# 23.2 Radioactive decay

Name: _____ Class: _____ Date: _____ **Total: 32 marks**

**KEY WORDS** activity • half-life • spontaneous • decay constant • random

## Objective

Build the skills to answer exam questions on **radioactive decay**.

**You must be able to:**

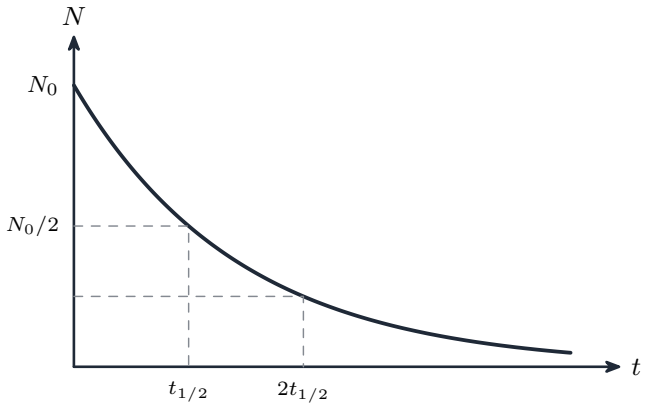
- describe decay as **spontaneous** and **random**
- use **activity** $A = \lambda N$ and $\lambda = 0.693/t_{1/2}$
- use the exponential decay $x = x_0 e^{-\lambda t}$ and **half-life**

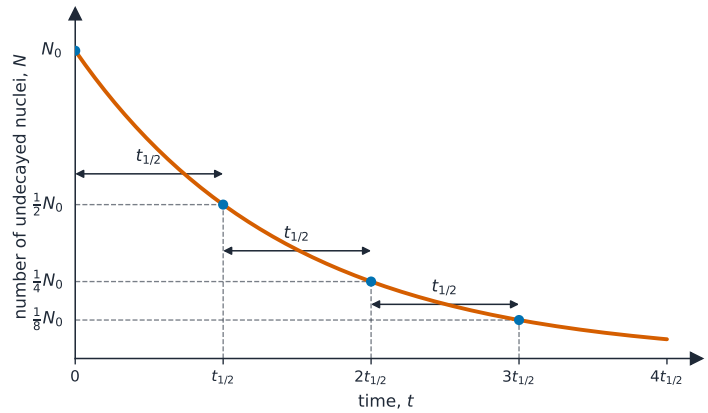
## 1 Worked examples

Study these first. Each one shows the method for a question type used later – follow the steps and you can do the Practice and Exam-style questions yourself.

### ■ Random and exponential

Decay is **spontaneous** (no trigger) and **random** (the count rate fluctuates). The number of undecayed nuclei falls **exponentially**, halving every **half-life**:




$N = N_0 e^{-\lambda t}$ · $T_{1/2} = \ln 2/\lambda$

*The exponential radioactive decay curve*

$$A = \lambda N, \qquad \lambda = \frac{0.693}{t_{1/2}}, \qquad N = N_0 e^{-\lambda t}.$$

- **activity** $A$ – decays per second (becquerel, Bq); **decay constant** $\lambda$ – probability of decay per second.

The same exponential law applies to the **activity** and the **count rate**: $A = A_0 e^{-\lambda t}$.

### ■ Finding the time – take logs

For a **whole number** of half-lives, just halve repeatedly (after $n$ half-lives a fraction $(\frac{1}{2})^n$ remains). For **any** time or fraction, take the natural log of $N = N_0 e^{-\lambda t}$:

$$\ln\frac{N}{N_0} = -\lambda t \quad \Longrightarrow \quad t = -\frac{1}{\lambda}\ln\frac{N}{N_0}.$$

**Worked example.** A source has half-life 8.0 days. How long until its activity falls to 20%?

- decay constant: $\lambda = \dfrac{0.693}{t_{1/2}} = \dfrac{0.693}{8.0} = 0.0866\ \text{day}^{-1}$
- time: $t = -\dfrac{1}{\lambda}\ln\dfrac{A}{A_0} = -\dfrac{1}{0.0866}\ln(0.20) = 18.6$ days

## 2 Practice

Now apply the methods above.

**2.1** State **two** features of radioactive decay. [2]

**2.2** Define **activity** and the **decay constant**. [2]

**2.3** Define **half-life**. [1]

**2.4** Write the equation linking **activity**, **decay constant** and **number** of nuclei. [1]

## 3 Exam-style questions

**3.1** The **half-life** is the time for: [1]

- ☐ **A** all the nuclei to decay
- **B** half the (undecayed) nuclei to decay
- **C** the activity to reach exactly zero
- **D** the decay constant to halve

**3.2** A source has a decay constant of $0.040\ \mathrm{s}^{-1}$. Calculate its **half-life**. [2]

**3.3** A sample has $2.0 \times 10^{20}$ undecayed nuclei and a decay constant of $1.5 \times 10^{-9}\ \mathrm{s}^{-1}$. Calculate its **activity**. [2]

**3.4** A radioactive isotope has a half-life of 6.0 hours. What **fraction** of the original nuclei remains after 18 hours? [2]

**3.5** A radioactive source has a half-life of 8.0 days. Calculate the **time** for its activity to fall to 20% of its initial value. [4]

**3.6** Oxygen-15 is used as a tracer in medical imaging. It decays by $\beta^+$ emission with a half-life of 122 s. A freshly prepared sample contains $4.8 \times 10^{14}$ nuclei.

(a) **State** what is meant by a **tracer**. [2]

(b) The decay is ${}^{15}_{8}\mathrm{O} \rightarrow {}^{P}_{Q}\mathrm{N} + {}^{R}_{S}\mathrm{Z}$. **State** the values of $P$, $Q$, $R$ and $S$, and **name** the particle Z. [3]

(c) **Define** the **activity** of a radioactive sample. [1]

(d) **Determine** the decay constant of oxygen-15. Give a **unit** with your answer. [2]

(e) **Determine** the initial activity of the sample. [2]

(f) **Determine** the activity 10 minutes after preparation. [3]

(g) **Suggest** why a tracer with a half-life of a few **days** would be unsuitable for this use. [2]

## Solutions

Each answer shows the steps, then the **result**.

**2.1**

- it is **spontaneous** (unaffected by external conditions)
- and **random** (cannot predict which nucleus decays when)

**2.2**

- **activity** – the number of decays per unit time
- **decay constant** – the probability per unit time that a nucleus decays

**2.3**

- the time for **half** the undecayed nuclei (or the activity) to decay

**2.4**

- $A = \lambda N$

**3.1 B**

- half-life is the time for **half the undecayed nuclei to decay**

**3.2**

- $t_{1/2} = 0.693/\lambda$

$$t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.040\ \text{s}^{-1}} = \mathbf{17}\ \text{s}$$

**3.3**

- $A = \lambda N$

$$A = \lambda N = (1.5 \times 10^{-9}\ \text{s}^{-1})(2.0 \times 10^{20}) = \mathbf{3.0 \times 10^{11}}\ \text{Bq}$$

**3.4**

- 18 h = 3 half-lives
- fraction $= (1/2)^3 = \mathbf{1/8}$

**3.5**

- $\lambda = 0.693/t_{1/2}$

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{8.0\ \text{day}} = 0.0866\ \text{day}^{-1}$$

- $t = -(1/\lambda)\ln(A/A_0)$

$$t = -\frac{1}{\lambda}\ln\frac{A}{A_0} = -\frac{1}{0.0866\ \text{day}^{-1}}\ln(0.20) = \mathbf{18.6}\ \text{days}$$

**3.6**

(a) a radioactive substance introduced into a system

- whose progress can be followed from outside by detecting the radiation it emits

(b) $P = 15$, $Q = 7$, $R = 0$, $S = +1$

- Z is a **positron**

(c) the **rate of decay** – the number of nuclei decaying per unit time

(d) $\lambda = \ln 2/t_{1/2}$

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{122\ \text{s}} = \mathbf{5.7 \times 10^{-3}}\ \text{s}^{-1}$$

(e) $A = \lambda N$

$$A = \lambda N = (5.68 \times 10^{-3}\ \text{s}^{-1})(4.8 \times 10^{14}) = \mathbf{2.7 \times 10^{12}}\ \text{Bq}$$

(f) 10 minutes is 600 s

$$A = A_0 e^{-\lambda t} = (2.73 \times 10^{12}\ \text{Bq})\, e^{-(5.68\times10^{-3}\ \text{s}^{-1})(600\ \text{s})} = \mathbf{9.1 \times 10^{10}}\ \text{Bq}$$

- about five half-lives, so roughly 1/32 of the start

(g) the patient would keep receiving **ionising radiation** long after the scan is over, raising the dose and the risk of cell damage

- the tracer should decay away soon after the measurement, which a half-life of minutes achieves and one of days does not