3.1 Momentum, force and Newton's laws

Name: _____ Class: _____ Date: _____ **Total: 35 marks**

KEY WORDS momentum • force • rate of change of momentum • resultant force • mass

Objective

Answer exam questions on **momentum** and **Newton's laws of motion**: what a force does to momentum, how to read force–time and momentum–time graphs, and the force from a stream of water or air. Syllabus 3.1, learning outcomes 1, 3, 4 and 5.

You must be able to:

- define **mass**, **linear momentum** and **force** in the examiner's words, and state Newton's three laws
- calculate a change in momentum with signs, and the average force from $F = \Delta p/\Delta t$
- read a **momentum–time graph** (the gradient is the force) and a **force–time graph** (the area is the change in momentum)
- find the force from a stream of water or air, and use Newton's third law to say which object pushes which

1 Key knowledge and worked examples

Key knowledge: the definitions

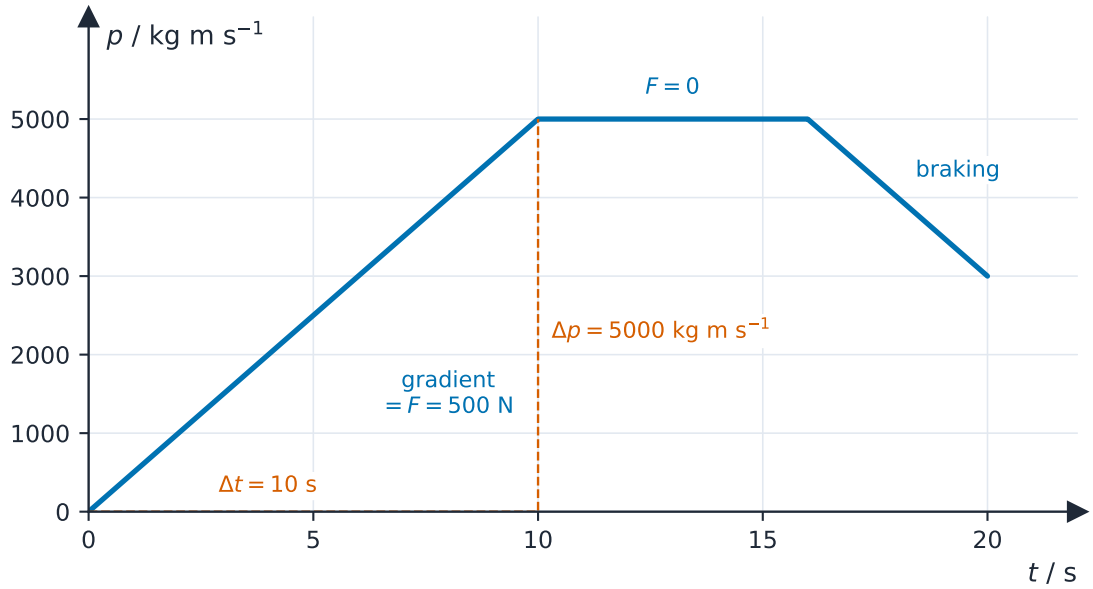
Quantity	Definition, in the words the mark scheme wants	Unit
mass	the property of an object that resists a change in motion	kg
linear momentum	the product of mass and velocity, $p = mv$; a **vector**	kg m s^{-1} (= N s)
force	the rate of change of momentum, $F = \Delta p/\Delta t$	N
weight	the force on a mass due to a gravitational field, $F_W = mg$	N

Newton's law	Statement	What the exam does with it
first	an object stays at rest, or moves at **constant velocity**, unless a **resultant force** acts on it	"explain why the velocity is constant": the resultant force is zero
second	the resultant force on an object is equal to its **rate of change of momentum**, in the direction of the change	$F = \Delta p / \Delta t$; for constant mass, $F = ma$
third	if A exerts a force on B, then B exerts a force on A that is **equal in magnitude, opposite in direction and of the same type**	rockets, hoses, collisions: the two forces act on **different** objects

■ **Key knowledge: change in momentum**

- choose a positive direction and give **every velocity a sign**; $\Delta p = p_{\text{after}} - p_{\text{before}} = m(v - u)$
- when the direction **reverses**, the two speeds **add**: a ball arriving at 8 and leaving at 6 $\frac{\text{m}}{\text{s}}$ has $\Delta v = 14\ \frac{\text{m}}{\text{s}}$
- **average force** during a contact lasting Δt: $F = \Delta p / \Delta t$; for the same Δp, a **longer** contact time means a **smaller** force (crumple zones, bending your knees when you land)
- by the third law, the object pushes back with the same force the other way

Key knowledge: the two graphs



Momentum–time graph: the gradient is the resultant force, here $F = \Delta p/\Delta t = 5000/10 = 500$ N; the flat part means zero resultant force

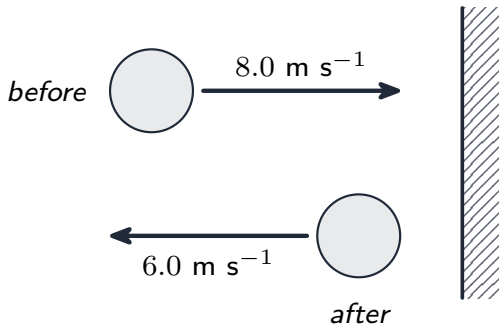
Graph	Read off	Because
momentum–time	the **gradient** is the resultant force; a straight line means a constant force; where the line crosses $p = 0$ the object is momentarily at rest and then moves the other way	$F = \Delta p/\Delta t$
force–time	the **area** under the line is the change in momentum, in N s	$\Delta p = F\,\Delta t$

- a **resistive force** cannot give a straight momentum–time line: it gets smaller as the object slows, and it is zero when the object is at rest
- speed at any instant: $v = p/m$

Key knowledge: the force from a stream of water or air

- the fluid's momentum changes at a steady rate: each second a mass $\Delta m/\Delta t$ (in kg s^{-1}) changes its velocity by Δv
- $F = \dfrac{\Delta p}{\Delta t} = \dfrac{\Delta m}{\Delta t} \times \Delta v$ is the force **on the fluid**; by the third law the fluid pushes the nozzle, fan or rocket with an **equal force the other way**
- a hose sends out 2.5 kg of water each second at 12 $\dfrac{\text{m}}{\text{s}}$: force on the water $F = \dfrac{\Delta m}{\Delta t} \times \Delta v = 2.5 \times 12 = 30$ N forwards, so the water pushes the hose **backwards** with 30 N and the person must push forwards with 30 N

Worked example: a ball bounces off a wall



A 0.20 kg ball hits a wall at 8.0 $\frac{\text{m}}{\text{s}}$ and rebounds at 6.0 $\frac{\text{m}}{\text{s}}$. The contact lasts 0.050 s. Find the average force on the ball.

1. **Known:** $m = 0.20$ kg, $\Delta t = 0.050$ s. Take **away from the wall** as positive: $u = -8.0\ \frac{\text{m}}{\text{s}}$, $v = +6.0\ \frac{\text{m}}{\text{s}}$.
2. **Change in momentum** – the definition of momentum, $p = mv$, applied before and after:

$$\Delta p = m(v - u) = 0.20 \times \big(6.0 - (-8.0)\big) = 2.8 \text{ kg m s}^{-1} \text{ (away from the wall)}$$

3. **Force** – Newton's second law, force is the rate of change of momentum:

$$F = \frac{\Delta p}{\Delta t} = \frac{2.8}{0.050} = 56 \text{ N on the ball, away from the wall}$$

4. **Third law:** the ball pushes the wall with 56 N **towards** the wall.

Worked example: the area under a force–time graph

A bat strikes a stationary 0.12 kg ball. The force rises steadily from zero to 240 N in 3.0 ms and falls steadily back to zero at 5.0 ms.

1. **Change in momentum** – the area under the force–time graph, from $\Delta p = F\,\Delta t$; the graph is a triangle:

$$\Delta p = \tfrac{1}{2} \times \text{base} \times \text{height} = \tfrac{1}{2} \times (5.0 \times 10^{-3}) \times 240 = 0.60 \text{ N s}$$

2. **Speed of the ball** – from $p = mv$, with $u = 0$: $v = \Delta p/m = 0.60/0.12 = 5.0\ \frac{\text{m}}{\text{s}}$.
3. **Force on the bat** – third law: the same triangle, the same 240 N peak, but in the **opposite** direction.

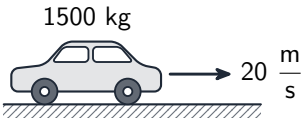
2 Practice

Now apply the methods above.

2.1 State **Newton's first law** of motion. [1]

Newton's first law

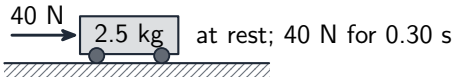
2.2 Calculate the **momentum** of a 1500 kg car travelling at 20 $\frac{\text{m}}{\text{s}}$. [2]



2.3 A resultant force of 40 N acts on a stationary trolley of mass 2.5 kg for 0.30 s.

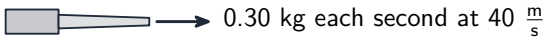
(a) Calculate the change in momentum of the trolley. [1]

(b) Calculate the velocity of the trolley after the 0.30 s. [2]



2.4 A leaf blower takes in still air and blows out 0.30 kg of air each second at 40 $\frac{\text{m}}{\text{s}}$.

(a) Calculate the force on the air. [1]



(b) State the size and the direction of the force of the air on the leaf blower. [1]

3 Exam-style questions

3.1 The unit kg m s^{-1} is the unit of which quantity? [1]

kg m s^{-1}

- **A** force
- **B** momentum
- **C** energy
- **D** power

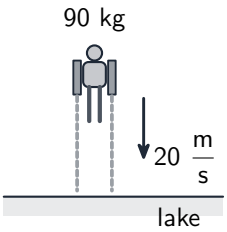
3.2 State **Newton's third law**, and give **one** example of a force pair. [2]

Newton's third law

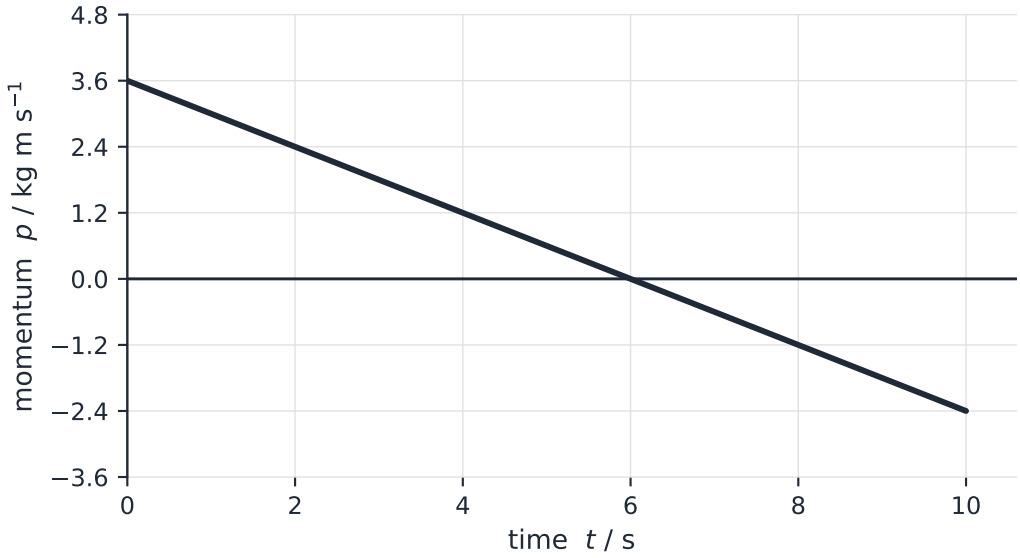
3.3 A water jet pack lifts a rider above a lake. The total mass of the rider and the pack is 90 kg. Water that starts from rest leaves the pack vertically downwards at 20 $\frac{m}{s}$.

(a) **Explain** how sending water downwards gives an upward force on the pack. [2]

(b) The rider stays at the same height. Calculate the mass of water that leaves the pack each second. [2]



3.4 A trolley of mass 1.2 kg moves in a straight line along a horizontal track. The graph shows how the momentum p of the trolley varies with time t.



(a) **Define** momentum. [1]

(b) **Determine** the speed of the trolley at time $t = 0$. [2]

(c) **Determine** the magnitude of the resultant force acting on the trolley. [3]

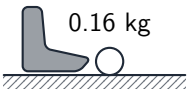
(d) **Describe** the motion of the trolley between $t = 0$ and $t = 10$ s. [2]

(e) By reference to the graph, **explain** why the resultant force acting on the trolley cannot be air resistance. [2]

(f) At time $t = 0$ the displacement of the trolley is zero. In the space below, **sketch** the variation with time of its displacement from $t = 0$ to $t = 10$ s. Numerical values are not required, but **label** both axes. [3]

3.5 A ball of mass 0.16 kg is at rest on the ground. A foot kicks it: the force on the ball rises uniformly from zero to a maximum of 180 N in 2.0 ms, then falls uniformly to zero at 6.0 ms.

(a) In the space below, **sketch** a graph of the force F on the ball against time t from $t = 0$ to $t = 6.0$ ms. Label the axes with values. [2]



(b) **Determine** the change in momentum of the ball. [2]

(c) Calculate the speed of the ball as it leaves the foot. [2]

(d) State the maximum force exerted by the ball on the foot, and name the law you used. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- Newton's first law: an object stays at **rest** or moves at **constant velocity** unless a **resultant (external) force** acts on it

2.2

- momentum is the product of mass and velocity, $p = mv$

$$p = mv = (1500 \text{ kg})(20 \frac{\text{m}}{\text{s}}) = \mathbf{3.0 \times 10^4} \text{ kg m s}^{\mathbf{-1}}$$

2.3

(a) from $F = \Delta p / \Delta t$: $\Delta p = F\,\Delta t = 40 \times 0.30 = \mathbf{12}$ kg m s$^{\mathbf{-1}}$

(b) from $p = mv$ with the trolley starting from rest

$$v = \frac{\Delta p}{m} = \frac{12 \text{ kg m s}^{-1}}{2.5 \text{ kg}} = \mathbf{4.8} \frac{\text{m}}{\text{s}}$$

2.4

(a) force is the rate of change of momentum: $F = \frac{\Delta m}{\Delta t}\,\Delta v = 0.30 \text{ kg s}^{-1} \times 40 \frac{\text{m}}{\text{s}} = \mathbf{12}$ N

(b) **12** N, **opposite** to the direction in which the air leaves the blower (Newton's third law)

3.1 B

- kg m s^{-1} is mass $\times$ velocity, which is **momentum** $p = mv$
- **A** force is kg m s^{-2} (rate of change of momentum)
- **C** energy is kg m^2 s^{-2}; **D** power is one more s^{-1}

3.2

- Newton's third law: when A exerts a force on B, B exerts an **equal and opposite** force of the **same type** on A
- example: a rocket pushes gas down, the gas pushes the rocket up (or foot / ground, hose / water)

3.3

(a) the pack pushes the water downwards, changing the momentum of the water

- by Newton's third law, the water pushes the pack **upwards** with an equal force

(b) at a steady height the upward force equals the weight: $F = F_W = mg = 90 \times 9.81 = 883$ N

- force is the rate of change of momentum, $F = \dfrac{\Delta m}{\Delta t}\Delta v$, so

$$\frac{\Delta m}{\Delta t} = \frac{F}{\Delta v} = \frac{883\ \text{N}}{20\ \frac{\text{m}}{\text{s}}} = \mathbf{44}\ \text{kg s}^{-1}$$

3.4

(a) momentum is the **product of mass and velocity**

(b) from $p = mv$

$$v = \frac{p}{m} = \frac{3.6\ \text{kg m s}^{-1}}{1.2\ \text{kg}} = \mathbf{3.0}\ \frac{\text{m}}{\text{s}}$$

(c) $\Delta p = p_{\text{after}} - p_{\text{before}} = (-2.4) - (+3.6) = -6.0$, a change of magnitude 6.0 kg m s^{-1}

- force is the rate of change of momentum, the gradient of the graph

$$F = \frac{\Delta p}{\Delta t} = \frac{6.0\ \text{kg m s}^{-1}}{10\ \text{s}} = \mathbf{0.60}\ \text{N}$$

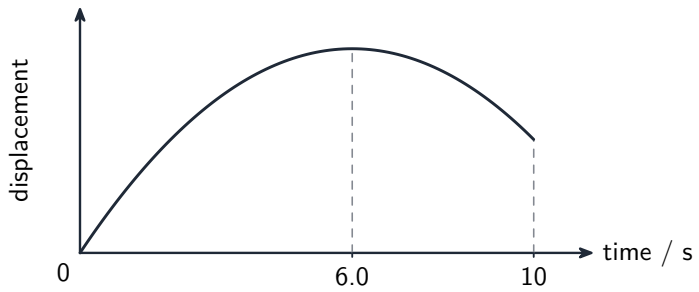
(d) the trolley slows **uniformly** (straight line, so a constant force) and is momentarily at rest at $t = 6.0$ s

- it then speeds up in the **opposite** direction

(e) the line is **straight**, so the resultant force is constant; air resistance would get **smaller** as the trolley slows

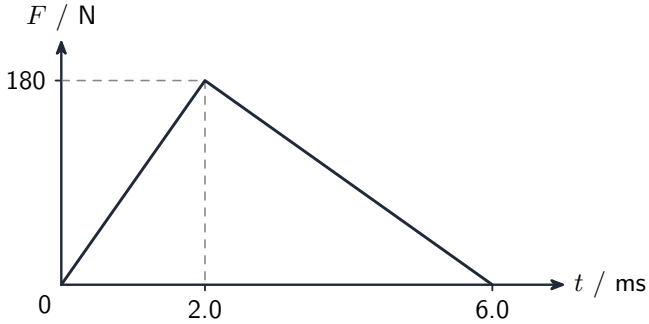
- at $t = 6.0$ s the trolley is at rest, so air resistance would be **zero**, yet the gradient is unchanged

(f)



3.5

(a)



(b) the change in momentum is the **area** under the force–time graph, from $\Delta p = F\,\Delta t$

$$\Delta p = \tfrac{1}{2} \times \text{base} \times \text{height} = \tfrac{1}{2} \times (6.0 \times 10^{-3}\ \text{s}) \times 180\ \text{N} = \mathbf{0.54}\ \text{N s}$$

(c) from $p = mv$ with $u = 0$

$$v = \frac{\Delta p}{m} = \frac{0.54\ \text{N s}}{0.16\ \text{kg}} = \mathbf{3.4}\ \frac{\text{m}}{\text{s}}$$

(d) **180** N, by Newton's **third** law (equal in magnitude, opposite in direction)