

4.3.1 Upthrust, floating and liquids in a U-tube

Name: _____ Class: _____ Date: _____

Total: **37 marks**

KEY WORDS upthrust • Archimedes' principle • displaced
 • floating • terminal velocity

Objective

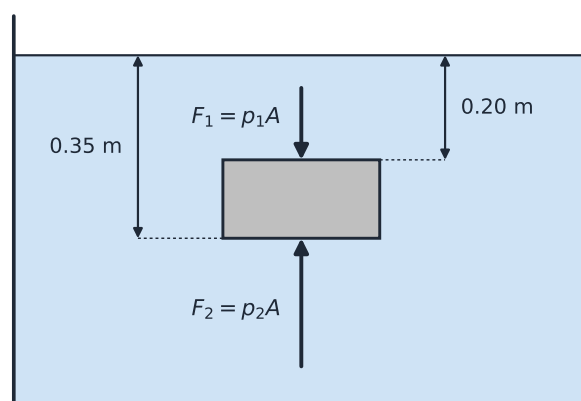
Answer exam questions on **upthrust** : where it comes from, $F = \rho g V$, floating, weighing an object in a liquid, an object falling through a liquid at terminal velocity, and a U-tube with two liquids. Syllabus 4.3, learning outcomes 5 and 6.

You must be able to:

- explain upthrust as the result of the greater hydrostatic pressure on the bottom of an object
- calculate upthrust with $F = \rho g V$, using the density of the **fluid** and the volume **displaced**
- balance the forces on an object that floats, hangs from a newton meter in a liquid, or falls at terminal velocity through a liquid
- use “same level, same pressure” in a U-tube that holds two liquids

1 Key knowledge and worked examples

■ Key knowledge: upthrust



- **origin** (2 marks): pressure increases with depth, so the pressure on the bottom face is greater than on the top face; the upward force on the bottom is greater than the downward force on the top, and the difference is the upthrust
- **Archimedes' principle**: upthrust = weight of the fluid displaced, $F = \rho_{\text{fluid}} g V_{\text{displaced}}$
- it depends on the **fluid** and on the **volume displaced**: not on the depth of a fully submerged object, not on what the object is made of; atmospheric pressure adds equally above and below, so it cancels

The situations the exam sets, each with its forces and its equation:

- **floating** – weight and upthrust: $F_U = F_W$; the fraction below the surface is $\frac{\rho_{\text{object}}}{\rho_{\text{fluid}}}$
- **hanging from a newton meter in a liquid** – tension F_T and F_U up, F_W down: $F_T + F_U = F_W$, so the upthrust is the drop in the reading
- **held under and released** – F_U up, F_W down: the resultant is $F_U - F_W$, so at the start $a = \frac{F_U - F_W}{m} = g \left(\frac{\rho_{\text{fluid}}}{\rho_{\text{object}}} - 1 \right)$
- **sinking at terminal velocity** – F_U and drag F_D up, F_W down: $F_W = F_U + F_D$, with $F_D = kv$ or $F_D = 6\pi\eta rv$
- **U-tube with two liquids** – the same pressure at the level of the boundary: $\rho_1 h_1 = \rho_2 h_2$, with the heights measured above that level

■ Worked example: where upthrust comes from

The cuboid above has top and bottom faces of area $A = 0.020 \text{ m}^2$, at depths 0.20 m and 0.35 m in water.

1. **Top face** – hydrostatic pressure, then force from $F = pA$:

$$p_1 = \rho g h_1 = 1000 \times 9.81 \times 0.20 = 1962 \text{ Pa}$$

$$F_1 = p_1 A = 1962 \times 0.020 = 39.2 \text{ N down}$$

2. **Bottom face** – the same two equations at the greater depth:

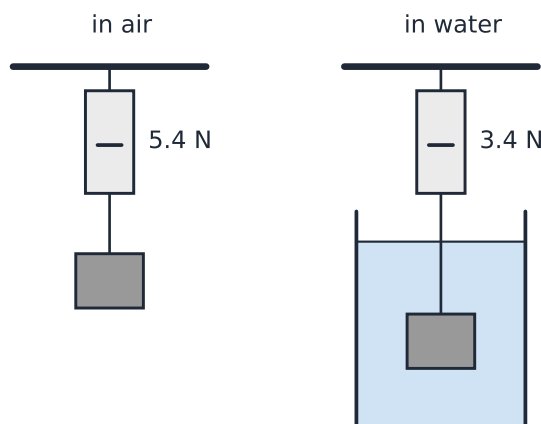
$$p_2 = \rho g h_2 = 1000 \times 9.81 \times 0.35 = 3434 \text{ Pa}$$

$$F_2 = p_2 A = 3434 \times 0.020 = 68.7 \text{ N up}$$

3. **Upthrust** – the side forces cancel: $F_U = F_2 - F_1 = 68.7 - 39.2 = 29.4 \text{ N upwards}$.
4. **Check with Archimedes** – the volume is $V = A(h_2 - h_1) = 0.020 \times 0.15 = 3.0 \times 10^{-3} \text{ m}^3$:

$$F_U = \rho g V = 1000 \times 9.81 \times 3.0 \times 10^{-3} = 29.4 \text{ N}$$

■ **Worked example: weighing a block in water**



1. **Upthrust** – equilibrium, $F_T + F_U = F_W$, so $F_U = F_W - F_T = 5.4 - 3.4 = 2.0$ N.
2. **Volume** – Archimedes, $F_U = \rho_{\text{water}} g V$: $V = \frac{F_U}{\rho g} = \frac{2.0}{1000 \times 9.81} = 2.04 \times 10^{-4} \text{ m}^3$.
3. **Density of the metal** – mass from $F_W = mg$: $m = \frac{F_W}{g} = \frac{5.4}{9.81} = 0.550$ kg,
then density:

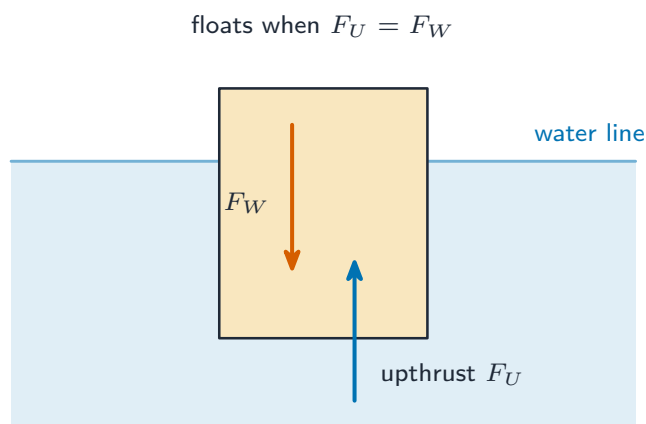
$$\rho = \frac{m}{V} = \frac{0.550 \text{ kg}}{2.04 \times 10^{-4} \text{ m}^3} = 2700 \text{ kg m}^{-3}$$

■ **Worked example: a sphere falling through oil**

A sphere of mass 1.6×10^{-3} kg and volume $2.0 \times 10^{-7} \text{ m}^3$ falls at terminal velocity through oil of density 900 kg m^{-3} . The drag is $F_D = kv$, with $k = 1.4 \times 10^{-2} \text{ kg s}^{-1}$.

1. **Weight** – $F_W = mg = 1.6 \times 10^{-3} \times 9.81 = 1.57 \times 10^{-2}$ N.
2. **Upthrust** – $F_U = \rho_{\text{oil}} g V = 900 \times 9.81 \times 2.0 \times 10^{-7} = 1.77 \times 10^{-3}$ N.
3. **Drag** – terminal velocity, the forces balance: $F_W = F_U + F_D$, so $F_D = F_W - F_U = 1.39 \times 10^{-2}$ N.
4. **Terminal speed** – from the drag model $F_D = kv$: $v = \frac{F_D}{k} = \frac{1.39 \times 10^{-2}}{1.4 \times 10^{-2}} = 0.99 \frac{\text{m}}{\text{s}}$.

■ Worked example: floating with a load



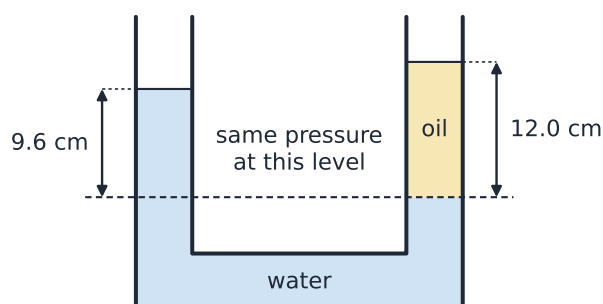
A foam raft of density 40 kg m^{-3} has a volume of 0.60 m^3 .

1. **Weight** – $m = \rho V = 40 \times 0.60 = 24 \text{ kg}$, so $F_W = mg = 235 \text{ N}$.
2. **Largest upthrust** – the raft just fully under: $F_U = \rho_{\text{water}} g V = 1000 \times 9.81 \times 0.60 = 5.89 \times 10^3 \text{ N}$.
3. **Largest load** – floating, $F_U = F_{W,\text{raft}} + F_{W,\text{load}}$, then $F_W = mg$ for its mass:

$$F_{W,\text{load}} = F_U - F_{W,\text{raft}} = 5890 - 235 = 5650 \text{ N}$$

$$m = \frac{F_{W,\text{load}}}{g} = \frac{5650 \text{ N}}{9.81 \text{ N kg}^{-1}} = 576 \text{ kg}$$

■ Worked example: two liquids in a U-tube



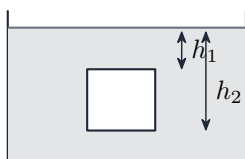
1. **Same level, same pressure** – at the level of the oil–water boundary both arms are joined by water, so the pressures there are equal; the air pressure on both tops cancels.
2. **Hydrostatic pressure on each side** – $\rho_{\text{water}} g h_{\text{water}} = \rho_{\text{oil}} g h_{\text{oil}}$; g cancels, and the heights may stay in cm:

$$\rho_{\text{oil}} = \frac{\rho_{\text{water}} h_{\text{water}}}{h_{\text{oil}}} = \frac{1000 \times 9.6}{12.0} = 800 \text{ kg m}^{-3}$$

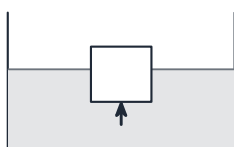
2 Practice

Now apply the methods above.

2.1 Explain why a submerged object experiences an upthrust. [2]



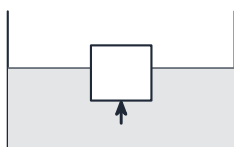
2.2 A block of volume $2.0 \times 10^{-3} \text{ m}^3$ is fully submerged in water. Calculate the upthrust on it. [2]



$2.0 \times 10^{-3} \text{ m}^3$

2.3 A stone weighs 3.6 N in air and 2.2 N when it hangs fully under water.

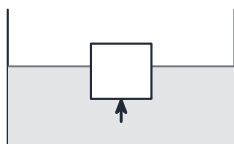
(a) Calculate the upthrust on the stone. [1]



3.6 N

2.2 N

2.4 A block of wood of density 650 kg m^{-3} floats in water. Calculate the fraction of its volume that lies below the surface. [2]



650 kg m^{-3}

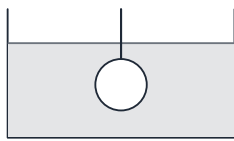
2.5 In a U-tube, a column of oil 10.0 cm high balances a column of water 8.0 cm high, both measured above the oil–water boundary. Calculate the density of the oil. [2]



10.0 cm

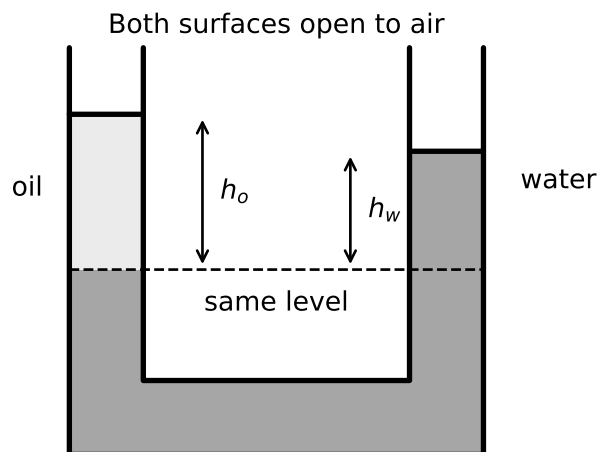
8.0 cm

2.6 A ball of density 300 kg m^{-3} is held completely under water and released. Calculate its initial acceleration. [2]



300 kg m^{-3}

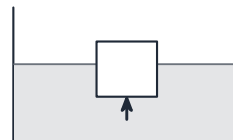
2.7 Both arms of a U-tube are open to air. A 0.15 m column of oil of density 820 kg/m^3 sits above water. The water density is 1000 kg/m^3 . Find the water-surface height above the oil–water boundary and the difference between the free-surface heights. [4]



3 Exam-style questions

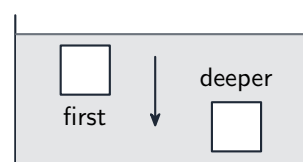
3.1 An object floats in water with three quarters of its volume below the surface. What is the density of the object? [1]

- ☐ **A** 250 kg m^{-3}
B 750 kg m^{-3}
C 1000 kg m^{-3}
D 1330 kg m^{-3}

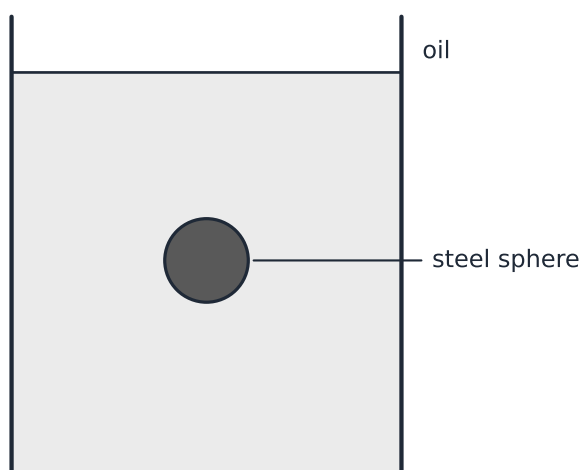


3.2 A solid cube is held completely under the surface of a liquid of uniform density. It is then pushed deeper into the liquid. What happens to the upthrust on the cube? [1]

- ☐ **A** It increases, because the pressure increases with depth.
B It decreases, because less liquid is above the cube.
C It stays the same, because the difference in pressure between the top and the bottom stays the same.
D It becomes zero when the cube reaches the bottom.



3.3 A small steel sphere of radius 3.0 mm falls vertically through a tall jar of oil of density 900 kg m^{-3} . The density of steel is 7800 kg m^{-3} .



(a) **Explain** how the pressure in the oil gives rise to an upthrust on the sphere. [2]

(b) On the diagram, **draw** labelled arrows to show the **three** forces acting on the sphere as it falls. [3]

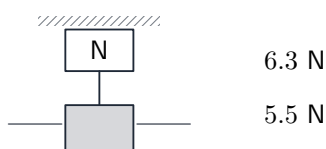
(c) The volume of a sphere is $\frac{4}{3}\pi r^3$. **Show that** the upthrust on the sphere is about 1.0×10^{-3} N. [2]

(d) The drag on the sphere is $F_D = 6\pi\eta rv$, with $\eta = 0.20 \text{ kg m}^{-1} \text{ s}^{-1}$. **Determine** the terminal speed of the sphere. [3]

3.4 A metal cylinder hangs from a newton meter. The reading is 6.3 N in air and 5.5 N when the cylinder is fully submerged in water.

(a) State **Archimedes' principle**. [1]

(b) **Determine** the volume of the cylinder. [2]



(c) **Determine** the density of the metal. [2]

(d) The cylinder is now fully submerged in a salt solution of density 1150 kg m^{-3} instead. Calculate the new reading of the newton meter. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the pressure in a fluid increases with depth, so the pressure on the bottom of the object is greater than on its top
- the upward force on the bottom is greater than the downward force on the top, and the resultant is upwards

2.2

- Archimedes' principle, $F = \rho g V$

$$F = \rho g V = 1000 \text{ kg m}^{-3} \times 9.81 \text{ m s}^{-2} \times 2.0 \times 10^{-3} \text{ m}^3 = \mathbf{20 \text{ N}}$$

2.3

(a) equilibrium, $F_T + F_U = F_W$: $F_U = F_W - F_T = 3.6 - 2.2 = \mathbf{1.4 \text{ N}}$

(b) Archimedes, $V = \frac{F_U}{\rho g} = \frac{1.4}{1000 \times 9.81} = 1.43 \times 10^{-4} \text{ m}^3$

- mass from $F_W = mg$: $m = \frac{F_W}{g} = \frac{3.6}{9.81} = 0.367 \text{ kg}$

$$\rho = \frac{m}{V} = \frac{0.367 \text{ kg}}{1.43 \times 10^{-4} \text{ m}^3} = \mathbf{2.6 \times 10^3 \text{ kg m}^{-3}}$$

2.4

- floating, $F_U = F_W$: $\rho_{\text{water}} V_{\text{below}} g = \rho_{\text{wood}} V g$
- fraction below = $\frac{\rho_{\text{wood}}}{\rho_{\text{water}}} = \frac{650}{1000} = \mathbf{0.65}$

2.5

- same pressure at the boundary level: $\rho_{\text{oil}} g h_{\text{oil}} = \rho_{\text{water}} g h_{\text{water}}$

$$\rho_{\text{oil}} = \frac{\rho_{\text{water}} h_{\text{water}}}{h_{\text{oil}}} = \frac{1000 \times 8.0}{10.0} = \mathbf{800 \text{ kg m}^{-3}}$$

2.6

- resultant force $F_U - F_W$, with $F_U = \rho_{\text{water}} V g$ and $m = \rho_{\text{ball}} V$

$$a = g \left(\frac{\rho_{\text{water}}}{\rho_{\text{ball}}} - 1 \right) = 9.81 \times \left(\frac{1000}{300} - 1 \right) = \mathbf{23 \text{ m s}^{-2} \text{ upwards}}$$

2.7

- At the common level, $\rho_o g h_o = \rho_w g h_w$.
- $h_w = \rho_o h_o / \rho_w = 820 \times 0.15 / 1000 = 0.123 \text{ m}$.
- Surface difference $\Delta h = h_o - h_w = 0.15 - 0.123 = 0.027 \text{ m}$.
- The oil surface is higher: the less dense liquid needs a taller column for the same pressure.

3.1 B

- floating: fraction below = $\rho_{\text{object}}/\rho_{\text{water}}$, so $\rho_{\text{object}} = (\text{fraction below}) \times \rho_{\text{water}} = 0.75 \times 1000 = 750 \text{ kg m}^{-3}$
- **D** inverts the fraction; **A** uses the fraction above the surface

3.2 C

- the pressure at the top and at the bottom both increase by the same amount, so their difference, and the upthrust $\rho g V$, is unchanged
- **A** confuses the pressure with the pressure difference

3.3

(a) the pressure in the oil increases with depth, so it is greater on the bottom of the sphere than on the top

- the resultant force of the oil on the sphere is therefore upwards: the upthrust

(b) weight, vertically **downwards**

- upthrust, vertically **upwards**
- drag (viscous force), vertically **upwards**

(c) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3.0 \times 10^{-3})^3 = 1.13 \times 10^{-7} \text{ m}^3$

$$F_U = \rho_{\text{oil}} g V = 900 \times 9.81 \times 1.13 \times 10^{-7} = \mathbf{1.0 \times 10^{-3} \text{ N}}$$

(d) weight $F_W = \rho_{\text{steel}} V g = 7800 \times 1.13 \times 10^{-7} \times 9.81 = 8.7 \times 10^{-3} \text{ N}$

- terminal velocity, $F_W = F_U + F_D$: $F_D = F_W - F_U = 8.7 \times 10^{-3} - 1.0 \times 10^{-3} = 7.7 \times 10^{-3} \text{ N}$

$$v = \frac{F_D}{6\pi\eta r} = \frac{7.7 \times 10^{-3}}{6\pi \times 0.20 \times 3.0 \times 10^{-3}} = \mathbf{0.68 \frac{m}{s}}$$

3.4

(a) the upthrust on an object in a fluid equals the **weight of the fluid it displaces**

(b) the reading falls by the upthrust, $F_U = F_W - F_T = 6.3 - 5.5 = 0.80 \text{ N}$; Archimedes, $F_U = \rho g V$

$$V = \frac{F_U}{\rho g} = \frac{0.80}{1000 \times 9.81} = \mathbf{8.2 \times 10^{-5} \text{ m}^3}$$

(c) $m = \frac{F_W}{g} = \frac{6.3}{9.81} = 0.642 \text{ kg}$, then $\rho = \frac{m}{V}$

$$\rho = \frac{m}{V} = \frac{0.642}{8.15 \times 10^{-5}} = \mathbf{7.9 \times 10^3 \text{ kg m}^{-3}}$$

(d) new upthrust $F_U = \rho g V = 1150 \times 9.81 \times 8.15 \times 10^{-5} = 0.92 \text{ N}$

- reading $F_T = F_W - F_U = 6.3 - 0.92 = \mathbf{5.4 \text{ N}}$