

3.3 Conservation of momentum in one dimension

Name: _____ Class: _____ Date: _____ Total: 34 marks

KEY WORDS conservation of momentum • elastic collision
• inelastic collision • kinetic energy • alpha particle α

Objective

Answer exam questions on **conservation of momentum** along a straight line: collisions in which objects bounce apart or stick together, explosions and decays, and what happens to the kinetic energy. Syllabus 3.3, learning outcomes 1, 2 and 4, and the meaning of an elastic collision from outcome 3.

You must be able to:

- state the principle of conservation of momentum in the examiner’s words
- apply it to a collision along a line, with a sign for every velocity, when the objects bounce apart or stick together
- apply it to an explosion or a decay that starts from rest
- calculate the kinetic energy before and after, and the percentage transferred to other forms
- decide whether a collision is **elastic** or **inelastic**, and read a collision from a velocity–time graph

1 Key knowledge and worked examples

■ Key knowledge: the principle

- statement** (2 marks): the total momentum of a system of objects is constant, provided **no resultant external force** acts on the system; in other words, total momentum before = total momentum after, for an isolated system
- why**: by Newton’s third law, the two objects push each other with equal and opposite forces for the same time, so their changes in momentum are equal and opposite, $\Delta p_A = -\Delta p_B$
- momentum is a **vector**: choose a positive direction and give **every velocity a sign**

For an isolated system:

Interaction	Total momentum	Total kinetic energy	Total energy
Elastic collision	Conserved	Conserved	Conserved
Inelastic collision	Conserved	Decreases: thermal energy, sound, energy in deformation	Conserved
Objects stick together	Conserved	Greatest decrease for the same masses and initial velocities	Conserved
Explosion or decay from rest	Conserved: stays zero	Increases: chemical or nuclear energy is released	Conserved

- Sticking together is a **perfectly inelastic collision**.
- for a head-on **elastic** collision, the **relative speed of approach equals the relative speed of separation**; sheet 3.3.1 uses this as a second equation

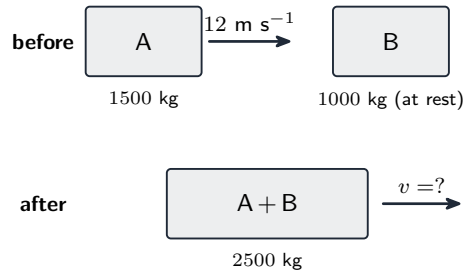
■ Key knowledge: the equations

- bounce apart**: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
- stick together**: $m_1u_1 + m_2u_2 = (m_1 + m_2)v$
- explosion or decay from rest**: $0 = m_1v_1 + m_2v_2$, so the pieces move off in **opposite** directions, the lighter one faster
- kinetic energy**: $E_k = \frac{1}{2}mv^2$; **percentage transferred** = $\frac{E_{k,before} - E_{k,after}}{E_{k,before}} \times 100$

■ Key knowledge: decays

- an **α -particle** α is a helium nucleus, ${}^4_2\alpha$; in a decay equation the top numbers (nucleon numbers) balance, and so do the bottom numbers (proton numbers): ${}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^4_2\alpha$
- a nuclear mass may stay in **u** (unified atomic mass units): it appears on both sides of the momentum equation and cancels

■ Worked example: two cars lock together



1. **Common velocity** – conservation of momentum, objects stick together, right positive: $m_1u_1 + m_2u_2 = (m_1 + m_2)v$, so

$$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{1500 \times 12 + 0}{1500 + 1000} = 7.2 \frac{\text{m}}{\text{s}} \text{ to the right}$$

2. **Kinetic energy** – $E_k = \frac{1}{2}mv^2$, before and after:

$$E_{\text{before}} = \frac{1}{2}m_1u_1^2 = \frac{1}{2} \times 1500 \times 12^2 = 1.08 \times 10^5 \text{ J}$$

$$E_{\text{after}} = \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2} \times 2500 \times 7.2^2 = 6.48 \times 10^4 \text{ J}$$

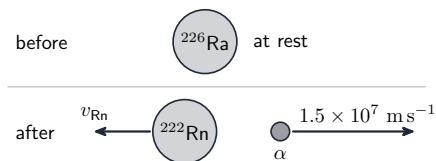
3. **Percentage transferred** – the energy lost as a fraction of the energy before:

$$\frac{E_{\text{before}} - E_{\text{after}}}{E_{\text{before}}} \times 100\% = \frac{1.08 \times 10^5 - 6.48 \times 10^4}{1.08 \times 10^5} \times 100\% = 40\%$$

to thermal energy, sound and the deformation of the cars. The collision is inelastic, yet the momentum is conserved exactly.

■ Worked example: a nucleus decays

A radium nucleus $^{226}_{88}\text{Ra}$ at rest emits an α -particle at $1.5 \times 10^7 \frac{\text{m}}{\text{s}}$.



1. **Equation** – nucleon numbers $226 = 222 + 4$, proton numbers $88 = 86 + 2$: $^{226}_{88}\text{Ra} \rightarrow ^{222}_{86}\text{Rn} + ^4_2\alpha$.
2. **Directions** – the nucleus was at rest, so the total momentum is zero before and after: the two momenta are equal in size and opposite in direction.

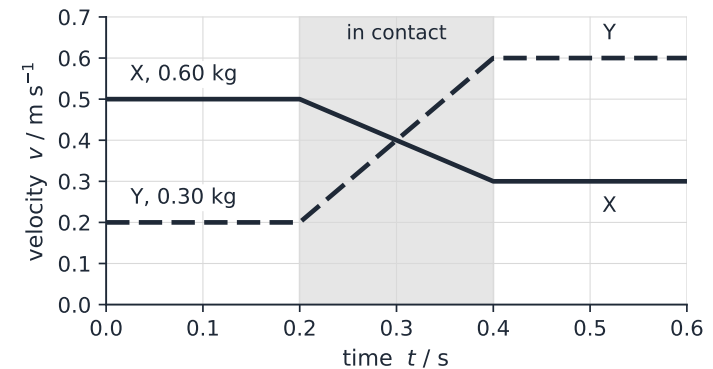
3. **Speed of the radon nucleus** – conservation of momentum, $m_{\text{Rn}}v_{\text{Rn}} = m_{\alpha}v_{\alpha}$ in size, masses in u:

$$v_{\text{Rn}} = \frac{m_{\alpha}v_{\alpha}}{m_{\text{Rn}}} = \frac{4 \times 1.5 \times 10^7}{222} = 2.7 \times 10^5 \frac{\text{m}}{\text{s}}, \text{ opposite to the } \alpha\text{-particle}$$

4. If instead the parent moves at speed u and the new nucleus ends at rest, all the momentum goes to the α -particle: $m_{\text{parent}}u = m_{\alpha}v_{\alpha}$.

■ Worked example: a collision on a velocity–time graph

The graph shows trolley X (0.60 kg) catching trolley Y (0.30 kg) on a horizontal track; they are in contact from $t = 0.20 \text{ s}$ to $t = 0.40 \text{ s}$.



1. **Acceleration of Y during contact** – acceleration is the gradient of the v – t line:
 $a = \frac{\Delta v}{\Delta t} = \frac{0.60 - 0.20}{0.40 - 0.20} = 2.0 \text{ m s}^{-2}$.
2. **The forces** – Newton's second law on Y, $F = ma = 0.30 \times 2.0 = 0.60 \text{ N}$ forwards; by Newton's third law X feels 0.60 N backwards for the **same** 0.20 s , so both lines change over the same interval.
3. **Final velocity of X** – conservation of momentum, bounce apart:

$$m_Xu_X + m_Yu_Y = m_Xv_X + m_Yv_Y : 0.60 \times 0.50 + 0.30 \times 0.20 = 0.60 v_X + 0.30 \times 0.60$$

so $0.36 = 0.60 v_X + 0.18$ and $v_X = 0.30 \frac{\text{m}}{\text{s}}$, as the graph shows.

4. **Elastic?** – add $E_k = \frac{1}{2}mv^2$ for the two trolleys, before and after:

$$E_{\text{before}} = \frac{1}{2}m_Xu_X^2 + \frac{1}{2}m_Yu_Y^2 = \frac{1}{2}(0.60)(0.50)^2 + \frac{1}{2}(0.30)(0.20)^2 = 0.081 \text{ J}$$

$$E_{\text{after}} = \frac{1}{2}m_Xv_X^2 + \frac{1}{2}m_Yv_Y^2 = \frac{1}{2}(0.60)(0.30)^2 + \frac{1}{2}(0.30)(0.60)^2 = 0.081 \text{ J}$$

The total kinetic energy is unchanged, so the collision is **elastic**.

5. **Distance Y moves during contact** – displacement is the area under its line from 0.20 to 0.40 s , a trapezium: $s = \frac{1}{2}(u + v)t = \frac{1}{2}(0.20 + 0.60) \times 0.20 = 0.080 \text{ m}$.

2 Practice

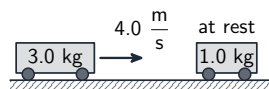
Now apply the methods above.

2.1 State the **principle of conservation of momentum**. [2]

conservation of momentum

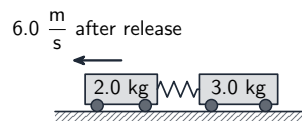
2.2 A trolley of mass 3.0 kg moving at $4.0 \frac{\text{m}}{\text{s}}$ collides with a stationary trolley of mass 1.0 kg. The trolleys stick together.

(a) Calculate their common velocity. [2]



(b) Calculate the percentage of the kinetic energy that is transferred to other forms in the collision. [2]

2.3 Two trolleys, of mass 2.0 kg and 3.0 kg, are held at rest with a compressed spring between them. When they are released, the 2.0 kg trolley moves off at $6.0 \frac{\text{m}}{\text{s}}$. Calculate the velocity of the 3.0 kg trolley. [2]



2.4 A ball of mass 0.50 kg moving at $4.0 \frac{\text{m}}{\text{s}}$ hits a stationary ball of the same mass head-on. Afterwards the first ball is at rest and the second moves at $4.0 \frac{\text{m}}{\text{s}}$.

(a) Show that momentum is conserved. [1]

(b) Show that the collision is elastic. [2]

before  $\rightarrow 4.0 \frac{\text{m}}{\text{s}}$  at rest

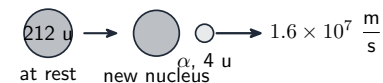
after  at rest  $\rightarrow 4.0 \frac{\text{m}}{\text{s}}$
both 0.50 kg

(c) Show that the relative speed of approach equals the relative speed of separation. [1]

2.5 A stationary nucleus of mass 212 u emits an α -particle of mass 4 u at a speed of $1.6 \times 10^7 \frac{\text{m}}{\text{s}}$.

(a) Explain why the new nucleus and the α -particle move off in opposite directions. [2]

(b) Calculate the speed of the new nucleus. [2]



3 Exam-style questions

3.1 A stationary nucleus of mass 220 u emits an α -particle of mass 4 u. What is the ratio $\frac{\text{speed of the } \alpha\text{-particle}}{\text{speed of the new nucleus}}$? [1]

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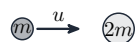
- A $\frac{4}{216}$
 B $\frac{4}{220}$
 C $\frac{4}{216}$
 D $\frac{4}{220}$



3.2 A ball of mass m moving at speed u hits a stationary ball of mass $2m$, and the two stick together. What fraction of the initial kinetic energy is transferred to other forms? [1]

☐

- A $\frac{1}{3}$
 B $\frac{1}{2}$
 C $\frac{2}{3}$
 D $\frac{3}{4}$

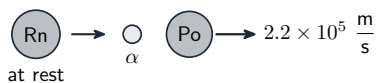


3.3 A radon-220 nucleus at rest breaks apart into a polonium nucleus and an α -particle. The polonium nucleus moves off at $2.2 \times 10^5 \frac{\text{m}}{\text{s}}$.

(a) **Complete** the equation for the decay. [1]

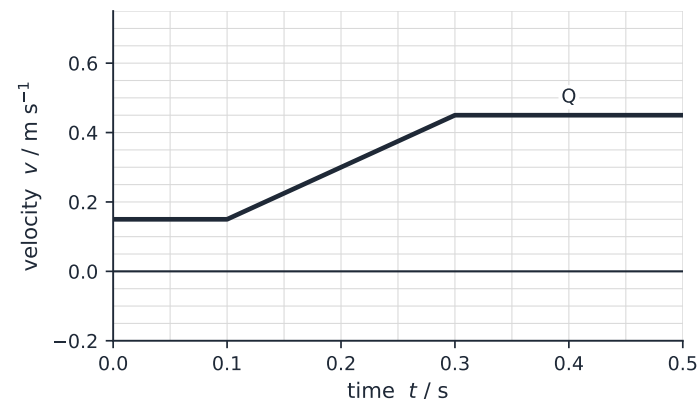


(b) By considering momentum, **calculate** the speed of the α -particle. [3]



(c) **Determine** the ratio $\frac{\text{kinetic energy of the } \alpha\text{-particle}}{\text{kinetic energy of the polonium nucleus}}$. [3]

3.4 Trolley P, of mass 0.40 kg, moves at $0.60 \frac{\text{m}}{\text{s}}$ along a horizontal track and catches trolley Q, of mass 0.80 kg, which is moving in the same direction. The graph shows how the velocity of Q varies with time t . The trolleys are in contact from $t = 0.10\text{ s}$ to $t = 0.30\text{ s}$.



(a) **Determine** the acceleration of Q during the collision. [2]

(b) Use conservation of momentum to **determine** the velocity of P after the collision. [2]

(c) On the graph, **sketch** the variation with time t of the velocity of P from $t = 0$ to $t = 0.50\text{ s}$. [3]

(d) **Show that** the collision is elastic. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the total momentum of a system of objects is constant (total momentum before = total momentum after)
- provided no resultant external force acts on the system

2.2

(a) conservation of momentum, objects stick together: $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{3.0 \times 4.0 + 0}{3.0 + 1.0} = \mathbf{3.0 \frac{m}{s}}$$

(b) $E_k = \frac{1}{2}mv^2$, before and after

$$E_{\text{before}} = \frac{1}{2}m_1u_1^2 = \frac{1}{2} \times 3.0 \times 4.0^2 = 24 \text{ J}$$

$$E_{\text{after}} = \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2} \times 4.0 \times 3.0^2 = 18 \text{ J}$$

- the energy lost as a percentage of the energy before

$$\frac{E_{\text{before}} - E_{\text{after}}}{E_{\text{before}}} \times 100\% = \frac{24 - 18}{24} \times 100\% = \mathbf{25\%}$$

2.3

- conservation of momentum from rest: $0 = m_1v_1 + m_2v_2$

$$v_2 = -\frac{m_1v_1}{m_2} = -\frac{2.0 \times 6.0}{3.0} = -4.0 \frac{\text{m}}{\text{s}}$$

- $\mathbf{4.0 \frac{m}{s}}$, in the **opposite** direction to the 2.0 kg trolley

2.4

(a) ball 1 moves and ball 2 is at rest; total momentum $p = mv$ before and after, the same m for both

$$p_{\text{before}} = mu_1 + mu_2 = 0.50 \times 4.0 + 0 = 2.0 \text{ N s}$$

$$p_{\text{after}} = mv_1 + mv_2 = 0 + 0.50 \times 4.0 = 2.0 \text{ N s}$$

(b) total $E_k = \frac{1}{2}mv^2$, before and after

$$E_{\text{before}} = \frac{1}{2}mu_1^2 + 0 = \frac{1}{2} \times 0.50 \times 4.0^2 = 4.0 \text{ J}$$

$$E_{\text{after}} = 0 + \frac{1}{2}mv_2^2 = \frac{1}{2} \times 0.50 \times 4.0^2 = 4.0 \text{ J}$$

- the total kinetic energy is conserved, so the collision is **elastic**

(c) relative speed of approach $= u_1 - u_2 = 4.0 - 0 = 4.0 \frac{\text{m}}{\text{s}}$

- relative speed of separation $= v_2 - v_1 = 4.0 - 0 = 4.0 \frac{\text{m}}{\text{s}}$, the same

2.5

(a) the nucleus was at rest, so the total momentum before is **zero**

- momentum is conserved, so the two momenta after are equal in size and **opposite** in direction

(b) conservation of momentum, $m_{\text{new}}v_{\text{new}} = m_{\alpha}v_{\alpha}$, with the new mass $m_{\text{new}} = 212 - 4 = 208 \text{ u}$

$$v_{\text{new}} = \frac{m_{\alpha}v_{\alpha}}{m_{\text{new}}} = \frac{4 \times 1.6 \times 10^7}{208} = \mathbf{3.1 \times 10^5 \frac{m}{s}}$$

3.1 C

- the momenta are equal in size, so speed $\propto 1/\text{mass}$: $\frac{v_{\alpha}}{v_{\text{new}}} = \frac{m_{\text{new}}}{m_{\alpha}} = \frac{216}{4}$, with the new nucleus of mass $m_{\text{new}} = 220 - 4 = 216 \text{ u}$

- D** uses the mass of the nucleus before the decay; **A** and **B** are upside down D

A B

3.2 C

- momentum: $mu = 3mv$, so $v = u/3$

- E_k after $= \frac{1}{2}(3m)(u/3)^2 = \frac{1}{3} \times \frac{1}{2}mu^2$, so one third remains and **two thirds** are transferred

- A** is the fraction that remains A

3.3

(a) nucleon numbers and proton numbers both balance: ${}^{220}_{86}\text{Rn} \rightarrow {}^{216}_{84}\text{Po} + {}^4_2\alpha$

(b) the nucleus was at rest, so the total momentum stays **zero**

- conservation of momentum, $m_\alpha v_\alpha = m_{\text{Po}} v_{\text{Po}}$, masses in u

$$v_\alpha = \frac{m_{\text{Po}} v_{\text{Po}}}{m_\alpha} = \frac{216 \times 2.2 \times 10^5}{4} = \mathbf{1.2 \times 10^7} \frac{\text{m}}{\text{s}}, \text{ opposite to the polonium}$$

(c) $E_k = \frac{1}{2}mv^2$, so $\frac{E_\alpha}{E_{\text{Po}}} = \frac{m_\alpha v_\alpha^2}{m_{\text{Po}} v_{\text{Po}}^2}$

- the momenta are equal in size, so $v_\alpha/v_{\text{Po}} = m_{\text{Po}}/m_\alpha$ and the ratio becomes $\frac{m_{\text{Po}}}{m_\alpha}$

$$\frac{E_\alpha}{E_{\text{Po}}} = \frac{m_{\text{Po}}}{m_\alpha} = \frac{216}{4} = \mathbf{54}$$

3.4

(a) acceleration is the gradient of the v - t line

$$a = \frac{\Delta v}{\Delta t} = \frac{0.45 - 0.15}{0.30 - 0.10} = \mathbf{1.5 \text{ m s}^{-2}}$$

(b) conservation of momentum, bounce apart

$$m_P u_P + m_Q u_Q = m_P v_P + m_Q v_Q : \quad 0.40 \times 0.60 + 0.80 \times 0.15 = 0.40 v_P + 0.80 \times 0.45$$

- so $0.36 = 0.40 v_P + 0.36$ and $v_P = \mathbf{0}$: P stops

(c) a horizontal line at $0.60 \frac{\text{m}}{\text{s}}$ from $t = 0$ to 0.10 s

- a straight line from $(0.10, 0.60)$ down to $(0.30, 0)$: the change happens over the same contact time as for Q
- a horizontal line along $v = 0$ from 0.30 to 0.50 s

(d) add $E_k = \frac{1}{2}mv^2$ for the two trolleys, before and after

$$E_{\text{before}} = \frac{1}{2}m_P u_P^2 + \frac{1}{2}m_Q u_Q^2 = \frac{1}{2}(0.40)(0.60)^2 + \frac{1}{2}(0.80)(0.15)^2 = 0.081 \text{ J}$$

$$E_{\text{after}} = \frac{1}{2}m_P v_P^2 + \frac{1}{2}m_Q v_Q^2 = 0 + \frac{1}{2}(0.80)(0.45)^2 = 0.081 \text{ J}$$

- the total kinetic energy is conserved, so the collision is elastic