10.2 Kirchhoff's laws and combining resistors

Name: _____ Class: _____ Date: _____ **Total: 18 marks**

KEY WORDS Kirchhoff's first law • Kirchhoff's second law • series • parallel • junction

Objective

Answer exam questions on **Kirchhoff's laws** : the two laws and the conservation law behind each, the derivations of the series and parallel resistance formulas, and simple series and parallel circuits. Syllabus 10.2, learning outcomes 1 to 6.

You must be able to:

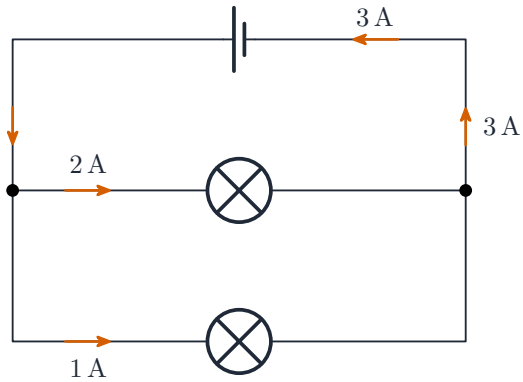
- state Kirchhoff's first and second laws, and the conservation law each follows from
- derive $R = R_1 + R_2$ for resistors in series and $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ for resistors in parallel, naming the law at each step
- use both formulas, and find the current at a junction
- find the current and the p.d.s in a series circuit

1 Key knowledge and worked examples

Key knowledge: use these earlier methods

- **NTC thermistor**: resistance falls as temperature rises. In a potential divider, $V_{\text{out}} = V_{\text{in}} R_{\text{out}}/(R_1 + R_2)$; identify which resistor the output is across before predicting a change.
- **Source with internal resistance**: $V = \mathcal{E} - Ir$; $\mathcal{E} = I(R+r)$ for a single external resistance R. A larger current causes a larger lost voltage Ir.
- Series resistances add; parallel branches share voltage and their currents add.

■ Key knowledge: Kirchhoff's laws



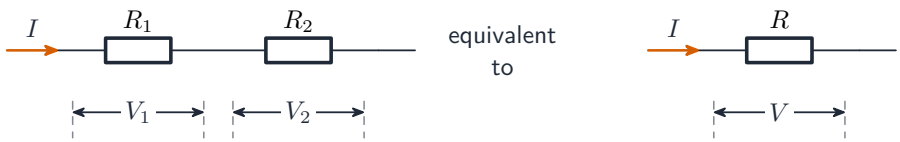
at each junction: $3\,\mathrm{A} = 2\,\mathrm{A} + 1\,\mathrm{A}$

- **First law.** At any **junction** , the total current in equals the total current out. Charge cannot pile up at a point, so this is conservation of charge.
- **Second law.** Round any closed loop, the total **electromotive force** equals the total p.d. across the components. A unit of charge gives up exactly the energy it gained, so this is conservation of energy.
- The pairing is examined almost every session, and the two are easy to swap. First law → charge. Second law → energy.

Combination	Same for each resistor	Combined resistance
series	current	$R = R_1 + R_2 + \ldots$: always larger than the largest
parallel	p.d.	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$: always smaller than the smallest

- two equal resistors R in parallel give $\frac{R}{2}$; n equal ones give $\frac{R}{n}$

■ Worked example: deriving the series formula



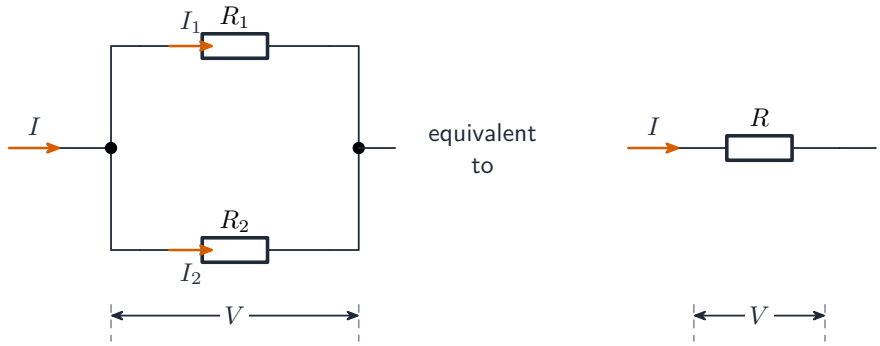
A derivation must name the law at each step; that is what earns the marks.

1. There is no junction between the resistors, so by the **first law** the same current

I is in both.

2. By the **second law**, the p.d. across the combination is the sum of the p.d.s: $V = V_1 + V_2$.
3. Each p.d. is IR, so $IR = IR_1 + IR_2$.
4. Divide every term by I: $R = R_1 + R_2$.

■ Worked example: deriving the parallel formula



1. Each resistor forms its own loop with the source, so by the **second law** the same p.d. V is across both.
2. By the **first law** the current arriving at the junction is shared: $I = I_1 + I_2$.
3. Each current is V/R, so $\dfrac{V}{R} = \dfrac{V}{R_1} + \dfrac{V}{R_2}$.
4. Divide every term by V: $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$.

■ Worked example: a series circuit

A 12 V battery of negligible internal resistance drives a current through a 3.0 Ω and a 6.0 Ω resistor in series.

1. **Combined resistance** – in series the resistances add:

$$R = R_1 + R_2 = 3.0\ \Omega + 6.0\ \Omega = 9.0\ \Omega$$

2. **Current** – from $R = \dfrac{V}{I}$; it is the same in both resistors (first law):

$$I = \frac{V}{R} = \frac{12\ \text{V}}{9.0\ \Omega} = 1.33\ \text{A}$$

3. **P.d.s** – $V = IR$ for each resistor; the two add to the supply's 12 V (second law):

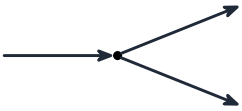
$$V_1 = IR_1 = (1.33\ \text{A})(3.0\ \Omega) = 4.0\ \text{V}$$

$$V_2 = IR_2 = (1.33\ \text{A})(6.0\ \Omega) = 8.0\ \text{V}$$

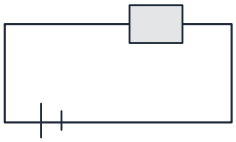
2 Practice

Now apply the methods above.

2.1 State **Kirchhoff's first law** and which conservation law it follows from. [2]



2.2 State **Kirchhoff's second law** and which conservation law it follows from. [2]

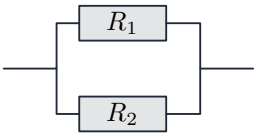


2.3 Find the combined resistance of 4.0 Ω and 6.0 Ω in **series**. [1]

4.0 Ω

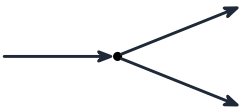
6.0 Ω

2.4 Find the combined resistance of two 4.0 Ω resistors in **parallel**. [2]



4.0 Ω

2.5 At a junction, 5.0 A enters and one branch carries 2.0 A away. Find the current in the other branch. [1]



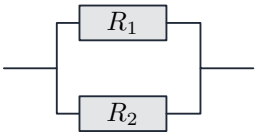
5.0 A

2.0 A

3 Exam-style questions

3.1 Three 6.0 Ω resistors in **parallel** have a combined resistance of: [1]

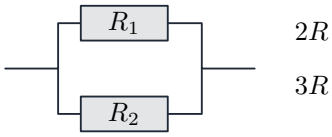
- **A** 2.0 Ω
- **B** 6.0 Ω
- **C** 18 Ω
- **D** 0.50 Ω



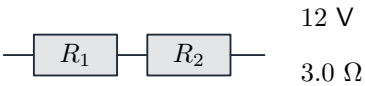
6.0 Ω

3.2 Two resistors of resistances $2R$ and $3R$ are connected in parallel. What is their combined resistance? [1]

- **A** $\frac{5}{6}R$
- **B** $\frac{6}{5}R$
- **C** $\frac{5}{2}R$
- **D** $5R$



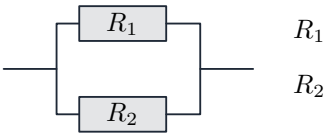
3.3 A 12 V battery drives current through a 3.0 Ω and a 6.0 Ω resistor in **series**. Calculate the current and the p.d. across each resistor. [4]



3.4 Two resistors of resistances R_1 and R_2 are connected in parallel. Use Kirchhoff's laws to show that the combined resistance R is given by

$$R = \frac{R_1 R_2}{R_1 + R_2}.$$

Name the law used at each step. [4]



Solutions

Each answer shows the steps, then the **result**.

2.1

- the **total current into a junction equals the total current out**
- it follows from conservation of **charge**

2.2

- around any closed loop, the **total e.m.f. equals the total p.d.**
- it follows from conservation of **energy**

2.3

- series: $R = R_1 + R_2$

$$R = R_1 + R_2 = 4.0\ \Omega + 6.0\ \Omega = \mathbf{10}\ \Omega$$

2.4

- parallel: $1/R = 1/R_1 + 1/R_2$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4.0\ \Omega} + \frac{1}{4.0\ \Omega} = \frac{1}{2.0\ \Omega} \Rightarrow R = \mathbf{2.0}\ \Omega$$

2.5

- Kirchhoff 1: current in = current out

$$I = I_{\text{in}} - I_1 = 5.0\ \text{A} - 2.0\ \text{A} = \mathbf{3.0}\ \text{A}$$

3.1 A

- three equal resistors in parallel: $R = R_1/3$

$$R = \frac{R_1}{3} = \frac{6.0\ \Omega}{3} = \mathbf{2.0}\ \Omega$$

- **B** is one resistor; **C** is series; **D** would be twelve in parallel

3.2 B

- parallel formula: $\dfrac{1}{R_{\text{total}}} = \dfrac{1}{2R} + \dfrac{1}{3R} = \dfrac{5}{6R}$, so $R_{\text{total}} = \frac{6}{5}R$
- **D** adds them as if in series; **A** forgets to take the reciprocal

3.3

- in series the resistances add

$$R = R_3 + R_6 = 3.0\ \Omega + 6.0\ \Omega = 9.0\ \Omega$$

- the current from $R = \dfrac{V}{I}$

$$I = \frac{V}{R} = \frac{12\ \text{V}}{9.0\ \Omega} = \mathbf{1.33}\ \text{A}$$

- the p.d. across each resistor from $V = IR$

$$V_3 = IR_3 = (1.33\ \text{A})(3.0\ \Omega) = \mathbf{4.0}\ \text{V}$$

$$V_6 = IR_6 = (1.33\ \text{A})(6.0\ \Omega) = \mathbf{8.0}\ \text{V}$$

3.4

- by **Kirchhoff's second law** each resistor is in its own loop with the source, so the same p.d. V is across both
- by **Kirchhoff's first law** the currents at the junction add: $I = I_1 + I_2$
- substitute $I = V/R$ for each: $\dfrac{V}{R} = \dfrac{V}{R_1} + \dfrac{V}{R_2}$, so $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$
- put the right-hand side over a common denominator and invert:

$$\frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2} \Rightarrow R = \frac{R_1 R_2}{R_1 + R_2}$$