4.2 Equilibrium, the principle of moments and vector triangles

Name: _____ Class: _____ Date: _____ **Total: 44 marks**

KEY WORDS equilibrium • principle of moments • vector triangle • coplanar • resultant force

Objective

Answer exam questions on **equilibrium** : the **principle of moments** , the two conditions for equilibrium, and the **vector triangle** of three forces. Syllabus 4.2, learning outcomes 1, 2 and 3.

You must be able to:

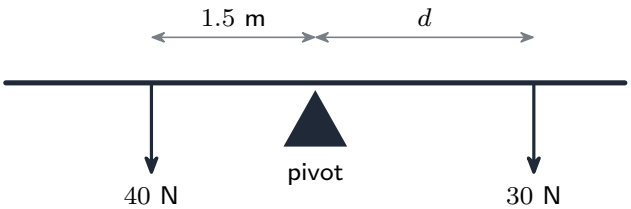
- state the principle of moments and the two conditions for equilibrium
- choose the point to take moments about, so that an unknown force drops out
- find the support forces on a beam, and the force from a hinge, including its direction
- solve three forces in equilibrium with a closed vector triangle or by resolving

1 Key knowledge and worked examples

■ Key knowledge: equilibrium

Statement	In the examiner's words
principle of moments (2 marks)	for an object in equilibrium, the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about the same point
equilibrium	the resultant force is zero **and** the resultant moment (torque) is zero
three **coplanar** forces in equilibrium	drawn tip to tail they form a **closed triangle**; their lines of action meet at one point

Method step	Why
draw every force on the object, the weight at the centre of gravity	a missing weight is a missing moment
take moments about the point where an **unknown** force acts	that force has zero moment there and drops out
then resolve: total up = total down, total left = total right	this gives the last unknown force
a hinge force may point **down**	a support between the hinge and the load can only balance if the hinge pulls down



A beam balanced on its centre: its own weight has no moment about the pivot

■ Worked example: balancing a beam

In the figure above the beam is uniform, so its weight acts at the pivot. A 40 N weight hangs 1.5 m to the left; where must a 30 N weight hang?

1. **Principle of moments** about the pivot – clockwise moment = anticlockwise moment, each $M = Fd$: $F_{\text{cw}}\, d = F_{\text{acw}}\, d_{\text{acw}}$.
2. **Distance** – rearrange for d:

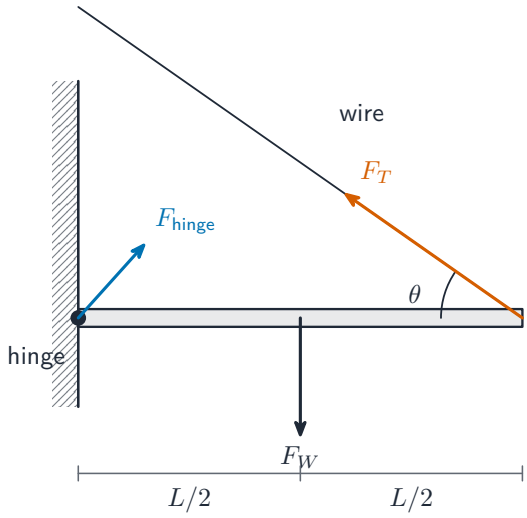
$$d = \frac{F_{\text{acw}}\, d_{\text{acw}}}{F_{\text{cw}}} = \frac{40\ \text{N} \times 1.5\ \text{m}}{30\ \text{N}} = 2.0\ \text{m to the right}$$

■ Worked example: a plank on two supports

A uniform plank of length 4.0 m and weight 200 N rests on supports at its ends A and B. A person of weight 500 N stands 1.0 m from A.

1. **Moments about A** – the force at A drops out; principle of moments, $\sum M_{\text{cw}} = \sum M_{\text{acw}}$ with $M = Fd$: $F_{N,\text{B}} \times 4.0 = 200 \times 2.0 + 500 \times 1.0 = 900$ N m, so $F_{N,\text{B}} = 225$ N.
2. **Resolve vertically** – no resultant force, $F_{N,\text{A}} + F_{N,\text{B}} = F_{W,\text{plank}} + F_{W,\text{person}}$, so $F_{N,\text{A}} = F_{W,\text{plank}} + F_{W,\text{person}} - F_{N,\text{B}} = 200 + 500 - 225 = 475$ N.

Worked example: a rod held by a wire



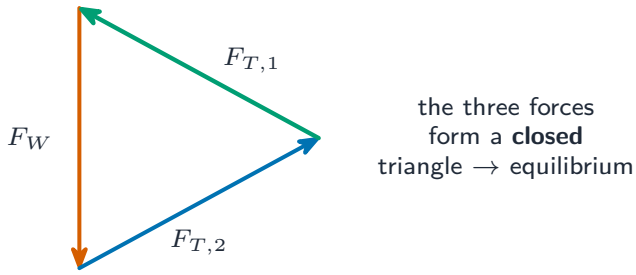
A uniform rod of length 1.2 m and weight 40 N is hinged to a wall and held horizontal by a wire from its far end, at 30° to the rod.

1. **Moments about the hinge** – only the component of the tension perpendicular to the rod, $F_T \sin 30°$, has a moment: $F_T \sin 30° \times 1.2 = 40 \times 0.60$, so $F_T = 40$ N.
2. **Hinge force, horizontal** – resolve horizontally: the wire pulls the rod towards the wall with $F_T \cos 30° = 35$ N, so the hinge pushes it away from the wall with $F_H = 35$ N.
3. **Hinge force, vertical** – resolve vertically: $F_V + F_T \sin 30° = F_W$, so $F_V = F_W - F_T \sin 30° = 40 - 20 = 20$ N upwards.

Worked example: a sign on two wires

A sign of weight 250 N hangs from a horizontal wire and a second wire at 40° to the vertical.

1. **Vertical** – resolve first where there is one unknown: $F_T \cos 40° = F_W$, so $F_T = \dfrac{F_W}{\cos 40°} = \dfrac{250}{\cos 40°} = 326$ N.
2. **Horizontal** – $F = F_T \sin 40° = 326 \times 0.643 = 210$ N in the horizontal wire.
3. **Check** – a closed triangle of the three forces, drawn to scale, gives the same two answers.

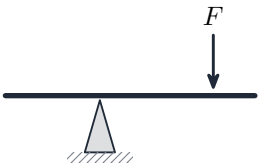


Three forces in equilibrium: tip to tail, a closed triangle

2 Practice

Now apply the methods above.

2.1 State the **principle of moments**. [2]



2.2 State the **two** conditions for an object to be in equilibrium. [2]

equilibrium

2.3 A uniform metre rule balances at its centre. A 0.60 N weight hangs 0.20 m to the left of the centre. Calculate where a 0.40 N weight must hang to balance it. [2]

0.60 N
0.20 m
0.40 N

2.4 A uniform shelf of length 1.0 m and weight 30 N rests on supports at its ends. A 50 N box stands 0.30 m from the left-hand end. Calculate the force from each support. [3]

1.0 m
30 N
50 N
0.30 m

2.5 A 20 N lamp hangs from two strings, each at 30° to the vertical. Calculate the tension in each string. [2]

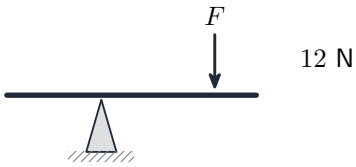
3 Exam-style questions

3.1 Three coplanar forces act on an object that is in equilibrium. Which statement **must** be true? [1]

- [ ] **A** The three forces are equal in magnitude.
B Drawn tip to tail, the three forces form a closed triangle.
C The three forces are parallel.
D The resultant of two of the forces is equal in magnitude and direction to the third.

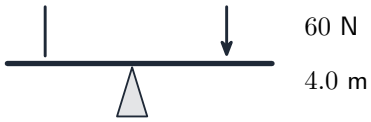
3.2 A uniform rod of weight 12 N is supported at one end by a pivot and held horizontal by a vertical string attached to its other end. What is the tension in the string? [1]

- [ ] **A** 3.0 N
B 6.0 N
C 12 N
D 24 N



3.3 A uniform beam of weight 60 N and length 4.0 m rests on supports A, at the left-hand end, and B, at the right-hand end. A 40 N load sits 1.0 m from A.

(a) **Show that** the upward force from support B is 40 N. [3]



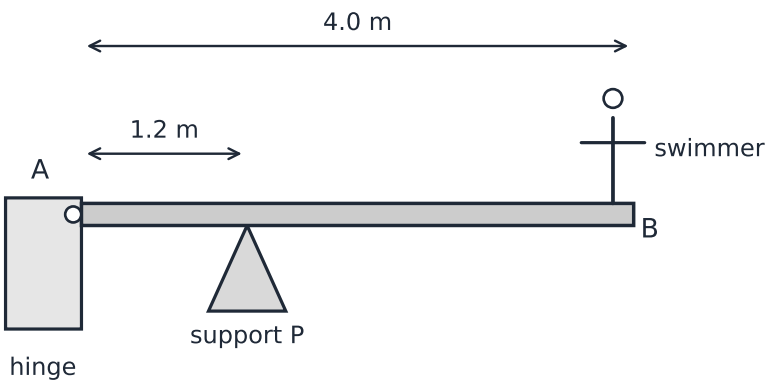
(b) Calculate the upward force from support A. [1]

3.4 A sign of weight 180 N hangs from two cables. Cable 1 makes an angle of 30° with the vertical and cable 2 an angle of 60° with the vertical, on the other side.

(a) State the two conditions for the sign to be in equilibrium. [2]

(b) By resolving horizontally and vertically, **determine** the tension in each cable. [4]

3.5 A uniform diving board AB has length 4.0 m and weight 800 N. It is hinged to a wall at end A and rests on a support P a distance 1.2 m from A. A swimmer of weight 620 N stands at end B, and the board is in equilibrium.



(a) **State** the principle of moments. [2]

(b) On the diagram above, **draw** arrows showing the **weight of the board**, the **force from the support P** and the **weight of the swimmer**. **Label** each arrow. [3]

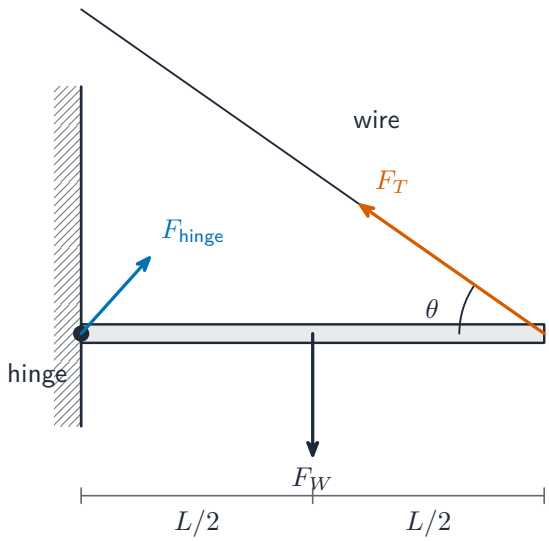
(c) By taking moments about A, **determine** the force that the support P exerts on the board. [3]

(d) **Determine** the magnitude and the direction of the force that the hinge at A exerts on the board. [3]

(e) **Explain** why taking moments about A is a better choice than taking them about P. [1]

(f) The swimmer now walks slowly back towards A. **Give** the distance from A at which she must stand for the hinge to exert **no** force on the board. [3]

3.6 A uniform horizontal beam is hinged at its left end. Its weight is 90 N and its length is 2.0 m. A wire from the right end pulls at 30° above the beam. Calculate the tension and the horizontal and vertical components of the hinge force. [6]



Solutions

Each answer shows the steps, then the **result**.

2.1

- for an object in equilibrium
- the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about the same point

2.2

- the resultant force is zero
- the resultant moment (torque) is zero

2.3

- principle of moments about the centre, $F_1 d_1 = F_2 d_2$: $0.40 \times d = 0.60 \times 0.20$

$$d = \frac{F_2 d_2}{F_1} = \frac{0.60 \text{ N} \times 0.20 \text{ m}}{0.40 \text{ N}} = \mathbf{0.30} \text{ m to the right of the centre}$$

2.4

- moments about the left-hand end: $F_{N,\text{right}} \times 1.0 = 30 \times 0.50 + 50 \times 0.30 = 30$ N m, so $F_{N,\text{right}} = \mathbf{30}$ N
- resolving vertically, $F_{N,\text{left}} + F_{N,\text{right}} = F_{W,\text{shelf}} + F_{W,\text{box}}$: $F_{N,\text{left}} = 30 + 50 - 30 = \mathbf{50}$ N

2.5

- resolving vertically: $2F_T \cos 30° = F_W$

$$F_T = \frac{F_W}{2\cos 30°} = \frac{20 \text{ N}}{2\cos 30°} = \mathbf{12} \text{ N}$$

3.1 B

- no resultant force means the three force vectors, added tip to tail, return to the start: a closed triangle
- **D** has the direction wrong: the resultant of two is equal and **opposite** to the third

3.2 B

- moments about the pivot: $F_T \times L = 12 \times \frac{L}{2}$, so $F_T = 6.0$ N
- the pivot supplies the other 6.0 N; **C** forgets the pivot takes a share

3.3

(a) take moments about A so that the force at A drops out

- principle of moments: $F_{N,\text{B}} \times 4.0 = 60 \times 2.0 + 40 \times 1.0 = 160$ N m

$$F_{N,\text{B}} = \frac{M}{d} = \frac{160 \text{ N m}}{4.0 \text{ m}} = \mathbf{40} \text{ N}$$

(b) resolving vertically, $F_{N,\text{A}} + F_{N,\text{B}} = F_{W,\text{beam}} + F_{W,\text{load}}$: $F_{N,\text{A}} = 60 + 40 - 40 = \mathbf{60}$ N

3.4

(a) the resultant force on the sign is zero

- the resultant moment on the sign is zero

(b) horizontally: $F_{T,1} \sin 30° = F_{T,2} \sin 60°$, so $F_{T,1} = \sqrt{3}\, F_{T,2}$

- vertically: $F_{T,1} \cos 30° + F_{T,2} \cos 60° = 180$
- substituting: $\sqrt{3}\, F_{T,2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} F_{T,2} = 2F_{T,2} = 180$, so $F_{T,2} = \mathbf{90}$ N
- $F_{T,1} = \sqrt{3}\, F_{T,2} = \sqrt{3} \times 90 = \mathbf{156}$ N

3.5

(a) for a body in **equilibrium**

- sum of **clockwise moments** about any point = sum of **anticlockwise moments** about that same point

(b)

- arrow **down** at the middle of the board (2.0 m from A) labelled *weight of board / 800 N*
- arrow **up** at P labelled *support force*
- arrow **down** at B labelled *weight of swimmer / 620 N*

(c) moments about A (hinge force has no moment)

- clockwise $\sum M = F_{W,1} d_1 + F_{W,2} d_2 = (800\text{ N})(2.0\text{ m}) + (620\text{ N})(4.0\text{ m}) = 4080\text{ N m}$
- anticlockwise $= F_P \times 1.2$ m

$$F_P = \frac{\sum M}{d} = \frac{4080\text{ N m}}{1.2\text{ m}} = \mathbf{3400}\text{ N}$$

(d) vertical equilibrium: up = down

- $\sum F_{\text{up}} = \sum F_{\text{down}}$: $3400\text{ N} + F_{\text{hinge}} = 800\text{ N} + 620\text{ N}$
- $F_{\text{hinge}} = -1980$ N, so **1980** N **downwards**

(e) the hinge force is unknown; moments **about** A give it zero moment, so it drops out

(f) no hinge: take moments about **P**

- board's weight is $d = 2.0\text{ m} - 1.2\text{ m} = 0.80$ m to the right of P
- clockwise: $M = Fd = (800\text{ N})(0.80\text{ m}) = 640\text{ N m}$
- swimmer must be on the **other** side of P, a distance $(1.2\text{ m} - x)$ from it

$$\sum M_{\text{anticlockwise}} = \sum M_{\text{clockwise}}: \quad (620\text{ N})(1.2\text{ m} - x) = 640\text{ N m} \Rightarrow x = \mathbf{0.17}\text{ m}$$

from A

3.6

- Take moments about the hinge: the hinge force has zero moment. $\sum M_{\text{cw}} = \sum M_{\text{acw}}$, so $F_W L/2 = F_T \sin\theta\, L$.
- $F_T = F_W/(2\sin\theta) = 90/(2\sin 30°) = 90$ N.
- Horizontal balance: $F_H = F_T \cos\theta = 90\cos 30° = 78$ N, away from the wall.
- Vertical balance: $F_V = F_W - F_T \sin\theta = 90 - 90\sin 30° = 45$ N upwards.