2.1.5 Projectile motion

Name: _____ Class: _____ Date: _____ **Total: 35 marks**

KEY WORDS projectile • component • range • horizontal • velocity

Objective

Answer exam questions on **projectile motion**: a **uniform velocity** and, **perpendicular** to it, a **uniform acceleration**. Syllabus 2.1, outcome 9.

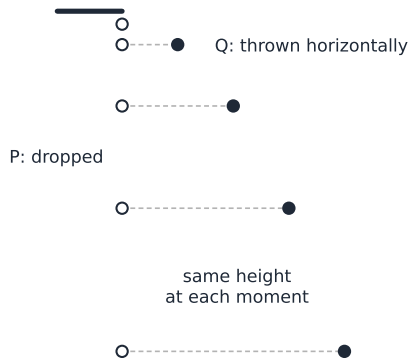
You must be able to:

- explain why the horizontal velocity of a **projectile** is constant
- **resolve** a launch velocity into **components**
- solve a horizontal launch, an angled launch, and a launch towards a wall or a target
- give a velocity as a size and a direction, and sketch its components against time
- say what a change of mass, or **air resistance**, does to the flight

1 Key knowledge and worked examples

■ Key knowledge: two independent motions

The only force is the weight, so the two motions are independent. They share only the time t.



P is dropped and Q is thrown horizontally at the same moment: the same height at every moment

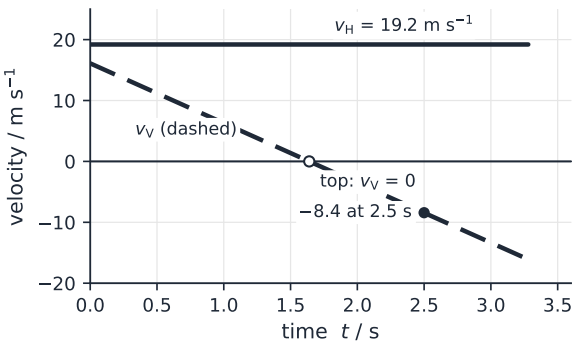
	horizontal	**vertical, up positive**
force, so acceleration	none, so 0	the weight, so $a = -9.81\ \mathrm{m\ s^{-2}}$
velocity	$v_H = u_H$, constant	$v_V = u_V + at$
displacement	$x = u_H t$	$y = u_V t + \frac{1}{2}at^2$
velocity–time graph	a horizontal line	a straight line, gradient $-9.81\ \mathrm{m\ s^{-2}}$

Moment	**What you know**
launch, u at θ to the horizontal	$u_H = u\cos\theta$ and $u_V = u\sin\theta$
horizontal launch	$u_V = 0$, so the time depends only on the height
highest point	$v_V = 0$; the velocity is u_H, **not zero**; the acceleration is still g
landing at the launch level	flight time $= 2\times$ time to the top; same speed; **range** $= u_H \times$ flight time

■ Key knowledge: the method, and what changes the flight

1. Resolve the launch velocity.
2. Find the **time**: from the vertical motion if it falls to the ground, from the horizontal motion if the distance to a wall or a target is given.
3. Use that time in the other direction.
4. Velocity: $v = \sqrt{v_H^2 + v_V^2}$ and $\tan\theta = v_V/v_H$, above or below the horizontal.

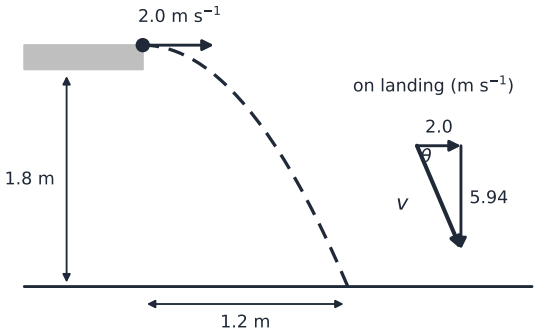
- **mass**: no effect, because g is the same for every mass
- air resistance: the horizontal velocity decreases, so a lower greatest height, a shorter range and a path that is not symmetrical



The exam asks you to sketch these: a horizontal line, and a straight line that crosses zero at the top (the ball of the second example)

■ Worked example: a horizontal launch

A ball rolls off a horizontal shelf 1.8 m above the floor at 2.0 $\frac{\text{m}}{\text{s}}$.



- vertical motion from rest, $s = \frac{1}{2}at^2$, gives the time; then the horizontal motion, $x = u_\text{H}t$, gives the distance

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 1.8\ \text{m}}{9.81\ \text{m s}^{-2}}} = 0.606\ \text{s}, \qquad x = u_\text{H}t = (2.0\ \frac{\text{m}}{\text{s}})(0.606\ \text{s}) = 1.2\ \text{m}$$

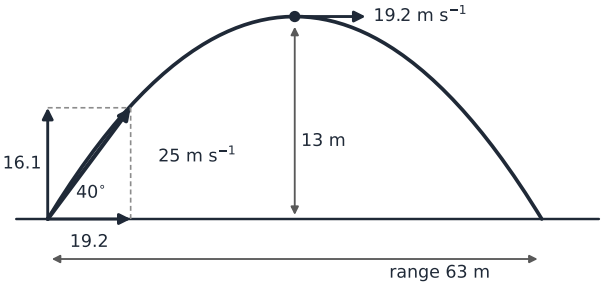
- on landing $v_\text{H} = 2.0\ \frac{\text{m}}{\text{s}}$, and $v = u + at$ gives $v_\text{V} = (9.81\ \text{m s}^{-2})(0.606\ \text{s}) = 5.94\ \frac{\text{m}}{\text{s}}$; combine them with $v = \sqrt{v_\text{H}^2 + v_\text{V}^2}$ and $\tan\theta = v_\text{V}/v_\text{H}$: $6.3\ \frac{\text{m}}{\text{s}}$ at 71° below the horizontal

$$v = \sqrt{v_\text{H}^2 + v_\text{V}^2} = \sqrt{(2.0\ \frac{\text{m}}{\text{s}})^2 + (5.94\ \frac{\text{m}}{\text{s}})^2} = 6.3\ \frac{\text{m}}{\text{s}}$$

$$\tan\theta = \frac{v_\text{V}}{v_\text{H}} = \frac{5.94\ \frac{\text{m}}{\text{s}}}{2.0\ \frac{\text{m}}{\text{s}}} \quad \Rightarrow \quad \theta = 71°$$

■ Worked example: an angled launch

A ball is kicked from level ground at 25 $\frac{\text{m}}{\text{s}}$ at 40° above the horizontal.



- components, $u_\text{H} = u\cos\theta$ and $u_\text{V} = u\sin\theta$

$$u_\text{H} = (25\ \frac{\text{m}}{\text{s}})\cos 40° = 19.2\ \frac{\text{m}}{\text{s}}, \qquad u_\text{V} = (25\ \frac{\text{m}}{\text{s}})\sin 40° = 16.1\ \frac{\text{m}}{\text{s}}$$

- at the top $v_\text{V} = 0$ (up positive): $v = u + at$ gives the time, and $v^2 = u^2 + 2as$ gives the height

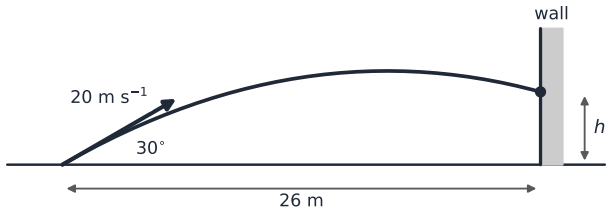
$$t_\text{top} = \frac{v_\text{V} - u_\text{V}}{a} = \frac{0 - 16.1\ \frac{\text{m}}{\text{s}}}{-9.81\ \text{m s}^{-2}} = 1.64\ \text{s}$$

$$H = \frac{v_\text{V}^2 - u_\text{V}^2}{2a} = \frac{0 - (16.1\ \frac{\text{m}}{\text{s}})^2}{2(-9.81\ \text{m s}^{-2})} = 13\ \text{m}$$

- same level, so the flight takes $t = 2t_\text{top} = 2 \times 1.64\ \text{s} = 3.28\ \text{s}$, and the range is $x = u_\text{H}t = (19.2\ \frac{\text{m}}{\text{s}})(3.28\ \text{s}) = 63\ \text{m}$
- at $t = 2.5$ s, the point marked on the graph, $v = u + at$ gives $v_\text{V} = 16.1\ \frac{\text{m}}{\text{s}} + (-9.81\ \text{m s}^{-2})(2.5\ \text{s}) = -8.4\ \frac{\text{m}}{\text{s}}$, so $v = \sqrt{19.2^2 + 8.4^2} = 21\ \frac{\text{m}}{\text{s}}$ at 24° below the horizontal

■ Worked example: reaching a wall or a target

A ball is kicked from level ground at 20 $\frac{\text{m}}{\text{s}}$ at 30° above the horizontal, towards a wall 26 m away.



- the distance to the wall is horizontal, so the **horizontal** motion, $x = u_H t$, gives the time

$$u_H = u\cos\theta = (20\ \frac{\text{m}}{\text{s}})\cos 30° = 17.3\ \frac{\text{m}}{\text{s}}, \qquad t = \frac{x}{u_H} = \frac{26\ \text{m}}{17.3\ \frac{\text{m}}{\text{s}}} = 1.50\ \text{s}$$

- the vertical motion, $y = u_V t + \frac{1}{2}at^2$, then gives the height, with $u_V = u\sin\theta = (20\ \frac{\text{m}}{\text{s}})\sin 30° = 10.0\ \frac{\text{m}}{\text{s}}$

$$h = u_V t + \tfrac{1}{2}at^2 = (10.0\ \frac{\text{m}}{\text{s}})(1.50\ \text{s}) + \tfrac{1}{2}(-9.81\ \text{m s}^{-2})(1.50\ \text{s})^2 = 4.0\ \text{m}$$

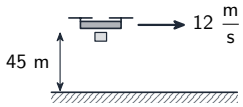
- at the wall $v = u + at$ gives $v_V = -4.7\ \frac{\text{m}}{\text{s}}$: the ball is already coming down

2 Practice

2.1 A delivery drone flies horizontally at 12 $\frac{\text{m}}{\text{s}}$, 45 m above flat ground, and releases a parcel. Air resistance is negligible.

(a) State the horizontal and vertical components of the parcel's velocity just after it is released. [1]

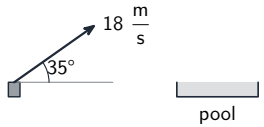
(b) Calculate the time taken for the parcel to reach the ground. [2]



(c) Calculate the speed of the parcel as it reaches the ground. [2]

2.2 Water leaves the **nozzle** of a fountain at 18 $\frac{\text{m}}{\text{s}}$ at 35° above the horizontal, and lands in a pool at the same height as the nozzle. Treat each drop of water as a projectile, and ignore the effect of the air.

(a) Calculate the horizontal and vertical components of the velocity of the water as it leaves the nozzle. [2]



(b) Calculate the greatest height of the water above the nozzle. [2]

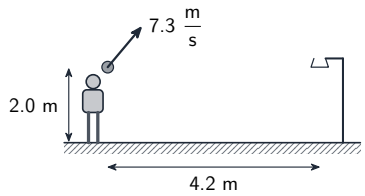
(c) Calculate the horizontal distance from the nozzle to where the water lands. [2]

2.3 For a drop of the water in **2.3**, sketch on one set of axes how the horizontal and vertical components of its velocity change with time, from the nozzle until it lands. Take up as positive, label each line, and mark the values at the start and at the end. [3]

2.4 A basketball player throws the ball at 7.3 $\frac{\text{m}}{\text{s}}$ at 50° above the horizontal. The ball leaves her hands 2.0 m above the floor, and the hoop is a horizontal distance of 4.2 m away. Air resistance is negligible.

(a) **Show that** the ball takes 0.90 s to reach the hoop. *(Hint: the distance to the hoop is horizontal.)* [2]

(b) Calculate the height of the ball above the floor at that moment. [3]



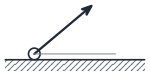
3 Exam-style questions

3.1 A ball is thrown at an angle above horizontal ground. Air resistance is negligible. Which row describes the ball at the highest point of its path? [1]

	velocity	acceleration
A	zero	zero
B	zero	9.81 m s^{-2} downwards
C	horizontal	zero
D	horizontal	9.81 m s^{-2} downwards

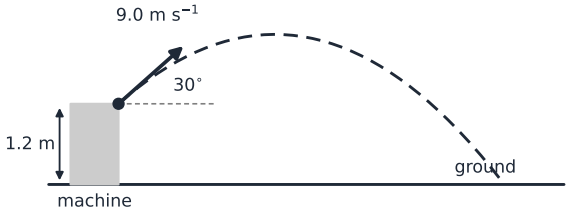
3.2 A golf ball is hit from level ground at an angle above the horizontal.

(a) A ball of greater mass is hit with the same velocity. Air resistance is still negligible. State and explain the effect, if any, on the time the ball spends in the air. [2]



(b) In practice, air resistance is not negligible. State and explain the effect of air resistance on the horizontal distance the ball travels. [2]

3.3 A ball-launching machine is 1.2 m above level ground. It fires a ball at 9.0 $\frac{\text{m}}{\text{s}}$ at 30° above the horizontal. Air resistance is negligible.



(a) Calculate the horizontal and vertical components of the ball's initial velocity. [2]

(b) Calculate the greatest height of the ball above the ground. [2]

(c) **Show that** the ball takes about 1.1 s to reach the ground. [3]

(d) Calculate the horizontal distance from the machine to where the ball lands. [1]

(e) Calculate the speed of the ball as it lands, and the angle its velocity makes with the ground. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

(a) horizontal 12 $\frac{\text{m}}{\text{s}}$ (the drone's velocity) and vertical 0

(b) vertical motion from rest, down positive; use $s = \frac{1}{2}at^2$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 45\ \text{m}}{9.81\ \text{m s}^{-2}}} = \mathbf{3.0}\ \text{s}$$

(c) vertically $v = u + at$ gives $v_\text{V} = (9.81\ \text{m s}^{-2})(3.03\ \text{s}) = 29.7\ \frac{\text{m}}{\text{s}}$; then combine the two components

$$v = \sqrt{v_\text{H}^2 + v_\text{V}^2} = \sqrt{(12\ \frac{\text{m}}{\text{s}})^2 + (29.7\ \frac{\text{m}}{\text{s}})^2} = \mathbf{32}\ \frac{\text{m}}{\text{s}}$$

2.2

(a) resolve: $u_\text{H} = u\cos\theta$ and $u_\text{V} = u\sin\theta$

$$u_\text{H} = (18\ \frac{\text{m}}{\text{s}})\cos 35^\circ = \mathbf{14.7}\ \frac{\text{m}}{\text{s}}, \qquad u_\text{V} = (18\ \frac{\text{m}}{\text{s}})\sin 35^\circ = \mathbf{10.3}\ \frac{\text{m}}{\text{s}}$$

(b) at the top $v_\text{V} = 0$; use $v^2 = u^2 + 2as$ vertically

$$H = \frac{v_\text{V}^2 - u_\text{V}^2}{2a} = \frac{0 - (10.3\ \frac{\text{m}}{\text{s}})^2}{2(-9.81\ \text{m s}^{-2})} = \mathbf{5.4}\ \text{m}$$

(c) time to the top from $v = u + at$ is $10.3/9.81 = 1.05$ s, so the flight takes 2.10 s; then $x = u_\text{H}t$

$$R = u_\text{H}t = (14.7\ \frac{\text{m}}{\text{s}})(2.10\ \text{s}) = \mathbf{31}\ \text{m}$$

2.3

- horizontal component: a horizontal line at 14.7 $\frac{\text{m}}{\text{s}}$ from 0 to 2.1 s
- vertical component: a straight line from +10.3 $\frac{\text{m}}{\text{s}}$ at $t = 0$
- through zero at 1.05 s to −10.3 $\frac{\text{m}}{\text{s}}$ at 2.1 s, with both lines labelled

2.4

(a) the horizontal velocity is constant, $u_\text{H} = (7.3\ \frac{\text{m}}{\text{s}})\cos 50^\circ = 4.69\ \frac{\text{m}}{\text{s}}$; use $x = u_\text{H}t$

$$t = \frac{x}{u_\text{H}} = \frac{4.2\ \text{m}}{4.69\ \frac{\text{m}}{\text{s}}} = 0.895\ \text{s} \approx 0.90\ \text{s}$$

(b) vertical motion, up positive, with $u_\text{V} = (7.3\ \frac{\text{m}}{\text{s}})\sin 50^\circ = 5.59\ \frac{\text{m}}{\text{s}}$; use $y = u_\text{V}t + \frac{1}{2}at^2$

$$y = u_\text{V}t + \tfrac{1}{2}at^2 = (5.59\ \frac{\text{m}}{\text{s}})(0.90\ \text{s}) + \tfrac{1}{2}(-9.81\ \text{m s}^{-2})(0.90\ \text{s})^2 = 1.06\ \text{m}$$

- height above the floor $h = h_\text{table} + y = 2.0\ \text{m} + 1.06\ \text{m} = \mathbf{3.1}\ \text{m}$

3.1 D

- at the top the vertical component is zero, but the horizontal component is not, so the velocity is horizontal
- the acceleration is g downwards during the whole flight, including at the top
- **A** and **B** forget the horizontal velocity; **C** thinks the acceleration stops at the top

3.2

(a)

- the time in the air does not change
- the acceleration of free fall is the same for every mass, and the vertical component of the launch velocity is the same

(b)

- the horizontal distance is smaller
- air resistance acts against the motion, so the horizontal component of the velocity decreases during the flight

3.3

(a) resolve: $u_\text{H} = u\cos\theta$ and $u_\text{V} = u\sin\theta$

$$u_\text{H} = (9.0\ \frac{\text{m}}{\text{s}})\cos 30^\circ = \mathbf{7.8}\ \frac{\text{m}}{\text{s}}, \qquad u_\text{V} = (9.0\ \frac{\text{m}}{\text{s}})\sin 30^\circ = \mathbf{4.5}\ \frac{\text{m}}{\text{s}}$$

(b) rise above the machine, with $v_\text{V} = 0$ at the top; use $v^2 = u^2 + 2as$

$$s = \frac{v^2 - u^2}{2a} = \frac{0 - (4.5\ \frac{\text{m}}{\text{s}})^2}{2(-9.81\ \text{m s}^{-2})} = 1.03\ \text{m}$$

- height above the ground $h = s + h_\text{machine} = 1.03\ \text{m} + 1.2\ \text{m} = \mathbf{2.2}\ \text{m}$

(c) up positive: $s = -1.2$ m, $u = +4.5\ \frac{\text{m}}{\text{s}}$, $a = -9.81\ \text{m s}^{-2}$ in $s = ut + \frac{1}{2}at^2$

$$-1.2 = 4.5t - 4.905t^2 \quad \Rightarrow \quad 4.905t^2 - 4.5t - 1.2 = 0$$

- quadratic formula, values in m and s

$$t = \frac{4.5 + \sqrt{20.25 + 23.5}}{9.81} = 1.13\ \text{s} \approx 1.1\ \text{s}$$

(d) $x = u_\text{H}t = (7.8\ \frac{\text{m}}{\text{s}})(1.13\ \text{s}) = \mathbf{8.8}\ \text{m}$

(e) $v = u + at$ gives $v_{\mathrm{V}} = 4.5\ \frac{\mathrm{m}}{\mathrm{s}} + (-9.81\ \mathrm{m\ s^{-2}})(1.13\ \mathrm{s}) = -6.6\ \frac{\mathrm{m}}{\mathrm{s}}$; then combine the components

$$v = \sqrt{v_{\mathrm{H}}^2 + v_{\mathrm{V}}^2} = \sqrt{(7.8\ \frac{\mathrm{m}}{\mathrm{s}})^2 + (6.6\ \frac{\mathrm{m}}{\mathrm{s}})^2} = \mathbf{10}\ \frac{\mathrm{m}}{\mathrm{s}}$$

- $\tan\theta = 6.6/7.8$, so the velocity is at $\mathbf{40}^\circ$ below the horizontal