Fv$ for vehicles on the level and on slopes. Syllabus 5.1, learning outcomes 3 to 7.

You must be able to:

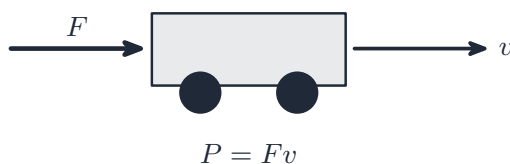
- calculate an efficiency from energies or from powers, including over several stages
- define power, and use $P = W/t$
- derive $P = Fv$ from the definitions of power and work
- use $P = Fv$ for a vehicle at a steady speed, on the level or up a slope, and when the drag grows with speed

1 Key knowledge and worked examples

■ Key knowledge: efficiency and power

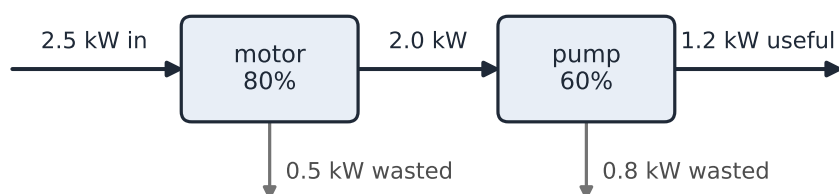
Quantity	Definition	Equation	Unit
efficiency	useful energy output ÷ total energy input	$\frac{\text{useful output}}{\text{total input}}$, as energies or as powers	none; never above 1
power	work done per unit time (rate of transfer of energy)	$P = \frac{W}{t}$	W = J s ⁻¹
power of a force	the force × the velocity along it	$P = Fv$	W

Deriving $P = Fv$ (2 marks): $P = \frac{W}{t}$ and $W = Fs$, so $P = \frac{Fs}{t}$; and $\frac{s}{t} = v$, so $P = Fv$.



Situation	Force to use in $P = Fv$
steady speed on a level road	driving force = total resistive force
steady speed up a slope at θ	driving force = resistive force + $mg \sin \theta$
lifting at a steady speed	force = weight mg , so $P = mgv$
drag $\propto v^2$	doubling the speed makes the drag $4\times$ and the power $8\times$

- efficiency over stages: multiply them, $\eta = \eta_1\eta_2$, e.g. $\eta = 0.80 \times 0.90 = 0.72$



- wasted energy = input – useful output, usually thermal energy and sound

■ Worked example: the efficiency of a pump

A pump raises 600 kg of water through 12 m in 30 s. It draws 3.0 kW.

1. **Useful power** – power is work done per unit time, the work being the gain in potential energy: $P = \frac{mg\Delta h}{t} = \frac{600 \times 9.81 \times 12}{30} = 2350 \text{ W}$.
2. **Efficiency** – efficiency = $\frac{\text{useful output}}{\text{total input}} = \frac{2350 \text{ W}}{3000 \text{ W}} = 0.78$.

■ Worked example: a car at a steady speed

A car moves at a steady $35 \frac{\text{m}}{\text{s}}$ along a level road against a total resistive force of 1750 N.

1. **Driving force** – steady speed, so no resultant force: $F = 1750 \text{ N}$.
2. **Power** – $P = Fv = 1750 \times 35 = 6.1 \times 10^4 \text{ W}$.

■ Worked example: up a slope

A car of mass 1200 kg climbs a slope at 5.0° to the horizontal at a steady $20 \frac{\text{m}}{\text{s}}$. The resistive force is 600 N.

1. **Driving force** – along the slope it balances the resistive force and the component of the weight: $F = 600 + mg \sin \theta = 600 + 1200 \times 9.81 \times \sin 5.0^\circ = 600 + 1030 = 1630 \text{ N}$.
2. **Power** – $P = Fv = 1630 \times 20 = 3.3 \times 10^4 \text{ W}$.

■ Worked example: drag that grows with speed

The resistive force on a car is proportional to the square of its speed: 800 N at $20 \frac{\text{m}}{\text{s}}$.

1. **Force at $40 \frac{\text{m}}{\text{s}}$** – $F \propto v^2$ and the speed doubles: $F = 800 \times \left(\frac{40}{20}\right)^2 = 3200 \text{ N}$.
2. **Power at a steady $40 \frac{\text{m}}{\text{s}}$** – $P = Fv = 3200 \times 40 = 1.28 \times 10^5 \text{ W}$, eight times the $1.6 \times 10^4 \text{ W}$ needed at $20 \frac{\text{m}}{\text{s}}$.

2 Practice

Now apply the methods above.

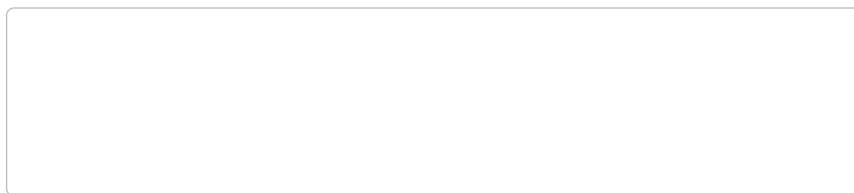
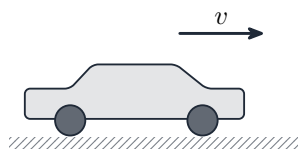
2.1 Define power.

[1]

power

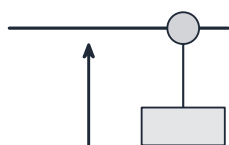
2.2 Show that the power of a force F moving an object at a constant velocity v in the direction of the force is $P = Fv$.

[2]



2.3 A motor lifts a load of mass 50 kg through 8.0 m in 10 s.

(a) Calculate the useful power output of the motor. [2]

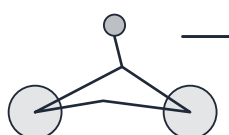


50 kg

8.0 m

(b) The input power is 500 W. Calculate the efficiency. [1]

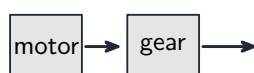
2.4 A cyclist rides at a steady $8.0 \frac{\text{m}}{\text{s}}$ along a level road against a total resistive force of 25 N. Calculate the useful power output of the cyclist. [2]



8.0 m s^{-1}

25 N

2.5 An electric motor is 80% efficient and drives a gearbox that is 90% efficient. The motor takes 5.0 kW. Calculate the useful power from the gearbox. [2]



80%

90%

3 Exam-style questions

3.1 What is the watt expressed in SI base units? [1]

☐

A $\text{kg m}^2 \text{s}^{-2}$

B $\text{kg m}^2 \text{s}^{-3}$

C kg m s^{-3}

D $\text{kg m}^2 \text{s}^{-1}$

watt

3.2 The resistive force on a boat is proportional to the square of its speed. At $15 \frac{\text{m}}{\text{s}}$ the resistive force is 600 N. What power is needed to keep the boat moving at a steady $30 \frac{\text{m}}{\text{s}}$? [1]

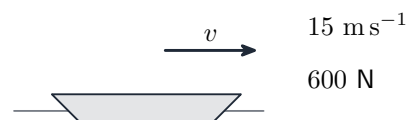
☐

A 18 kW

B 36 kW

C 72 kW

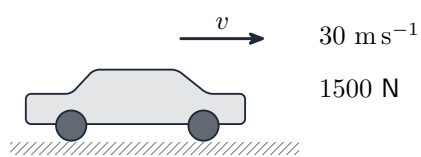
D 144 kW



3.3 An electric car travels at a constant speed of $30 \frac{\text{m}}{\text{s}}$ along a straight level road. The total resistive force on the car is 1500 N.

(a) **Define** power. [1]

(b) Calculate the power delivered to the wheels. [2]

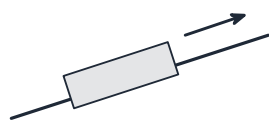


(c) Calculate the useful work done by the motor as the car travels 12 km. [2]

(d) The motor is 85% efficient. Calculate the electrical power input to the motor. [2]

3.4 A lorry of mass 1.5×10^4 kg climbs a straight road inclined at 4.0° to the horizontal, at a steady speed of $12 \frac{\text{m}}{\text{s}}$. The total resistive force on the lorry is 3.0×10^3 N.

(a) **Show that** the driving force on the lorry is about 1.3×10^4 N. [2]



1.5×10^4 kg

4.0°

(b) Calculate the useful output power of the engine. [2]

(c) The engine's efficiency is 35%. Calculate the rate at which the engine uses energy from its fuel. [2]

(d) At the top of the hill the road becomes level. The engine keeps the same output power and the resistive force stays the same. **Explain** why the lorry speeds up, and calculate its greatest possible speed on the level. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- power is the work done per unit time (the rate of transfer of energy)

2.2

- $P = \frac{W}{t}$ and $W = Fs$, so $P = \frac{Fs}{t}$
- $\frac{s}{t} = v$, so $P = Fv$

2.3

- (a) power is work done per unit time, $P = \frac{mg\Delta h}{t}$

$$P = \frac{mg\Delta h}{t} = \frac{50 \times 9.81 \times 8.0}{10} = \mathbf{390 \text{ W}}$$

- (b) efficiency = $\frac{\text{useful output}}{\text{total input}} = \frac{392}{500} = \mathbf{0.78}$

2.4

- steady speed, so the driving force equals the resistive force; $P = Fv$

$$P = Fv = 25 \text{ N} \times 8.0 \frac{\text{m}}{\text{s}} = \mathbf{200 \text{ W}}$$

2.5

- efficiencies in stages multiply: $\eta = \eta_1\eta_2 = 0.80 \times 0.90 = 0.72$
- useful power = $\eta \times P_{\text{in}} = 0.72 \times 5.0 \text{ kW} = \mathbf{3.6 \text{ kW}}$

3.1 B

- $\text{W} = \text{J s}^{-1} = \text{kg m}^2 \text{s}^{-2} \times \text{s}^{-1} = \text{kg m}^2 \text{s}^{-3}$
- A** is the joule

3.2 C

- $F \propto v^2$, so at twice the speed $F = 2^2 \times 600 = 2400 \text{ N}$
- $P = Fv = 2400 \times 30 = 72 \text{ kW}$; **B** forgets that the force grows, **D** squares the power instead

3.3

- (a) the work done per unit time

- (b) steady speed, so the driving force equals the resistive force; $P = Fv$

$$P = Fv = 1500 \text{ N} \times 30 \frac{\text{m}}{\text{s}} = \mathbf{4.5 \times 10^4 \text{ W}}$$

(c) work done, $W = Fs$

$$W = Fs = 1500 \text{ N} \times 12 \times 10^3 \text{ m} = \mathbf{1.8 \times 10^7 \text{ J}}$$

(d) efficiency = $\frac{\text{useful output}}{\text{total input}}$, so input = $\frac{\text{output}}{\text{efficiency}}$

$$P_{\text{in}} = \frac{P_{\text{out}}}{\text{efficiency}} = \frac{4.5 \times 10^4 \text{ W}}{0.85} = \mathbf{5.3 \times 10^4 \text{ W}}$$

3.4

(a) steady speed: driving force = resistive force + $mg \sin \theta$

$$mg \sin \theta = 1.5 \times 10^4 \times 9.81 \times \sin 4.0^\circ = 1.03 \times 10^4 \text{ N}$$

$$F = F_r + mg \sin \theta = 3.0 \times 10^3 + 1.03 \times 10^4 = \mathbf{1.3 \times 10^4 \text{ N}}$$

(b) $P = Fv$

$$P = Fv = 1.33 \times 10^4 \text{ N} \times 12 \frac{\text{m}}{\text{s}} = \mathbf{1.6 \times 10^5 \text{ W}}$$

(c) input = $\frac{\text{output}}{\text{efficiency}}$

$$P_{\text{in}} = \frac{P_{\text{out}}}{\text{efficiency}} = \frac{1.6 \times 10^5 \text{ W}}{0.35} = \mathbf{4.6 \times 10^5 \text{ W}}$$

(d) on the level the driving force needed is only the resistive force, so the same power gives a driving force greater than the resistive force and a resultant force forwards

- top speed when the driving force equals the resistive force: $P = Fv$, so $v = \frac{P}{F}$

$$v = \frac{P}{F} = \frac{1.6 \times 10^5 \text{ W}}{3.0 \times 10^3 \text{ N}} = \mathbf{53 \frac{\text{m}}{\text{s}}}$$