4.1.1 Centre of gravity, moments at an angle, non-uniform objects

Name: _____ Class: _____ Date: _____ **Total: 33 marks**

KEY WORDS centre of gravity • moment • uniform • perpendicular distance • torque

Objective

Answer the harder moment questions: the **centre of gravity** and how to find it, a **moment of a force at an angle**, the moment of the weight of a tilted object, and objects that are **not uniform**. Syllabus 4.1, learning outcome 1, with outcomes 2 to 4 applied.

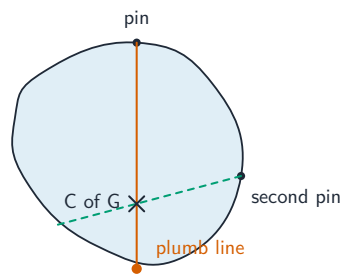
You must be able to:

- define the centre of gravity, and describe how to find it for an irregular flat sheet
- calculate the moment of a force at an angle, by the perpendicular distance or by resolving the force
- find the moment of the weight of a tilted object, such as a trapdoor, about its hinge
- find the position of the centre of gravity, or the weight, of a non-uniform object from its moments

1 Key knowledge and worked examples

Key knowledge: the centre of gravity

- **centre of gravity**: the point at which the **whole weight** of an object may be taken to act
- a **uniform** object: at its geometric centre (the middle of a rod, the centre of a disc); a **non-uniform** object: nearer its heavier end
- an object hanging freely comes to rest with its centre of gravity **vertically below** the point of suspension; otherwise its weight has a moment about that point and it swings

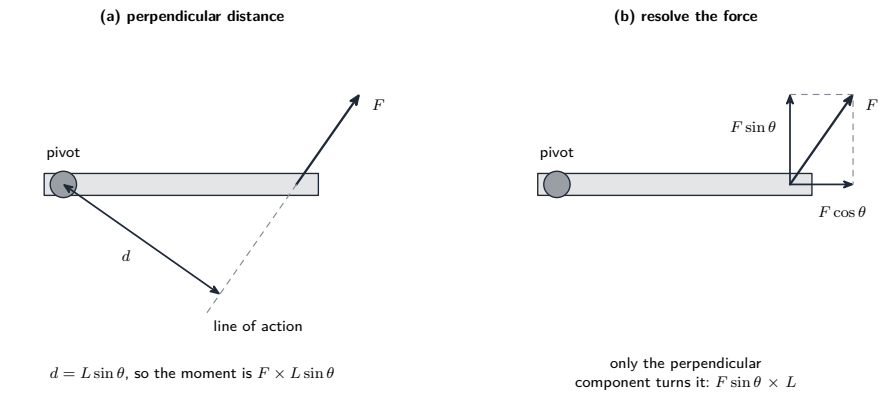


hanging freely, the sheet settles with its centre of gravity vertically below the pin

hang it from a second point: the two plumb lines cross at the centre of gravity

Finding it for an irregular sheet: (1) hang it from a pin near its edge so it swings freely; (2) hang a plumb line from the pin and draw the vertical line; (3) repeat from a second hole; (4) the centre of gravity is where the lines cross; a third hole checks it.

Key knowledge: a moment at an angle



The two routes give the same answer, and either earns full marks. The component *along* the bar, $F\cos\theta$, has a line of action passing **through the pivot**, so its perpendicular distance is zero and it produces no turning effect at all.

A force F at an angle θ to a bar, a distance L from the pivot:

Route	What to do	Moment
perpendicular distance	extend the line of action; the perpendicular from the pivot to it is $d = L\sin\theta$	$FL\sin\theta$
resolve the force	only the component $F\sin\theta$ **perpendicular** to the bar turns it; $F\cos\theta$ along the bar passes through the pivot	$F\sin\theta \times L$

- a tilted uniform object: its weight acts **vertically** through its centre, so about

the hinge the perpendicular distance is $\frac{L}{2}\cos\theta$ for an object at θ to the horizontal; equivalently, use the component of the weight perpendicular to the object, $F_W \cos\theta$, at a distance $\frac{L}{2}$

■ Worked example: a spanner pulled at an angle

A force of 50 N acts at the end of a spanner 0.40 m long, at 30° to the spanner.

1. **By the perpendicular distance** – $d = L\sin\theta = 0.40\sin 30° = 0.20$ m; moment of a force, $M = Fd = 50 \times 0.20 = 10$ N m.
2. **By resolving** – perpendicular component $F\sin\theta = 50\sin 30° = 25$ N; $M = (F\sin\theta)L = 25 \times 0.40 = 10$ N m. Both routes give the same moment.

■ Worked example: a trapdoor

A uniform trapdoor of length 0.80 m and weight 75 N is hinged at one end and held open at 30° to the horizontal by a force F at its other end, perpendicular to the door.

1. **Component of the weight perpendicular to the door** – $F_W \cos\theta = 75\cos 30° = 65$ N, acting at the centre, 0.40 m from the hinge.
2. **Moment of the weight about the hinge** – $M = Fd = 65 \times 0.40 = 26$ N m.
3. **The force** – the door is still, so the moments about the hinge balance: $F \times 0.80 = 26$, so $F = 32$ N.

■ Worked example: a non-uniform rod

A non-uniform rod AB, 1.00 m long, balances on a pivot 0.40 m from A. When a 2.0 N weight hangs from A, the rod balances on a pivot 0.30 m from A.

1. **Centre of gravity** – balanced with no load, so it is directly above the first pivot: 0.40 m from A.
2. **Moments about the new pivot** – moment of a force, $M = Fd$: the 2.0 N load is 0.30 m on one side, $M = Fd = 2.0 \times 0.30 = 0.60$ N m; the rod's weight F_W acts 0.10 m on the other side, $F_W \times 0.10$.
3. **Weight of the rod** – balanced, so the moments are equal: $F_W \times 0.10 = 0.60$, so $F_W = \dfrac{M}{d} = \dfrac{0.60}{0.10} = 6.0$ N.

■ Worked example: holding a non-uniform plate still

A non-uniform plate of mass 2.0 kg is fixed to a horizontal rod through a point that is not its centre of gravity. A torque of 1.6 N m applied to the rod holds it still.

1. **Moment of the weight** – $M = mgd$, where d is the horizontal distance from the rod to the centre of gravity; it must equal the applied torque.
2. **Distance** – $d = \dfrac{\tau}{mg} = \dfrac{1.6}{2.0 \times 9.81} = 0.082$ m.
3. **Direction** – if the centre of gravity is to the right of the rod, the weight turns the plate clockwise, so the applied torque is anticlockwise.

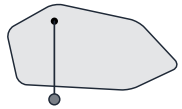
2 Practice

Now use the methods above.

2.1 Define the **centre of gravity** of an object. [1]

centre of gravity

2.2 Describe how you would find the centre of gravity of an irregularly shaped flat sheet of card. [3]



2.3 A force of 40 N acts at the end of a spanner 0.25 m long, at 60° to the spanner. Calculate its moment about the nut. [2]

40 N
0.25 m
60°

2.4 Explain why the component of a force **along** a bar has no moment about a pivot at the end of the bar. [1]

2.5 A uniform trapdoor of length 1.2 m and weight 90 N is hinged at one end and held at 40° to the horizontal. Calculate the moment of its weight about the hinge. [2]

1.2 m
90 N
40°

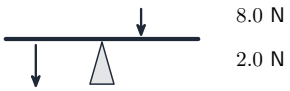
3 Exam-style questions

3.1 A force F acts at a point a distance L from a pivot, at an angle θ to the line joining the pivot to that point. What is the moment of the force about the pivot? [1]

☐ **A** FL
B $FL\sin\theta$
C $FL\cos\theta$
D $FL\tan\theta$

3.2 A non-uniform rod of weight 8.0 N is balanced horizontally on a pivot at its midpoint when a 2.0 N weight hangs 0.30 m to one side of the pivot. How far is the centre of gravity of the rod from the pivot? [1]

☐ **A** 0.075 m
B 0.12 m
C 0.30 m
D 1.2 m

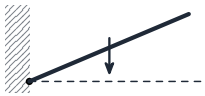


3.3 A uniform trapdoor of length 0.90 m and weight 120 N is hinged at end H. It is held open at 35° to the horizontal by a rope attached to its other end.

(a) **Define** the moment of a force. [2]

(b) Calculate the component of the weight of the trapdoor that is perpendicular to the trapdoor. [1]

(c) Calculate the moment of the weight of the trapdoor about H. [2]



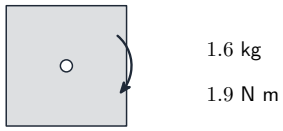
(d) The rope pulls perpendicular to the trapdoor. Calculate the tension in the rope. [2]

(e) The rope is now moved so that it pulls **vertically** upwards on the same end. State and explain whether the tension needed is greater than, equal to or less than in (d). [2]

3.4 A square sheet of **non-uniform** metal has a mass of 1.6 kg. A thin rod through the centre of the sheet holds it in a vertical plane. To stop the sheet turning, a torque of 1.9 N m must be applied to the rod.

(a) **Define** the centre of gravity. [1]

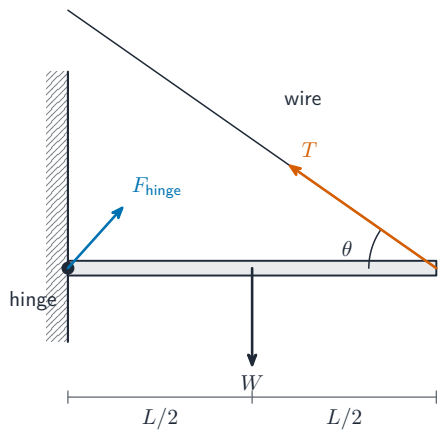
(b) **Show that** the centre of gravity of the sheet is a horizontal distance of about 0.12 m from the rod. [2]



(c) The centre of gravity is to the left of the rod. **State and explain** whether the torque applied to the rod is clockwise or anticlockwise. [2]

(d) The rod is removed and the sheet hangs freely from one corner. **State** where its centre of gravity lies relative to that corner, and explain why. [2]

3.5 A uniform horizontal beam is hinged at its left end. Its weight is 90 N and its length is 2.0 m. A wire from the right end pulls at 30° above the beam. Calculate the tension and the horizontal and vertical components of the hinge force. [6]



Solutions

Each answer shows the steps, then the **result**.

2.1

- the point at which the **whole weight** of the object may be taken to act

2.2

- hang the sheet from a pin through a hole near its edge so that it swings freely
- hang a plumb line from the pin and mark the vertical line on the sheet
- repeat from a second hole: the centre of gravity is where the two lines cross

2.3

- moment of a force, $M = Fd$ with $d = L\sin\theta$

$$M = FL\sin\theta = 40\ \text{N} \times 0.25\ \text{m} \times \sin 60^\circ = \mathbf{8.7}\ \text{N m}$$

2.4

- its line of action passes through the pivot, so its perpendicular distance from the pivot, and its moment, are zero

2.5

- the weight acts vertically at the centre; perpendicular distance from the hinge $d = \frac{L}{2}\cos\theta = 0.60\cos 40^\circ = 0.46$ m

$$M = F_W d = 90\ \text{N} \times 0.46\ \text{m} = \mathbf{41}\ \text{N m}$$

3.1 B

- the perpendicular distance from the pivot to the line of action is $L\sin\theta$
- **C** takes the component along the bar, which passes through the pivot

3.2 A

- balance: $8.0 \times d = 2.0 \times 0.30$, so $d = 0.075$ m, on the other side of the pivot from the load
- **D** divides the wrong way round; **C** is the distance of the load

3.3

(a) the force multiplied by the perpendicular distance

- from the point to the line of action of the force

(b) $F_W\cos\theta = 120\cos 35^\circ = \mathbf{98}$ N

(c) moment of a force, $M = Fd$, with the perpendicular component at the centre, 0.45 m from H

$$M = Fd = 98\ \text{N} \times 0.45\ \text{m} = \mathbf{44}\ \text{N m}$$

(d) the trapdoor is still, so the moments about H balance: $F_T \times 0.90 = 44$

$$F_T = \frac{M}{d} = \frac{44\ \text{N m}}{0.90\ \text{m}} = \mathbf{49}\ \text{N}$$

(e) **greater**

- a vertical rope has a smaller perpendicular distance from H ($0.90\cos 35^\circ$), so it needs a larger force for the same moment

3.4

(a) the point at which the whole weight of the sheet may be taken to act

(b) the moment of the weight balances the applied torque: $mgd = \tau$

$$d = \frac{\tau}{mg} = \frac{1.9\ \text{N m}}{1.6\ \text{kg} \times 9.81\ \text{m s}^{-2}} = \mathbf{0.12}\ \text{m}$$

(c) the weight, to the left of the rod, turns the sheet **anticlockwise**

- so the applied torque must be **clockwise** to balance it

(d) **vertically below** the corner

- in equilibrium the weight has no moment about the point of suspension, so its line of action passes through that point

3.5

- Take moments about the hinge: the hinge force has zero moment. $\sum M_{\text{cw}} = \sum M_{\text{acw}}$, so $F_W L/2 = F_T \sin\theta\, L$.
- $F_T = F_W/(2\sin\theta) = 90/(2\sin 30^\circ) = 90$ N.
- Horizontal balance: $F_H = F_T\cos\theta = 90\cos 30^\circ = 78$ N, away from the wall.
- Vertical balance: $F_V = F_W - F_T\sin\theta = 90 - 90\sin 30^\circ = 45$ N upwards.