

Exercise walk-through

6.1.1

Stress, strain and the Young modulus

A-Level Physics

Sheet 6.1.1 – what you need to know

You must be able to:

- define stress, strain and the Young modulus, with units
- calculate E for a wire from its load, extension, length and diameter
- find E from the gradient of a stress–strain or load–extension graph
- describe the wire experiment, and which measurement limits its accuracy

What the exam asks

“Define the Young modulus of a material.” [1]

“Show that the spring constant k_0 of the wire is given by $k_0 = \frac{EA}{L}$.” [2]

“State and explain the effect, if any, that this has on the strain in the wire.” [2]

Practise on exercise sheet 6.1.1, then try the real exam questions on past-paper sheet 6.1.1.

Stress and strain

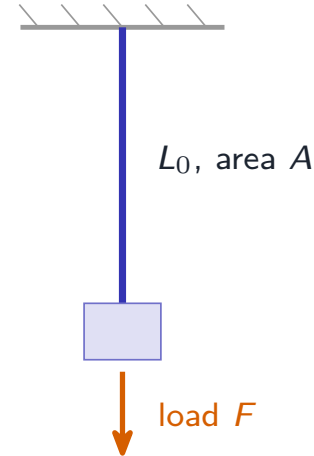
- **Stress** is the force per unit cross-sectional area.

$$\sigma = \frac{F}{A} \quad (\text{Pa} = \text{N m}^{-2})$$

In base units, $\text{Pa} = \text{kg m}^{-1} \text{s}^{-2}$.

- **Strain** is the extension per unit original length.

$$\varepsilon = \frac{x}{L_0} \quad (\text{no unit})$$



Area, and where the stress is larger

- A round wire: $A = \pi d^2/4 = \pi r^2$. Using the diameter **as the radius** makes the area four times too big. Halve d first.
- $1 \text{ mm}^2 = 10^{-6} \text{ m}^2$.
- 18 N on 3.2 mm^2 : $\sigma = \frac{F}{A} = \frac{18 \text{ N}}{3.2 \times 10^{-6} \text{ m}^2} = 5.6 \times 10^6 \text{ Pa}$.
- The same force acts all along the wire. A bolt of diameter $2d$ at one end and d at the other has **four times** the stress at the narrow end ($\sigma \propto 1/d^2$).
- Three times the radius means **one ninth** of the stress.

The Young modulus

The **Young modulus**

is stress divided by strain, in the Hooke's-law region:

$$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{x/L_0} = \frac{FL_0}{Ax}.$$

- Unit: Pa. Metals are about 10^{11} Pa. Steel is 2.0×10^{11} Pa.
- E belongs to the **material**. A thicker wire of the same metal has the same E .
- The spring constant also depends on size:
 $k = EA/L_0$.

It stretches less because the area is larger, not because the metal is stiffer.

A steel wire

$$L_0 = 2.0 \text{ m}, A = 1.5 \times 10^{-7} \text{ m}^2, x = 1.0 \text{ mm under } 15 \text{ N}$$

The extension must be in metres: $1.0 \text{ mm} = 1.0 \times 10^{-3} \text{ m}$.

$$E = \frac{FL_0}{Ax} = \frac{(15 \text{ N})(2.0 \text{ m})}{(1.5 \times 10^{-7} \text{ m}^2)(1.0 \times 10^{-3} \text{ m})} = 2.0 \times 10^{11} \text{ Pa.}$$

A thicker steel wire has this same Young modulus.

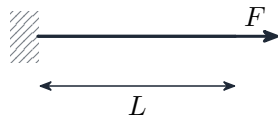
Exercises 2.1·2.2

2.1 Define **stress** and **strain**.

[2]

stress / strain

2.2 State the unit of the **Young modulus**, and explain why it is the same as the unit of stress. [2]



Exercises 2.1·2.2

2.1 Define **stress** and **strain**.

[2]

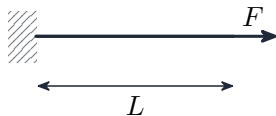
stress / strain

2.1

- stress: the force per unit cross-sectional area
 - strain: the extension per unit original length
-

2.2 State the unit of the **Young modulus**, and explain why it is the same as the unit of stress.

[2]

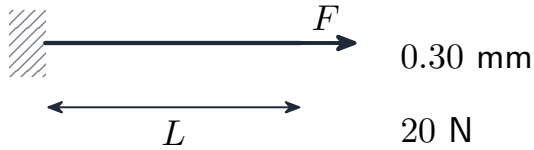


2.2

- the pascal, Pa
- strain is a ratio of two lengths and has no unit, so stress $\div$ strain has the unit of stress

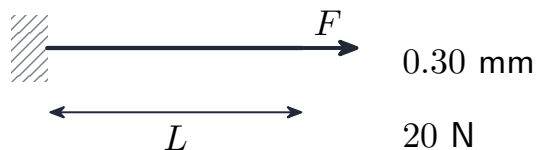
Exercise 2.3

2.3 A wire of diameter 0.30 mm carries a load of 20 N. Calculate the stress in the wire.[2]



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2.3

$$\bullet \quad A = \frac{\pi d^2}{4} = \frac{\pi(0.30 \times 10^{-3})^2}{4} = 7.07 \times 10^{-8} \text{ m}^2$$

$$\sigma = \frac{F}{A} = \frac{20 \text{ N}}{7.07 \times 10^{-8} \text{ m}^2} = \mathbf{2.8 \times 10^8 \text{ Pa}}$$

Exercise 2.4

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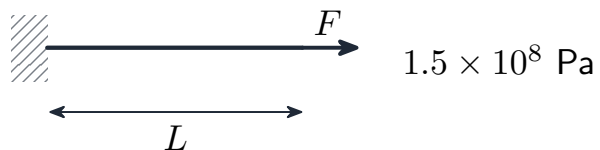


2.4

- $$\epsilon = \frac{x}{L} = \frac{1.2 \times 10^{-3}}{1.80} = \mathbf{6.7 \times 10^{-4}}$$

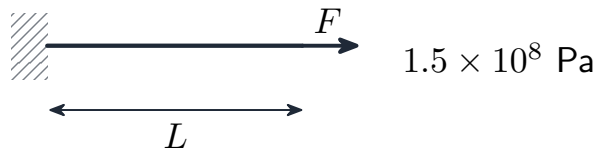
Exercise 2.5

2.5 The stress in the wire of **2.4** is $1.5 \times 10^8 \text{ Pa}$. Calculate the Young modulus of its metal. [2]



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2.5

- Young modulus, $E = \frac{\sigma}{\varepsilon}$

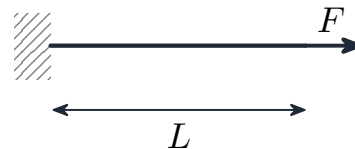
$$E = \frac{\sigma}{\varepsilon} = \frac{1.5 \times 10^8 \text{ Pa}}{6.67 \times 10^{-4}} = \mathbf{2.3 \times 10^{11} \text{ Pa}}$$

Exercise 3.1

3.1 Which unit is the unit of the Young modulus, written in SI base units? [1]

☐

- A** kg m s^{-2}
B $\text{kg m}^{-1} \text{s}^{-2}$
C kg m^{-2}
D kg s^{-2}

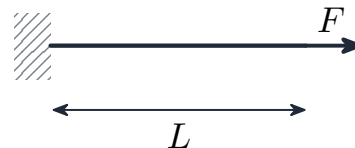


Exercise 3.1

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B $\text{kg m}^{-1} \text{s}^{-2}$
C kg m^{-2}
D kg s^{-2}



3.1 B

- the pascal: $\text{N m}^{-2} = \text{kg m s}^{-2} \times \text{m}^{-2} = \text{kg m}^{-1} \text{s}^{-2}$
- A** is the newton

Exercise 3.2

3.2 Two wires, P and Q, are made of the same metal. Q has twice the length and twice the diameter of P. Both carry the same load. What is $\frac{\text{extension of Q}}{\text{extension of P}}$? [1]

☐

A $\frac{1}{4}$

B $\frac{1}{2}$

C 1

D 2

X

Y

Exercise 3.2

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☐
A $\frac{1}{4}$ B $\frac{1}{2}$

C 1

D 2

X Y

3.2 B

- $x = \frac{FL}{AE}$: twice the length doubles x ; twice the diameter makes the area four times as large and divides x by 4
- so $x_Q/x_P = 2/4 = \frac{1}{2}$

Exercise 3.3 (a)

3.3 A brass wire has an original length of 2.40 m and a diameter of 0.62 mm. A load of 45 N extends it by 2.5 mm, within its limit of proportionality.

(a) **Define** the Young modulus.

[1]

Exercise 3.3 (a)

3.3 A brass wire has an original length of 2.40 m and a diameter of 0.62 mm. A load of 45 N extends it by 2.5 mm, within its limit of proportionality.

(a) **Define** the Young modulus.

[1]

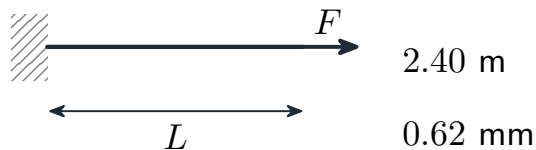
(a) the stress divided by the strain (within the limit of proportionality)

Exercise 3.3 (b)

3.3 A brass wire has an original length of 2.40 m and a diameter of 0.62 mm. A load of 45 N extends it by 2.5 mm, within its limit of proportionality.

(b) Calculate the stress in the wire.

[2]

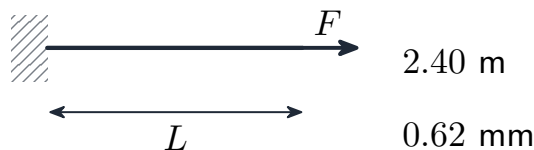


Exercise 3.3 (b)

3.3 A brass wire has an original length of 2.40 m and a diameter of 0.62 mm. A load of 45 N extends it by 2.5 mm, within its limit of proportionality.

(b) Calculate the stress in the wire.

[2]



$$(b) \ A = \frac{\pi d^2}{4} = \frac{\pi(0.62 \times 10^{-3})^2}{4} = 3.02 \times 10^{-7} \text{ m}^2$$

$$\sigma = \frac{F}{A} = \frac{45 \text{ N}}{3.02 \times 10^{-7} \text{ m}^2} = \mathbf{1.5 \times 10^8 \text{ Pa}}$$

Exercise 3.3 (c)

3.3 A brass wire has an original length of 2.40 m and a diameter of 0.62 mm. A load of 45 N extends it by 2.5 mm, within its limit of proportionality.

(c) **Determine** the Young modulus of brass.

[3]

Exercise 3.3 (c)

3.3 A brass wire has an original length of 2.40 m and a diameter of 0.62 mm. A load of 45 N extends it by 2.5 mm, within its limit of proportionality.

(c) **Determine** the Young modulus of brass. [3]

$$(c) \text{ strain } \varepsilon = \frac{x}{L} = \frac{2.5 \times 10^{-3}}{2.40} = 1.04 \times 10^{-3}$$

- Young modulus, $E = \frac{\sigma}{\varepsilon}$

$$E = \frac{\sigma}{\varepsilon} = \frac{1.49 \times 10^8 \text{ Pa}}{1.04 \times 10^{-3}} = \mathbf{1.4 \times 10^{11} \text{ Pa}}$$

Exercise 3.3 (d)

3.3 A brass wire has an original length of 2.40 m and a diameter of 0.62 mm. A load of 45 N extends it by 2.5 mm, within its limit of proportionality.

(d) A second wire of the same brass has the same length but half the diameter. It carries the same load. Calculate its extension. [2]

Exercise 3.3 (d)

3.3 A brass wire has an original length of 2.40 m and a diameter of 0.62 mm. A load of 45 N extends it by 2.5 mm, within its limit of proportionality.

(d) A second wire of the same brass has the same length but half the diameter. It carries the same load. Calculate its extension. [2]

(d) half the diameter gives a quarter of the area, so four times the stress and four times the extension

- $x_{\text{new}} = 4x = 4 \times 2.5 \text{ mm} = \mathbf{10 \text{ mm}}$

Exercise 3.4 (a)

3.4 A student measures the Young modulus of a copper wire.

(a) **Describe** how the student should measure the diameter of the wire, and explain why. [2]

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(a) **Describe** how the student should measure the diameter of the wire, and explain why. [2]

(a) with a micrometer, at several places along the wire and in two perpendicular directions, then average

- the wire may not be uniform or perfectly round, and the diameter has the largest percentage uncertainty

Exercise 3.4 (b)

3.4 A student measures the Young modulus of a copper wire.

(b) The student plots a graph of load against extension. **Explain** how the Young modulus is found from the graph. [2]

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3.4 A student measures the Young modulus of a copper wire.

(b) The student plots a graph of load against extension. **Explain** how the Young modulus is found from the graph. [2]

(b) the gradient of the straight part of the load–extension graph is $\frac{F}{x} = \frac{EA}{L}$

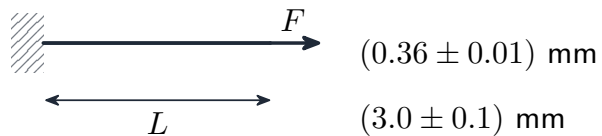
- so $E = \text{gradient} \times \frac{L}{A}$

Exercise 3.4 (c)

3.4 A student measures the Young modulus of a copper wire.

(c) The diameter is (0.36 ± 0.01) mm and the extension at the largest load is (3.0 ± 0.1) mm.

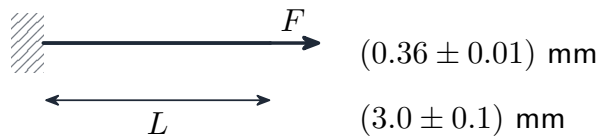
Determine the percentage uncertainty in the Young modulus, assuming the length and the load are exact. [2]



Exercise 3.4 (c)

3.4 A student measures the Young modulus of a copper wire.

(c) The diameter is (0.36 ± 0.01) mm and the extension at the largest load is (3.0 ± 0.1) mm. **Determine** the percentage uncertainty in the Young modulus, assuming the length and the load are exact. [2]



(c) diameter: $\frac{\Delta d}{d} = \frac{0.01}{0.36} = 2.8\%$, doubled to 5.6% because $A \propto d^2$

- extension: $\frac{\Delta x}{x} = \frac{0.1}{3.0} = 3.3\%$

- $E = \frac{FL}{Ax}$, so the percentages add: $\frac{\Delta E}{E} = 5.6\% + 3.3\% = \mathbf{8.9\%}$

Exercise 3.4 (d)

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(d) State **one** change to the wire that would make the percentage uncertainty in the extension smaller.

[1]

Exercise 3.4 (d)

3.4 A student measures the Young modulus of a copper wire.

(d) State **one** change to the wire that would make the percentage uncertainty in the extension smaller. [1]

(d) use a longer wire (or a thinner one), so that the extension is larger