3.2 Friction, drag and terminal velocity

Name: _____ Class: _____ Date: _____ **Total: 37 marks**

KEY WORDS drag • terminal velocity • air resistance • resultant force • upthrust

Objective

Use **friction**, **drag** and resultant force to explain **non-uniform motion** and **terminal velocity**. Read motion graphs and calculate accelerations in air and liquids. Syllabus 3.2, learning outcomes 1, 2 and 3.

You must be able to:

- describe friction and drag: drag is zero at rest and increases with speed
- explain falling from rest to terminal velocity using forces
- read velocity-time and acceleration-time graphs: terminal velocity, gradients for acceleration and areas for distance
- calculate acceleration when a parachute opens or an object moves upwards against gravity and drag
- balance the three forces at terminal velocity in a liquid

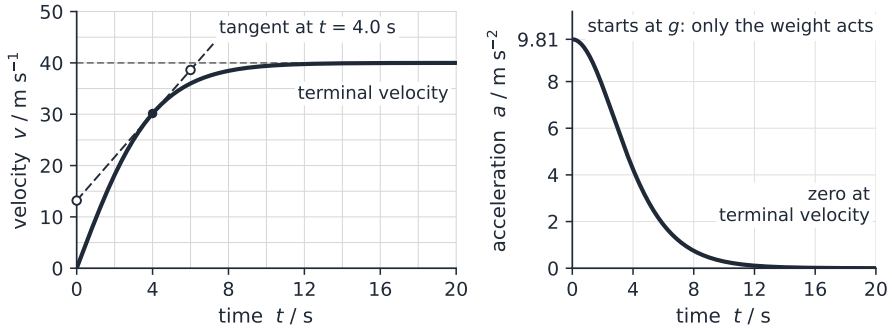
1 Key knowledge and worked examples

Key knowledge: friction and drag

Force	Where it acts	Size
friction	between two surfaces in contact, against their relative motion	does not depend on the speed
drag (air resistance, viscous force)	on an object moving through a fluid (a gas or a liquid), against its velocity	**zero at rest**, and **increases as the speed increases**

- a resistive force opposes the **motion**, so it reverses when the motion reverses
- drag also depends on the shape and on the area facing the flow: spreading the arms or opening a parachute increases it
- work done against friction and drag is transferred to **thermal energy** of the surroundings

Key knowledge: falling from rest through air



A fall from rest through air: the velocity levels off at the terminal velocity; the acceleration, the gradient of the velocity–time graph, falls from g to zero

Stage	Forces	$F_W - F_D$	Acceleration
just released, $v = 0$	weight only; drag $= 0$	F_W	$g = 9.81$ m s^{-2}, the largest
speeding up	drag increases with speed	decreases	decreases: the v–t curve flattens
terminal velocity	drag = weight	zero	zero: constant velocity (first law)

The "describe and explain" answer, one mark per line: at first only the weight acts, so $a = g$; as the speed increases, the drag increases; so the resultant force and the acceleration decrease; when the drag equals the weight, the resultant force is zero and the velocity is constant: the terminal velocity.

Key knowledge: falling through a liquid

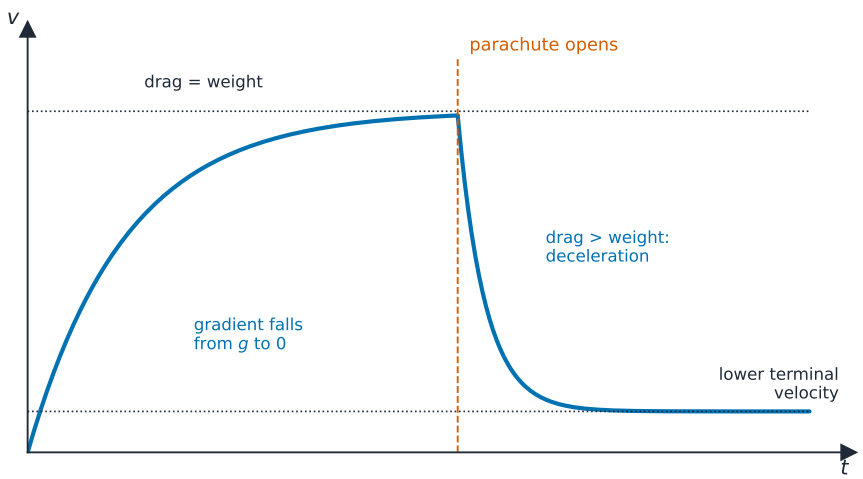
Three forces act on an object sinking through a liquid:

- weight $F_W = mg$, downwards
- **upthrust** F_U, upwards: the weight of the liquid pushed aside, $F_U = \rho_{\text{liquid}} V g$; it does not depend on the speed, and in a liquid of uniform density it is the same at every depth
- viscous drag F_D, upwards, increasing with speed

At terminal velocity the resultant force is zero: $F_W = F_U + F_D$. Write this line first; it earns the method mark.

Key knowledge: other motions against drag

Situation	Forces	What happens
thrown **upwards**	weight **and** drag both act **downwards**	resultant $F_W + F_D$: the deceleration is **greater** than g; as the object slows, the drag and the resultant decrease; at the top $v = 0$, drag $= 0$ and $a = g$
parachute opens	drag suddenly **greater** than the weight	resultant **upwards**: the skydiver still moves down but **slows**; the drag falls with the speed until drag = weight at a new, **lower** terminal velocity
heavier object, same size and shape	needs more drag to balance its weight	**higher** terminal velocity
car at top speed on a level road	driving force = total resistive force	a larger driving force gives a new, higher top speed
at terminal velocity	kinetic energy constant	gravitational potential energy → thermal energy (work done against the drag)



Opening a parachute: drag exceeds weight, so the skydiver decelerates to a lower terminal velocity

Worked example: reading the graph of a fall

The skydiver in the graphs above has a mass of 70 kg.

1. **Terminal velocity** – where the curve levels off: $v_\mathrm{T} = 40\ \frac{\mathrm{m}}{\mathrm{s}}$.
2. **Acceleration at** $t = 4.0$ **s** – acceleration is the gradient of the v–t graph; the line is curved, so take the gradient of the **tangent**, read at two points far apart, (0, 13) and (6.0, 39):

$$a = \frac{\Delta v}{\Delta t} = \frac{39 - 13}{6.0 - 0} = 4.3\ \mathrm{m\ s^{-2}}$$

3. **Drag at that instant** – Newton's second law, $F_\mathrm{res} = ma$ with $F_\mathrm{res} = F_W - F_D$:

$$F_D = mg - ma = 70 \times 9.81 - 70 \times 4.3 = 390\ \mathrm{N}$$

4. **Distance fallen in the first** 10 **s** – displacement is the area under the v–t graph. The equations of uniform acceleration do **not** apply here: the acceleration keeps changing. Count the squares under the curve from 0 to 10 s, a part square as a half: about 29, each 2.0 s wide and 5.0 $\frac{\mathrm{m}}{\mathrm{s}}$ tall, an area of 10 m:

$$s = \text{area under the graph} \approx 29 \times 10\ \mathrm{m} = 290\ \mathrm{m}$$

Worked example: a parachute opens

A skydiver of mass 75 kg falls at a terminal velocity of 50 $\frac{\mathrm{m}}{\mathrm{s}}$. She opens her parachute, and at that instant the drag becomes 2100 N.

1. **Before it opens** – terminal velocity, so the resultant force is zero (first law): $F_D = F_W = mg = 75 \times 9.81 = 736$ N.
2. **Just after it opens** – take downwards as positive; Newton's second law, $F_\mathrm{res} = ma$ with $F_\mathrm{res} = F_W - F_D$:

$$a = \frac{F_W - F_D}{m} = \frac{736 - 2100}{75} = -18\ \mathrm{m\ s^{-2}}$$

   The acceleration is 18 m s^{-2} **upwards** while she still moves **downwards**: she slows down.
3. **After that** – as she slows, the drag decreases, so the resultant force and the deceleration decrease, until drag = weight again at a new, lower terminal velocity.

2 Practice

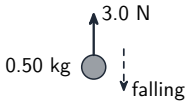
Now apply the methods above.

2.1 An object is released from rest in air. State, just after it is released:

(a) the size of the drag force on it [1]

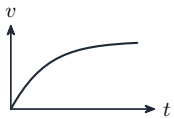
(b) its acceleration. [1]

2.2 A ball of mass 0.50 kg falls vertically through air. At one instant the air resistance on it is 3.0 N. Calculate the acceleration of the ball at this instant. [2]



2.3 Use the velocity–time graph of the fall in the Key knowledge.

(a) Draw a tangent to the curve at $t = 2.0$ s, and use it to determine the acceleration at that time. [2]



(b) Estimate the distance fallen in the first 6.0 s. [2]

2.4 A ball is thrown vertically upwards. Air resistance is not negligible. State and explain whether the deceleration of the ball on its way up is greater than, equal to or less than g. [2]

2.5 A skydiver of mass 80 kg falls at terminal velocity.

(a) State the size of the drag force on her. [1]

(b) She opens her parachute, and the drag force becomes 2000 N. Calculate her acceleration, and state its direction. [3]

2.6 A steel ball falls at terminal velocity through oil.

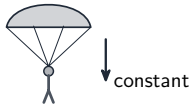
(a) Name the **three** forces acting on the ball, and give the direction of each. [2]

(b) The weight of the ball is 2.0×10^{-2} N and the upthrust on it is 2.5×10^{-3} N. Calculate the drag force on the ball. [1]

3 Exam-style questions

3.1 A parachutist falls at constant (terminal) velocity. Which statement is **not** correct? [1]

☐ **A** The resultant force on the parachutist is zero.
B Gravitational potential energy is converted to kinetic energy of the parachutist.
C Gravitational potential energy is converted to thermal energy.
D The air resistance on the parachutist is equal to the weight.



3.2 A car moves along a straight, level road at its top speed, with the engine at full power. Which statement is correct? [1]

A The air resistance on the car is zero.

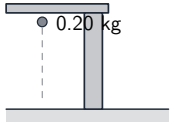
B The driving force is greater than the total resistive force.

C The driving force is equal to the total resistive force.

D The resultant force on the car is forwards.

3.3 A stone of mass 0.20 kg is dropped from a high bridge. Air resistance on the stone is significant.

(a) **Describe and explain** the motion of the stone from the moment it is released until it reaches terminal velocity. [4]



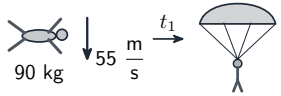
(b) In the space below, **sketch** a graph to show the variation with time t of the acceleration a of the stone, from its release until after it has reached terminal velocity. Mark the value of a at $t = 0$. [2]

(c) At one instant, the acceleration of the stone is 6.0 m s^{-2}. Calculate the air resistance on the stone at this instant. [2]

3.4 A skydiver and her equipment have a total mass of 90 kg. She falls vertically at a terminal velocity of 55 $\frac{\text{m}}{\text{s}}$. At time t_1 she opens her parachute, and at that instant the drag force on her is 2400 N. Later she reaches a new terminal velocity of 6.0 $\frac{\text{m}}{\text{s}}$.

(a) State the size of the drag force on her just **before** t_1. Explain your answer. [2]

(b) Calculate the magnitude of her acceleration just after t_1, and state its direction. [3]



(c) **Describe and explain** the variation of the magnitude of her acceleration from t_1 until she reaches the new terminal velocity. [3]

(d) Calculate the change in her momentum from t_1 until she reaches the new terminal velocity. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

(a) the speed is zero, so the drag is **zero**

(b) only the weight acts, so $a = g = \mathbf{9.81}$ m s^{-2}

2.2

- Newton's second law, $F_{\text{res}} = ma$, with $F_{\text{res}} = F_W - F_D = 0.50 \times 9.81 - 3.0 = 1.9$ N

$$a = \frac{F_{\text{res}}}{m} = \frac{1.9 \text{ N}}{0.50 \text{ kg}} = \mathbf{3.8} \text{ m s}^{-2}$$

2.3

(a) acceleration is the gradient of the v–t graph: the gradient of the tangent at 2.0 s, from two points far apart on it

- $a \approx \mathbf{7.8}$ m s^{-2} (any value from 7.0 to 8.5 m s^{-2} from a drawn tangent)

(b) displacement is the area under the v–t graph: about 13 to 14 squares of 10 m from 0 to 6.0 s

- $s \approx \mathbf{140}$ m (accept 130 to 145 m)

2.4

- **greater** than g
- on the way up the weight **and** the air resistance both act downwards, so the resultant force is greater than the weight and $a = (F_W + F_D)/m > g$

2.5

(a) at terminal velocity the resultant force is zero, so $F_D = F_W = mg = 80 \times 9.81 = \mathbf{785}$ N

(b) Newton's second law, $F_{\text{res}} = ma$, with $F_{\text{res}} = F_D - F_W = 2000 - 785 = 1215$ N upwards

$$a = \frac{F_{\text{res}}}{m} = \frac{1215 \text{ N}}{80 \text{ kg}} = \mathbf{15} \text{ m s}^{-2}$$

- directed **upwards** (she slows down)

2.6

(a) weight, downwards

- upthrust **and** drag (viscous force), both upwards

(b) terminal velocity, so $F_W = F_U + F_D$: $F_D = F_W - F_U = 2.0 \times 10^{-2} - 2.5 \times 10^{-3} = \mathbf{1.75 \times 10^{-2}}$ N

3.1 B

- at terminal velocity the speed, and so the kinetic energy of the parachutist, is **constant**: the lost potential energy is not converted to her kinetic energy
- it becomes thermal energy (work done against the air resistance), so **C** is correct; **A** and **D** describe terminal velocity

3.2 C

- at top speed the velocity is constant, so the resultant force is zero (first law): driving force = resistive force
- **D** would make the car speed up; **B** is the same mistake; the air resistance is at its **largest** at top speed, so **A** is wrong

3.3

(a) at first only the weight acts, so the acceleration is g

- as the speed increases, the air resistance increases
- so the resultant force and the acceleration **decrease**
- when the air resistance equals the weight, the resultant force is zero and the stone moves at constant velocity, the terminal velocity

(b) a curve starting at $a = 9.81$ m s^{-2} at $t = 0$, with a gradient that is negative and becomes less steep

- falling to $a = 0$ and then continuing along the time axis

(c) Newton's second law, $F_{\text{res}} = ma$, with $F_{\text{res}} = F_W - F_D$

$$F_D = mg - ma = 0.20 \text{ kg} \times (9.81 - 6.0) \text{ m s}^{-2} = \mathbf{0.76} \text{ N}$$

3.4

(a) $F_D = F_W = mg = 90 \times 9.81 = \mathbf{880}$ N

- at terminal velocity the velocity is constant, so the resultant force is zero (first law)

(b) Newton's second law, $F_{\text{res}} = ma$, with $F_{\text{res}} = F_D - F_W = 2400 - 883 = 1517$ N

$$a = \frac{F_{\text{res}}}{m} = \frac{1517 \text{ N}}{90 \text{ kg}} = \mathbf{17} \text{ m s}^{-2}$$

- directed **upwards**

(c) as she slows down, the drag force decreases

- so the resultant force, $F_D - F_W$, decreases
- so the magnitude of the acceleration decreases, to zero when the drag equals the weight again

(d) momentum $p = mv$, so $\Delta p = m(v - u)$ with downwards positive

$$\Delta p = m(v - u) = 90 \text{ kg} \times (6.0 - 55)\ \frac{\text{m}}{\text{s}} = -4400 \text{ N s}$$

- a change of **4400** N s, directed upwards