

10.1.1 Internal resistance and the V-I graph

Name: _____ Class: _____ Date: _____

Total: 27 marks

KEY WORDS internal resistance • terminal potential difference
 • electromotive force • gradient • intercept

Objective

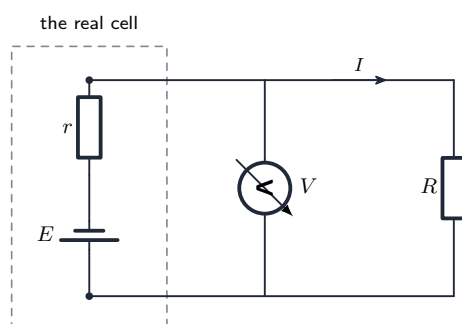
Answer exam questions on **internal resistance** : why the **terminal potential difference** of a real source is less than its e.m.f., the equation $E = V + Ir$, the limiting cases, and the V - I graph that measures E and r . Syllabus 10.1, learning outcome 5.

You must be able to:

- explain the effect of internal resistance on the terminal p.d., in terms of energy
- use $E = V + Ir = I(R + r)$ to find a current, a terminal p.d. or an internal resistance
- state what happens when no current is drawn and when the terminals are shorted
- find E and r from the intercept and gradient of a graph of V against I , and the power wasted inside the source

1 Key knowledge and worked examples

■ Key knowledge: internal resistance



V is the **terminal p.d.**:
what the rest of the
circuit actually receives

Each coulomb is given E joules by the source, gives up Ir joules inside the cell and V joules outside it, so $E = V + Ir$, or $V = E - Ir$. Nothing is lost – the missing energy is dissipated *inside* the cell, which is why a battery gets warm.

Follow one coulomb round the circuit:

$$E = V + Ir \quad \Longleftrightarrow \quad V = E - Ir \quad \Longleftrightarrow \quad E = I(R + r)$$

- It is given E joules by the source.
- It gives up Ir joules **inside** the cell, warming it – these are sometimes called the “lost volts”.
- It gives up V joules **outside**, in the rest of the circuit. V is the **terminal p.d.**: what the circuit actually receives.

Two limiting cases the exam likes:

situation	what happens
no current drawn ($I = 0$)	$V = E$: the terminal p.d. equals the e.m.f., because nothing is lost inside
terminals shorted ($R = 0$)	$I = \frac{E}{r}$, the largest current the cell can deliver, and all of the energy is wasted inside it

■ Worked example: current and terminal p.d.

A cell of e.m.f. 1.5 V and internal resistance 0.50 Ω is connected to a 2.5 Ω resistor.

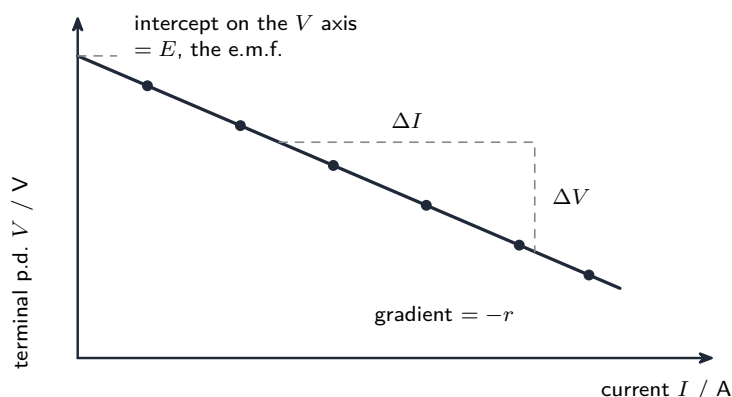
- the e.m.f. drives the current through **both** resistances

$$I = \frac{E}{R + r} = \frac{1.5 \text{ V}}{2.5 \Omega + 0.50 \Omega} = 0.50 \text{ A}$$

- then the terminal p.d. is what is left after the internal drop

$$V = E - Ir = 1.5 \text{ V} - (0.50 \text{ A})(0.50 \Omega) = \mathbf{1.25 \text{ V}}$$

■ Worked example: the experiment, one graph, two answers



Vary the external resistance, plot V against I , and $V = E - Ir$ gives a straight line: the **intercept** is the e.m.f. and the **gradient** is minus the internal resistance. Extrapolating back to $I = 0$ is how you measure an e.m.f. you can never read directly, because any real voltmeter draws some current.

Vary the external resistance, record V and I each time, and plot V against I . Since

$$V = E - rI$$

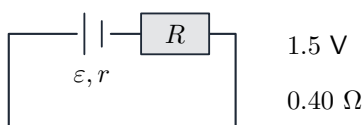
is $y = c + mx$:

- the **intercept** on the V axis is the **e.m.f.** E ;
- the **gradient** is $-r$, so the internal resistance is **minus** the gradient, and it is positive.
- This is how an e.m.f. is measured, because a real voltmeter always draws a **small current**, so the reading straight across the cell is always slightly **less** than E . The graph extrapolates to the $I = 0$ case that cannot quite be reached.
- **Power:** the source supplies EI in total, of which I^2r is wasted inside and VI is delivered outside.

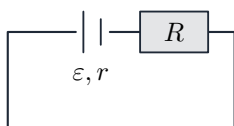
2 Practice

Now use the methods above.

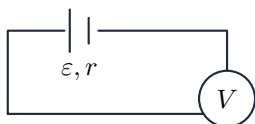
2.1 A cell of e.m.f. 1.5 V and internal resistance $0.40 \, \Omega$ delivers a current of 0.60 A . Calculate the terminal p.d. [2]



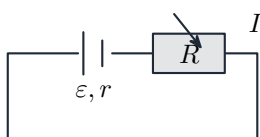
2.2 Explain why the terminal p.d. of a cell falls as the current drawn from it increases.[2]



2.3 State the terminal p.d. of a cell when **no** current is drawn from it, and explain your answer. [2]



2.4 Calculate the largest current the cell in **2.3** could deliver, and state the condition under which this happens. [2]



3 Exam-style questions

3.1 As the current drawn from a cell increases, the terminal potential difference [1]

☐

A increases, because more energy is transferred each second.

B stays the same, because the e.m.f. of the cell is fixed.

C decreases, because more energy is dissipated in the internal resistance.

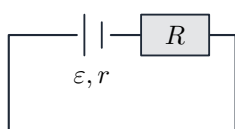
D decreases, because the e.m.f. of the cell falls.



3.2 A battery of e.m.f. 6.0 V and internal resistance $0.80\ \Omega$ is connected to a lamp of resistance $4.0\ \Omega$.

(a) Explain why the p.d. across the terminals of the battery is less than its e.m.f. when the lamp is lit. [2]

(b) Calculate the current in the lamp. [2]



6.0 V

$0.80\ \Omega$

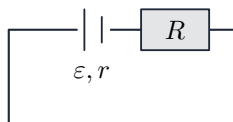
(c) Calculate the terminal p.d. [1]

(d) Calculate the power wasted inside the battery. [2]

3.3 A student varied the resistance of the external circuit and recorded the terminal p.d. V and the current I .

I / A	0.20	0.45	0.70	0.95	1.20
V / V	5.72	5.45	5.18	4.91	4.64

(a) State the equation relating V and I for this circuit, and state what the gradient and the intercept of a graph of V against I represent. [3]



(b) Use the first and last pairs of readings to calculate the internal resistance of the source. [3]

(c) Calculate the e.m.f. of the source. [2]

(d) Calculate the power dissipated inside the source when the current is 1.20 A. [2]

(e) Explain why connecting a voltmeter directly across the terminals of the source does not give exactly its e.m.f. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- use $V = E - Ir$

$$V = E - Ir = 1.5 \text{ V} - (0.60 \text{ A})(0.40 \Omega) = \mathbf{1.26 \text{ V}}$$

2.2

- a larger current means a larger p.d. Ir across the **internal resistance**
- the e.m.f. is fixed, so with more of it used inside the cell, less is left for the external circuit and $V = E - Ir$ falls

2.3

- the terminal p.d. **equals the e.m.f.**
- with $I = 0$ there is no p.d. across the internal resistance, so nothing is lost inside the cell

2.4

- the largest current flows when the external resistance is **zero** – the terminals are short-circuited

$$I_{\max} = \frac{E}{r} = \frac{1.5 \text{ V}}{0.40 \Omega} = \mathbf{3.75 \text{ A}}$$

3.1 C

- the internal resistance is unchanged, but a larger current gives a larger Ir inside the cell, so less of the fixed e.m.f. is left as terminal p.d.
- **D** is the standard error: the e.m.f. does **not** change with current

3.2

(a) the battery has internal resistance

- some of the energy given to each coulomb is transferred to thermal energy inside the battery, Ir per coulomb, so less is left for the lamp

(b) the e.m.f. drives the current through both resistances: $I = \frac{E}{R + r}$

$$I = \frac{E}{R + r} = \frac{6.0 \text{ V}}{4.0 \Omega + 0.80 \Omega} = \mathbf{1.25 \text{ A}}$$

(c) the terminal p.d. is the e.m.f. less the p.d. across the internal resistance, $V = E - Ir$

$$V = E - Ir = 6.0 \text{ V} - (1.25 \text{ A})(0.80 \Omega) = \mathbf{5.0 \text{ V}}$$

(d) power in the internal resistance, $P = I^2 r$

$$P = I^2 r = (1.25 \text{ A})^2 (0.80 \Omega) = \mathbf{1.25 \text{ W}}$$

3.3

(a)

- $V = E - Ir$
- the **intercept** on the V axis is the **e.m.f.** E
- the **gradient** is $-r$, so the internal resistance is minus the gradient

(b)

- the internal resistance is minus the gradient of V against I

$$\text{gradient} = \frac{\Delta V}{\Delta I} = \frac{4.64 \text{ V} - 5.72 \text{ V}}{1.20 \text{ A} - 0.20 \text{ A}} = -1.08 \Omega$$

$$r = -\text{gradient} = -(-1.08 \Omega) = \mathbf{1.08 \Omega}$$

(c)

- substitute one pair of readings into $E = V + Ir$

$$E = V + Ir = 5.72 \text{ V} + (0.20 \text{ A})(1.08 \Omega) = \mathbf{5.94 \text{ V}}$$

(d)

- the power dissipated inside the source is I^2r

$$P = I^2r = (1.20 \text{ A})^2(1.08 \Omega) = \mathbf{1.6 \text{ W}}$$

(e)

- a real voltmeter draws a small current, so there is a small p.d. Ir inside the cell and the reading is slightly **less** than the e.m.f.