

3.3.1 Elastic collisions and momentum in two dimensions

Name: _____ Class: _____ Date: _____

Total: **37 marks****KEY WORDS** elastic collision

• conservation of momentum

• relative speed

• kinetic energy

• component

Objective

Answer the harder momentum questions: **elastic** collisions solved with the relative-speed rule, and conservation of momentum in **two dimensions**. Syllabus 3.3, learning outcomes 2 and 3.

You must be able to:

- recall the two facts about an elastic collision, and use the relative-speed rule with conservation of momentum to find two unknown velocities
- resolve a momentum into two **perpendicular components**, and apply conservation of momentum along each direction on its own
- combine two perpendicular momenta into a magnitude and a direction, and find the missing fragment of an explosion
- decide whether a collision in two dimensions is elastic, using the full speed of each object

1 Key knowledge and worked examples

■ Key knowledge: elastic collisions

Quantity or rule	Before collision	After collision
Total momentum	$m_1u_1 + m_2u_2$	$m_1v_1 + m_2v_2$
Total kinetic energy	$\frac{1}{2}(m_1u_1^2 + m_2u_2^2)$	$\frac{1}{2}(m_1v_1^2 + m_2v_2^2)$
Relative speed (head-on only)	Approach: $u_1 - u_2$	Separation: $v_2 - v_1$

In each row, **before** = **after** for an elastic collision. Choose one positive direction and use **signed velocities**: u before, v after.

With negligible external impulse, total momentum is conserved in **all** collisions. Total kinetic energy is conserved only in an **elastic** collision.

- relative speed: objects moving **towards** each other, **add** the speeds; moving the **same way**, **subtract** them
- to find two unknown velocities, use momentum **and** the relative-speed rule: two equations with no squares, quicker than the energy equation
- equal masses, one at rest, head-on and elastic: the two **exchange velocities**
- the relative-speed rule works only along the line of the collision; to test a collision in two dimensions, compare the total kinetic energy before and after

■ Key knowledge: momentum in two dimensions

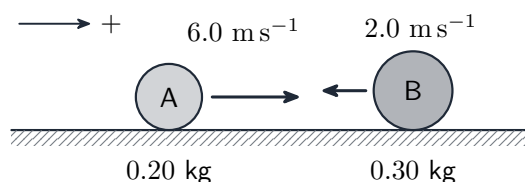
Momentum is a **vector**, so it is conserved along each of two perpendicular directions **separately**.

Step	Do this
1 choose axes	x along the incoming object, y across it: the total y -momentum before is then zero
2 resolve	a momentum p at angle θ to the x -axis has components $p \cos \theta$ along x and $p \sin \theta$ along y
3 two equations	total before = total after, once along x and once along y ; label each equation with its axis
4 recombine	$p = \sqrt{p_x^2 + p_y^2}$ and $\theta = \tan^{-1}(p_y/p_x)$; give the direction with its sense, such as "36° south of east"

- an **explosion** from rest: the momenta of the fragments add to zero as vectors, so the last fragment cancels the resultant of all the others
- kinetic energy is a **scalar**: always use the full speed, never a component

■ Worked example: an elastic head-on collision

Ball A (0.20 kg) moves right at $6.0 \frac{\text{m}}{\text{s}}$ and meets ball B (0.30 kg) moving left at $2.0 \frac{\text{m}}{\text{s}}$. The collision is elastic. Find both velocities afterwards.



1. **Signs** – right positive: $u_A = +6.0 \frac{\text{m}}{\text{s}}$, $u_B = -2.0 \frac{\text{m}}{\text{s}}$.
2. **Momentum** – conservation of momentum, $m_A u_A + m_B u_B = m_A v_A + m_B v_B$:
 $0.20 \times 6.0 + 0.30 \times (-2.0) = 0.20 v_A + 0.30 v_B$, so $0.60 = 0.20 v_A + 0.30 v_B$
3. **Relative speeds** – elastic, so approach = separation, $u_A - u_B = v_B - v_A$: $6.0 -$

$(-2.0) = 8.0$, so $v_B = v_A + 8.0$.

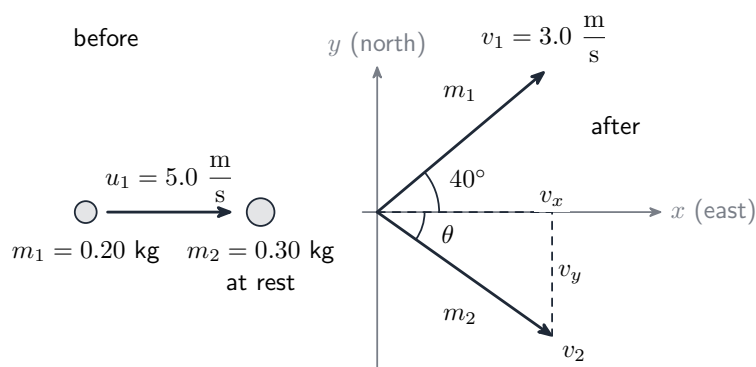
4. **Solve** – substitute into step 2: $0.60 = 0.20 v_A + 0.30 (v_A + 8.0) = 0.50 v_A + 2.4$,
so $v_A = -3.6 \frac{\text{m}}{\text{s}}$ (A moves **left** at $3.6 \frac{\text{m}}{\text{s}}$) and $v_B = +4.4 \frac{\text{m}}{\text{s}}$ (B moves **right**).

5. **Check** – the total $E_k = \frac{1}{2}mv^2$ must be the same before and after:

$$E_{\text{before}} = \frac{1}{2}m_A u_A^2 + \frac{1}{2}m_B u_B^2 = \frac{1}{2}(0.20)(6.0)^2 + \frac{1}{2}(0.30)(2.0)^2 = 4.2 \text{ J}$$

$$E_{\text{after}} = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}(0.20)(3.6)^2 + \frac{1}{2}(0.30)(4.4)^2 = 4.2 \text{ J}$$

■ Worked example: a glancing collision



A 0.20 kg ball moving east at $5.0 \frac{\text{m}}{\text{s}}$ strikes a stationary 0.30 kg ball. Afterwards the 0.20 kg ball moves at $3.0 \frac{\text{m}}{\text{s}}$ at 40° north of east. Find the velocity of the 0.30 kg ball, and test the collision.

1. **Along x (east)** – momentum along x is conserved, $m_1 u_1 = m_1 v_1 \cos 40^\circ + m_2 v_x$, with ball 1 the 0.20 kg ball:

$$v_x = \frac{m_1 u_1 - m_1 v_1 \cos 40^\circ}{m_2} = \frac{0.20 \times 5.0 - 0.20 \times 3.0 \cos 40^\circ}{0.30} = 1.80 \frac{\text{m}}{\text{s}}$$

2. **Along y (north)** – the total before is **zero**, so $0 = m_1 v_1 \sin 40^\circ + m_2 v_y$:

$$v_y = -\frac{m_1 v_1 \sin 40^\circ}{m_2} = -\frac{0.20 \times 3.0 \sin 40^\circ}{0.30} = -1.29 \frac{\text{m}}{\text{s}} \text{ (south)}$$

3. **Recombine** – Pythagoras for ball 2's speed v_2 , $\tan^{-1}$ for the angle:

$$v_2 = \sqrt{v_x^2 + v_y^2} = \sqrt{1.80^2 + 1.29^2} = 2.2 \frac{\text{m}}{\text{s}}$$

$$\theta = \tan^{-1} \frac{|v_y|}{v_x} = \tan^{-1} \frac{1.29}{1.80} = 36^\circ \text{ south of east}$$

4. **Elastic?** – the total $E_k = \frac{1}{2}mv^2$, with the full speeds:

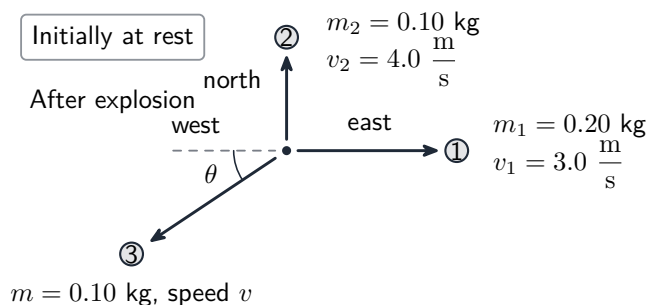
$$E_{\text{before}} = \frac{1}{2}m_1 u_1^2 = \frac{1}{2}(0.20)(5.0)^2 = 2.5 \text{ J}$$

$$E_{\text{after}} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}(0.20)(3.0)^2 + \frac{1}{2}(0.30)(2.2)^2 = 1.6 \text{ J}$$

Kinetic energy is lost, so the collision is **inelastic**.

■ Worked example: a firework in three pieces

A stationary firework explodes into three pieces that move in a horizontal plane: 0.20 kg at $3.0 \frac{\text{m}}{\text{s}}$ east and 0.10 kg at $4.0 \frac{\text{m}}{\text{s}}$ north. The third piece has a mass of 0.10 kg.



1. **The total stays zero** – conservation of momentum from rest, so the third piece cancels the momentum $p = mv$ of the other two along each axis:

$$p_{\text{east}} = m_1 v_1 = 0.20 \times 3.0 = 0.60 \text{ N s}$$

$$p_{\text{north}} = m_2 v_2 = 0.10 \times 4.0 = 0.40 \text{ N s}$$

2. **Magnitude** – the third piece's momentum has the size of their resultant (Pythagoras), and $p = mv$:

$$p = \sqrt{p_{\text{east}}^2 + p_{\text{north}}^2} = \sqrt{0.60^2 + 0.40^2} = 0.72 \text{ N s}$$

$$v = \frac{p}{m} = \frac{0.72 \text{ N s}}{0.10 \text{ kg}} = 7.2 \frac{\text{m}}{\text{s}}$$

3. **Direction** – opposite to the resultant of the other two: $\theta = \tan^{-1} \frac{p_{\text{north}}}{p_{\text{east}}} = \tan^{-1} \frac{0.40}{0.60} = 34^\circ$ **south of west**.

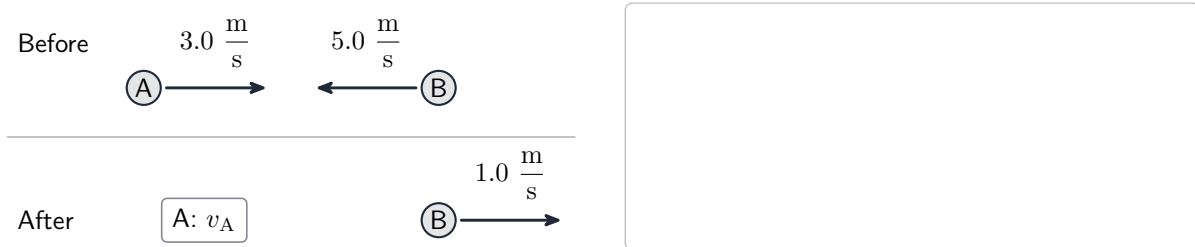
2 Practice

Now use the methods above.

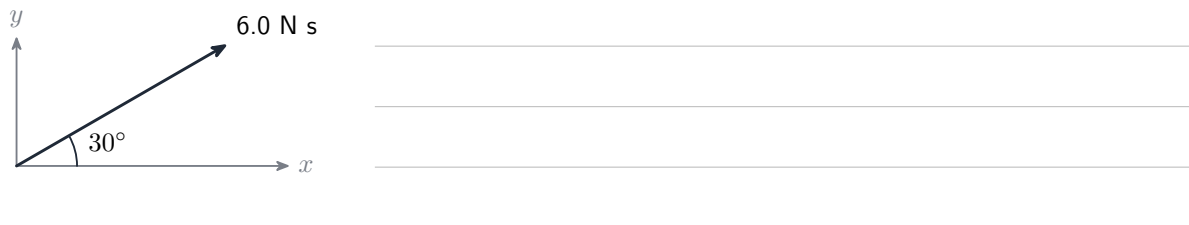
2.1 State **two** things that are true of an elastic collision, apart from the conservation of momentum. [2]

elastic collision

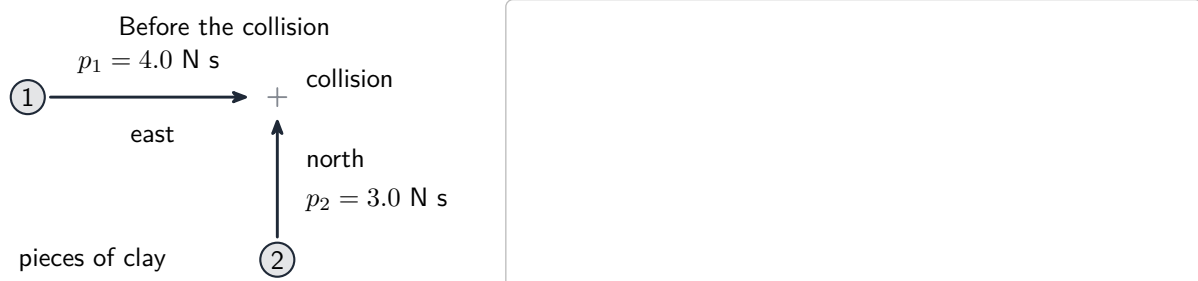
2.2 Ball A moves right at $3.0 \frac{\text{m}}{\text{s}}$ and ball B moves left at $5.0 \frac{\text{m}}{\text{s}}$. They collide head-on and elastically. Afterwards B moves right at $1.0 \frac{\text{m}}{\text{s}}$. Use the relative-speed rule to find the velocity of A after the collision. [2]



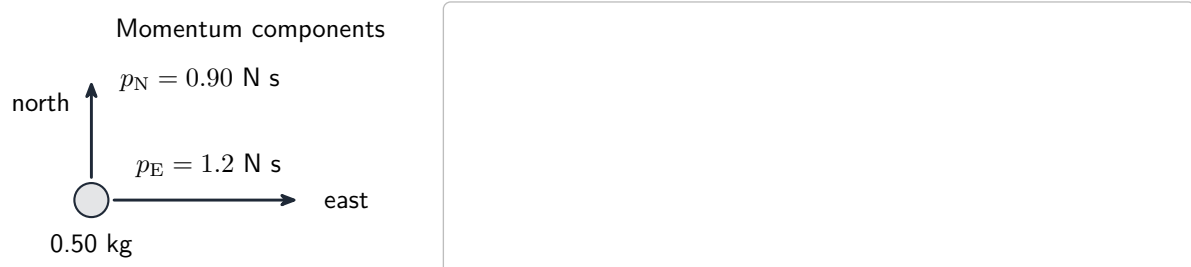
2.3 A momentum of 6.0 kg m s^{-1} is directed at 30° above the horizontal. Calculate its horizontal and vertical components. [2]



2.4 Two pieces of clay collide and stick together. Just before the collision one has momentum 4.0 N s due east and the other 3.0 N s due north. Calculate the magnitude and the direction of the momentum of the combined lump. [3]



2.5 After a collision, a 0.50 kg ball has momentum components 1.2 N s east and 0.90 N s north. Calculate its speed, and state why its kinetic energy must be found from this speed and not from a component. [3]



3 Exam-style questions

3.1 Two balls of equal mass move towards each other along a straight line, one at speed $2v$ and the other at speed v . They collide head-on and elastically. What happens after the collision? [1]

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A Both balls move at speed $1.5v$.

B Each ball moves back the way it came, the first at speed v and the second at speed $2v$.

C The first ball stops and the second moves back at speed $3v$.

D Each ball moves back the way it came, the first at speed $2v$ and the second at speed v .

before: equal masses



3.2 A stationary object explodes into three fragments that move in a horizontal plane. One fragment has momentum 6.0 N s due east and another has momentum 8.0 N s due north. What is the momentum of the third fragment? [1]

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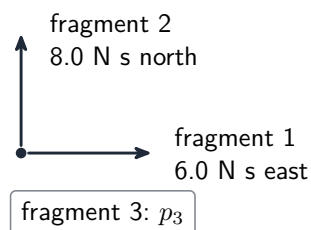
A 2.0 N s due south-west

B 10 N s , directed south of west

C 14 N s , directed south of west

D 10 N s , directed north of east

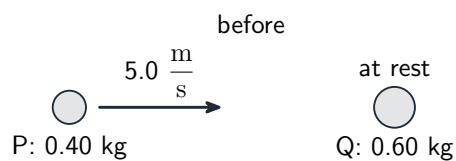
After the explosion



3.3 A ball P of mass 0.40 kg moving at $5.0 \frac{\text{m}}{\text{s}}$ collides head-on and elastically with a stationary ball Q of mass 0.60 kg.

(a) State the **two** conditions, other than the conservation of momentum, that an elastic collision satisfies. [2]

(b) Use conservation of momentum and the relative-speed rule to **determine** the velocities of P and Q after the collision. [4]

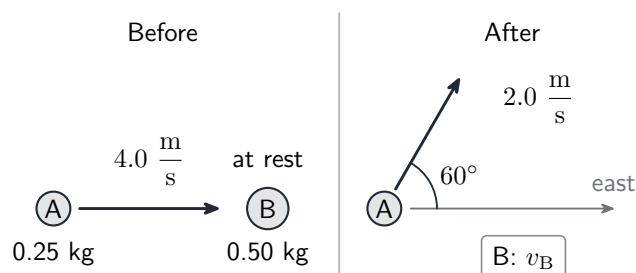


(c) Calculate the percentage of the initial kinetic energy of P that is transferred to Q. [2]

3.4 On a horizontal frictionless surface, a puck A of mass 0.25 kg moving due east at $4.0 \frac{\text{m}}{\text{s}}$ strikes a stationary puck B of mass 0.50 kg. After the collision, A moves at $2.0 \frac{\text{m}}{\text{s}}$ at 60° north of east.

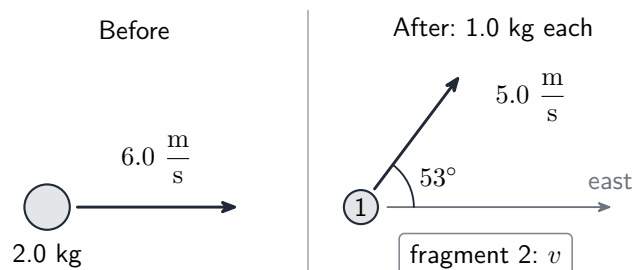
(a) State the principle of conservation of momentum. [2]

(b) Calculate the magnitude and the direction of the velocity of B after the collision. [5]



(c) **Determine** whether the collision is elastic. Show your working. [3]

3.5 An object of mass 2.0 kg moving due east at $6.0 \frac{\text{m}}{\text{s}}$ explodes into two fragments, each of mass 1.0 kg. One fragment moves at $5.0 \frac{\text{m}}{\text{s}}$ at 53° north of east. Calculate the magnitude and the direction of the velocity of the other fragment. [5]



Solutions

Each answer shows the steps, then the **result**.

2.1

- the total **kinetic energy** is conserved
- the **relative speed of approach** equals the **relative speed of separation**

2.2

- right positive; elastic, so $u_A - u_B = v_B - v_A$: $3.0 - (-5.0) = 8.0 = 1.0 - v_A$
- $v_A = -7.0 \frac{\text{m}}{\text{s}}$: A moves **left** at **7.0** $\frac{\text{m}}{\text{s}}$ A

2.3

- horizontal: $p \cos \theta = 6.0 \cos 30^\circ = \mathbf{5.2} \text{ kg m s}^{-1}$
- vertical: $p \sin \theta = 6.0 \sin 30^\circ = \mathbf{3.0} \text{ kg m s}^{-1}$

2.4

- momentum is conserved along each axis, so the lump has 4.0 N s east and 3.0 N s north
- Pythagoras: $p = \sqrt{p_{\text{east}}^2 + p_{\text{north}}^2} = \sqrt{4.0^2 + 3.0^2} = \mathbf{5.0} \text{ N s}$
- $\theta = \tan^{-1} \frac{p_{\text{north}}}{p_{\text{east}}} = \tan^{-1} \frac{3.0}{4.0} = \mathbf{37^\circ}$ north of east

2.5

- Pythagoras: $p = \sqrt{p_{\text{east}}^2 + p_{\text{north}}^2} = \sqrt{1.2^2 + 0.90^2} = 1.5 \text{ N s}$
- from $p = mv$: $v = p/m = 1.5/0.50 = \mathbf{3.0} \frac{\text{m}}{\text{s}}$
- kinetic energy is a **scalar** that depends on the full speed, $\frac{1}{2}mv^2$; a component gives a value that is too small

3.1 B

- equal masses in a head-on elastic collision **exchange velocities**: the first takes the second's velocity (v , backwards) and the second takes the first's ($2v$, backwards)
- check: approach $2v + v = 3v$, separation $v + 2v = 3v$; **D** has the right directions but breaks momentum, **C** breaks the relative-speed rule and **A** is a sticking-type answer
D C A

3.2 B

- from rest, the three momenta add to zero, so the third cancels 6.0 N s east and 8.0 N s north
- $p = \sqrt{p_{\text{east}}^2 + p_{\text{north}}^2} = \sqrt{6.0^2 + 8.0^2} = 10 \text{ N s}$, directed south of west; **D** is the resultant of the other two, not its opposite, and **C** adds the magnitudes

D C

3.3

(a) the total kinetic energy is conserved

- the relative speed of approach equals the relative speed of separation

(b) conservation of momentum, Q at rest: $m_P u_P = m_P v_P + m_Q v_Q$

P

$$0.40 \times 5.0 = 0.40 v_P + 0.60 v_Q, \quad \text{so} \quad 2.0 = 0.40 v_P + 0.60 v_Q$$

- elastic, so $u_P - u_Q = v_Q - v_P$: $5.0 - 0 = v_Q - v_P$, so $v_Q = v_P + 5.0$
- $2.0 = 0.40 v_P + 0.60 v_P + 3.0$, so $v_P = -1.0 \frac{\text{m}}{\text{s}}$: P moves **back** at **1.0** $\frac{\text{m}}{\text{s}}$
- $v_Q = 4.0 \frac{\text{m}}{\text{s}}$ forwards Q

(c) $E_k = \frac{1}{2}mv^2$ for P before and for Q after

P Q

$$E_P = \frac{1}{2}m_P u_P^2 = \frac{1}{2} \times 0.40 \times 5.0^2 = 5.0 \text{ J}$$

$$E_Q = \frac{1}{2}m_Q v_Q^2 = \frac{1}{2} \times 0.60 \times 4.0^2 = 4.8 \text{ J}$$

- the share of P's energy that Q receives

Q P

$$\frac{E_Q}{E_P} \times 100\% = \frac{4.8}{5.0} \times 100\% = \mathbf{96\%}$$

3.4

(a) the total momentum of a system of objects is constant

- provided no resultant external force acts on the system

(b) momentum along east is conserved, $m_A u_A = m_A v_A \cos 60^\circ + m_B v_x$

B

$$v_x = \frac{m_A u_A - m_A v_A \cos 60^\circ}{m_B} = \frac{0.25 \times 4.0 - 0.25 \times 2.0 \cos 60^\circ}{0.50} = 1.5 \frac{\text{m}}{\text{s}}$$

- along north the total before is zero, $0 = m_A v_A \sin 60^\circ + m_B v_y$
- B

$$v_y = -\frac{m_A v_A \sin 60^\circ}{m_B} = -\frac{0.25 \times 2.0 \sin 60^\circ}{0.50} = -0.87 \frac{\text{m}}{\text{s}} \text{ (south)}$$

- Pythagoras: $v = \sqrt{v_x^2 + v_y^2} = \sqrt{1.5^2 + 0.87^2} = \mathbf{1.7 \frac{m}{s}}$
- $\theta = \tan^{-1} \frac{|v_y|}{v_x} = \tan^{-1} \frac{0.87}{1.5} = \mathbf{30^\circ \text{ south of east}}$

(c) the total $E_k = \frac{1}{2}mv^2$, with the full speeds

$$E_{\text{before}} = \frac{1}{2}m_A u_A^2 = \frac{1}{2}(0.25)(4.0)^2 = 2.0 \text{ J}$$

$$E_{\text{after}} = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}(0.25)(2.0)^2 + \frac{1}{2}(0.50)(1.73)^2 = 1.25 \text{ J}$$

- the kinetic energy decreases, so the collision is **inelastic**

3.5

- momentum along east is conserved, $Mu = m_1 v_1 \cos 53^\circ + m_2 v_x$

$$v_x = \frac{Mu - m_1 v_1 \cos 53^\circ}{m_2} = \frac{2.0 \times 6.0 - 1.0 \times 5.0 \cos 53^\circ}{1.0} = 9.0 \frac{\text{m}}{\text{s}}$$

- along north the total before is zero, $0 = m_1 v_1 \sin 53^\circ + m_2 v_y$

$$v_y = -\frac{m_1 v_1 \sin 53^\circ}{m_2} = -\frac{1.0 \times 5.0 \sin 53^\circ}{1.0} = -4.0 \frac{\text{m}}{\text{s}} \text{ (south)}$$

- Pythagoras: $v = \sqrt{v_x^2 + v_y^2} = \sqrt{9.0^2 + 4.0^2} = \mathbf{9.8 \frac{m}{s}}$
- $\theta = \tan^{-1} \frac{|v_y|}{v_x} = \tan^{-1} \frac{4.0}{9.0} = \mathbf{24^\circ \text{ south of east}}$