

Exercise walk-through

5.2

Gravitational potential energy and kinetic energy

A-Level Physics

Sheet 5.2 – what you need to know

You must be able to:

- derive $\Delta E_P = mg\Delta h$ from $W = Fs$
- derive $E_K = \frac{1}{2}mv^2$ from the equations of motion
- solve problems where potential and kinetic energy exchange
- follow the energy through a chain of transfers

What the exam asks

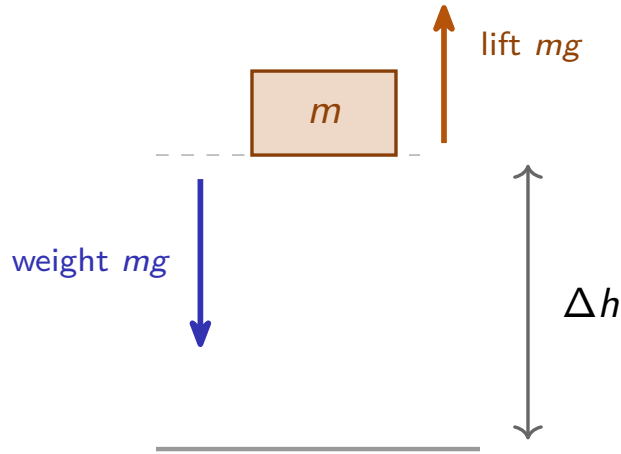
“Use the concept of work done to show that $\Delta E_P = mg\Delta h$.” [2]

“Calculate the kinetic energy of the block as it leaves the spring.” [2]

“Determine the gain in kinetic energy of the truck between A and B.” [3]

Practise on exercise sheet 5.2, then try the real exam questions on past-paper sheet 5.2.

Where $mg\Delta h$ comes from



Raising a **mass** m stores
gravitational potential energy

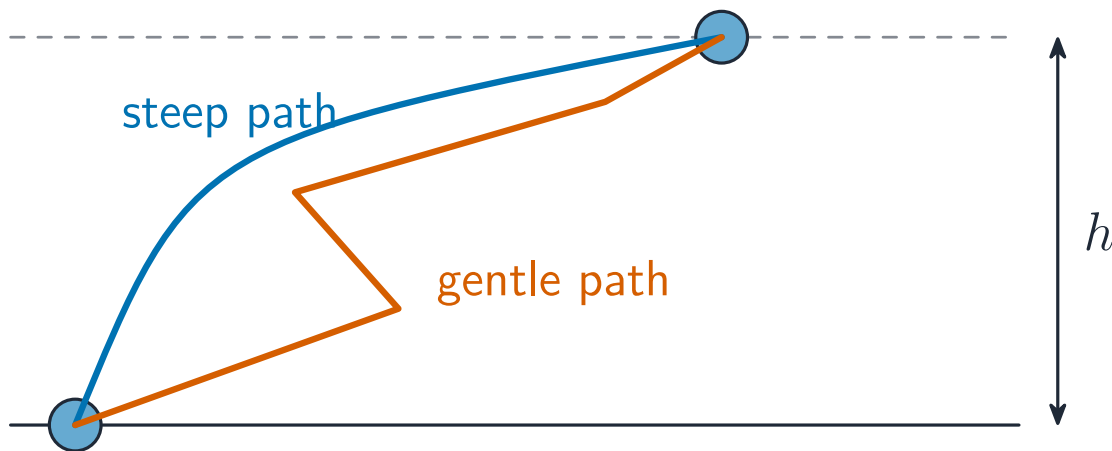
$$\Delta E_P = mg\Delta h$$

Lift it slowly, so no kinetic energy is gained. The upward force is mg , the work it does is $mg \cdot \Delta h$, and that work *is* the stored energy.

You need m , Δh and g – **never** speed or time.

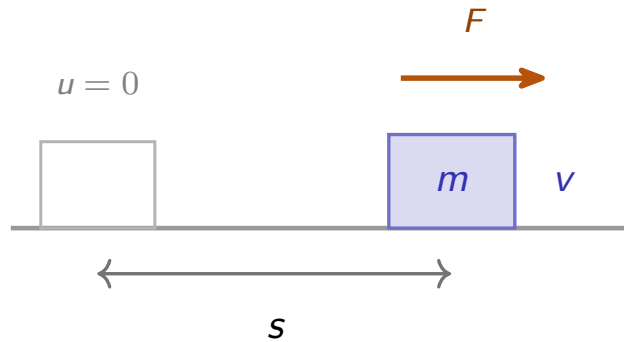
Only the height matters, not the path

same $\Delta E_P = mgh$ – only the height h matters



Steep or zig-zag, the gain is mgh either way: gravitational PE counts only the height risen.

Kinetic energy: where $\frac{1}{2}mv^2$ comes from



A push from rest, through a distance s , up to speed v .

$$v^2 = u^2 + 2as \text{ with } u = 0$$
$$\text{so } s = v^2 / (2a)$$

$$W = Fs = ma \cdot s$$

the a cancels

$$E_K = \frac{1}{2}mv^2$$

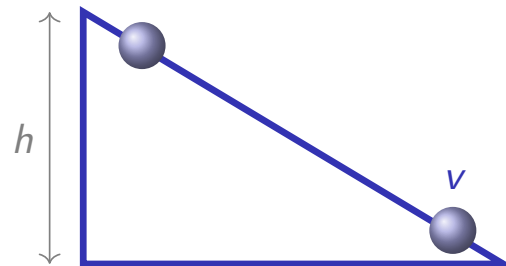
Worked example – speed down a ramp

From rest, smooth ramp, $h = 1.2 \text{ m}$

All the GPE becomes KE. Mass cancels.

$$mgh = \frac{1}{2}mv^2$$

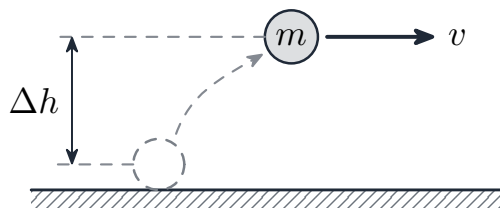
$$v = \sqrt{2gh} = \sqrt{2(9.81 \text{ m s}^{-2})(1.2 \text{ m})} \approx 4.9 \text{ m s}^{-1}$$



Exercise 2.1

2.1 Write the formula for **GPE change** and for **kinetic energy**.

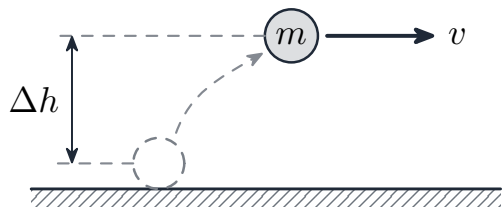
[2]



Exercise 2.1

2.1 Write the formula for **GPE change** and for **kinetic energy**.

[2]



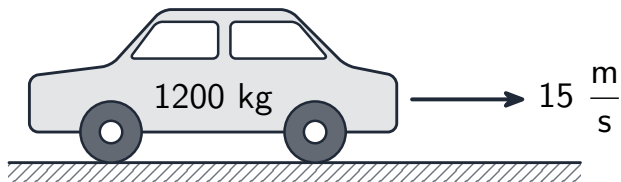
2.1

- $\Delta E_P = mg\Delta h$
- $E_K = \frac{1}{2}mv^2$

Exercise 2.2

2.2 Find the **kinetic energy** of a 1200 kg car moving at $15 \frac{\text{m}}{\text{s}}$.

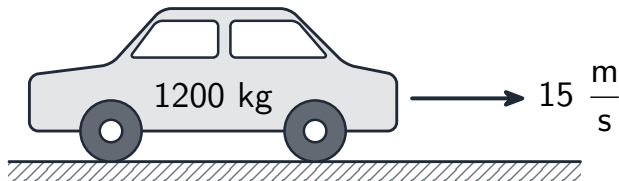
[2]



Exercise 2.2

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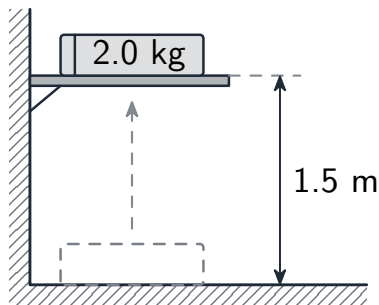
2.2

- $E_K = \frac{1}{2}mv^2$

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}(1200 \text{ kg})(15 \frac{\text{m}}{\text{s}})^2 = \mathbf{1.35 \times 10^5 \text{ J}}$$

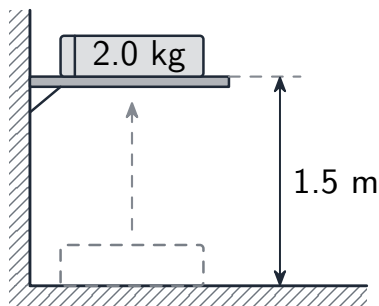
Exercise 2.3

2.3 Find the **GPE gained** by a 2.0 kg book lifted 1.5 m ($g = 9.81 \text{ m s}^{-2}$). [2]



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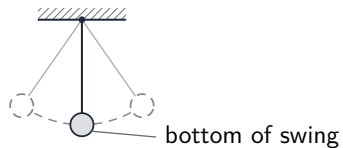
- $\Delta E_P = mg\Delta h$

$$\Delta E_P = mg\Delta h = (2.0 \text{ kg})(9.81 \text{ m s}^{-2})(1.5 \text{ m}) = \mathbf{29 \text{ J}}$$

Exercises 2.4·3.1

2.4 At the **bottom** of a pendulum swing, which energy is at a maximum?

[1]



3.1 An object's speed **doubles**. Its kinetic energy:

[1]

☐

A doubles

B halves

C becomes four times larger

D stays the same

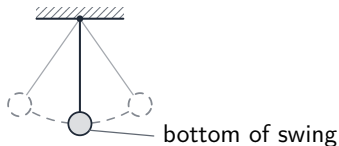
before  $\rightarrow v$

after  $\rightarrow 2v$

Exercises 2.4·3.1

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2.4

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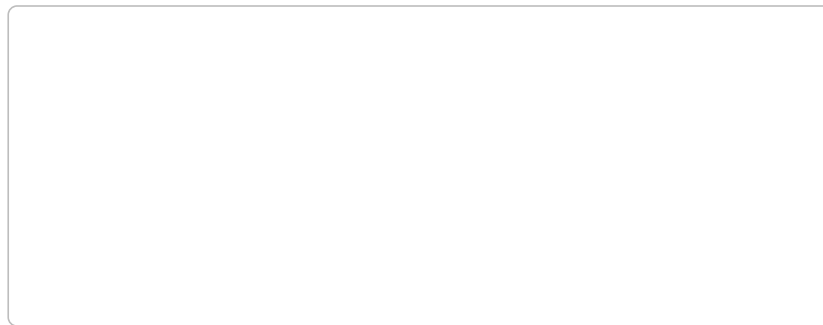
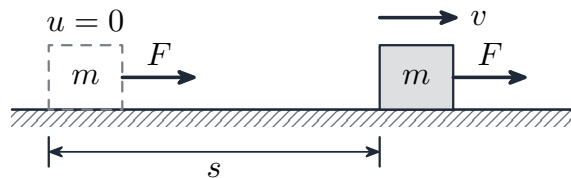
3.1 C

- $E_K \propto v^2$, so doubling v makes KE **four** times larger

Exercise 3.2

3.2 Derive $E_K = \frac{1}{2}mv^2$ using the equations of motion.

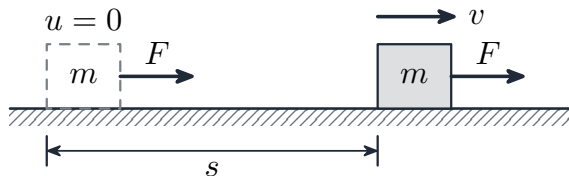
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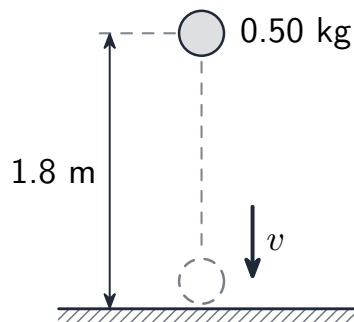
3.2

- work done $W = Fs = mas$
- from rest, $v^2 = 2as \Rightarrow s = v^2/(2a)$

$$W = ma \cdot \frac{v^2}{2a} = \frac{1}{2}mv^2 = E_K$$

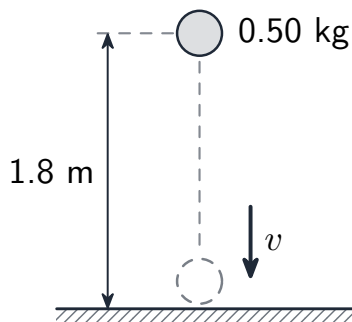
Exercise 3.3

3.3 A 0.50 kg ball is dropped from a height of 1.8 m. Calculate its speed just before it hits the ground (ignore air resistance). [3]



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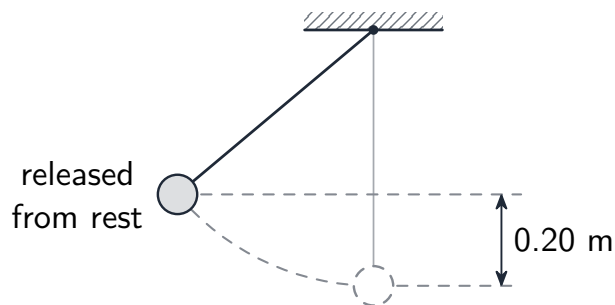
3.3

- $\frac{1}{2}mv^2 = mgh$

$$v = \sqrt{2gh} = \sqrt{2(9.81 \text{ m s}^{-2})(1.8 \text{ m})} = \mathbf{5.9 \frac{m}{s}}$$

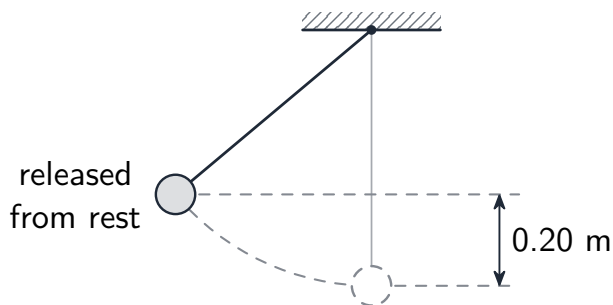
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3.4 A pendulum bob is released from a point 0.20 m above its lowest point. Calculate its **maximum speed**. [2]



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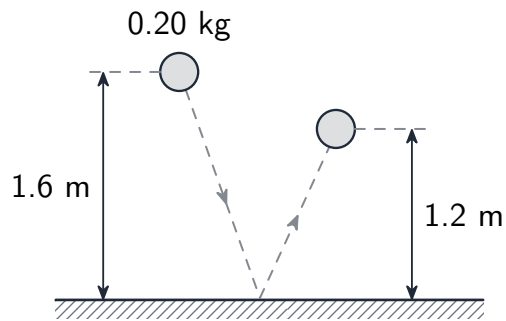
3.4

- from rest, $v^2 = 2gh$

$$v = \sqrt{2gh} = \sqrt{2(9.81 \text{ m s}^{-2})(0.20 \text{ m})} = \mathbf{2.0} \frac{\text{m}}{\text{s}}$$

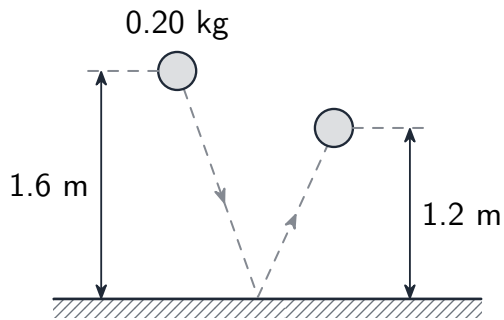
Exercise 3.5

3.5 A 0.20 kg ball is dropped from a height of 1.6 m and rebounds to a height of 1.2 m . Calculate the energy **transferred to other forms** during the bounce. [3]



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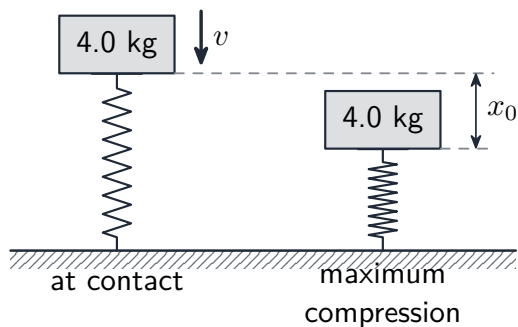
3.5

- GPE before: $mgh_1 = (0.20 \text{ kg})(9.81 \text{ m s}^{-2})(1.6 \text{ m}) = 3.1 \text{ J}$
- GPE after: $mgh_2 = (0.20 \text{ kg})(9.81 \text{ m s}^{-2})(1.2 \text{ m}) = 2.4 \text{ J}$
- energy transferred $\Delta E = mgh_1 - mgh_2 = 3.1 \text{ J} - 2.4 \text{ J} = \mathbf{0.79 \text{ J}}$

Exercise 3.6 (a)

3.6 A vertical spring stands on a horizontal surface with its lower end fixed. Its mass is negligible. A block of mass 4.0 kg falls onto the spring and is brought to rest as the spring compresses. The block has kinetic energy 72 J at the instant it touches the spring.

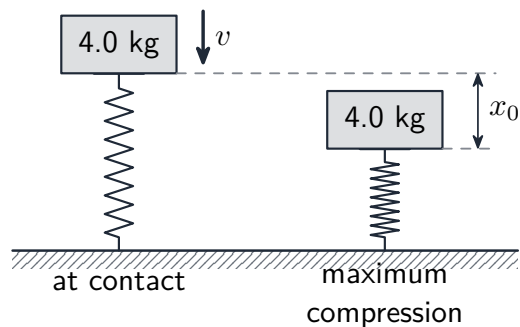
(a) **Calculate** the speed of the block as it touches the spring. [2]



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(a) **Calculate** the speed of the block as it touches the spring. [2]



(a) $E_K = \frac{1}{2}mv^2$

$$72 \text{ J} = \frac{1}{2}(4.0 \text{ kg})v^2 \Rightarrow v = \sqrt{36 \text{ m}^2 \text{ s}^{-2}} = \mathbf{6.0} \frac{\text{m}}{\text{s}}$$

Exercise 3.6 (b)

3.6 A vertical spring stands on a horizontal surface with its lower end fixed. Its mass is negligible. A block of mass 4.0 kg falls onto the spring and is brought to rest as the spring compresses. The block has kinetic energy 72 J at the instant it touches the spring.

(b) While the spring compresses, the gravitational potential energy of the block falls by a further 12 J. **Show that** the maximum compression x_0 of the spring is about 0.31 m.[2]

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(b) While the spring compresses, the gravitational potential energy of the block falls by a further 12 J. **Show that** the maximum compression x_0 of the spring is about 0.31 m.[2]

(b) the block falls a further x_0 while compressing the spring, so $\Delta E_P = mgx_0$

$$x_0 = \frac{E_P}{mg} = \frac{12 \text{ J}}{(4.0 \text{ kg})(9.81 \text{ m s}^{-2})} = \mathbf{0.31 \text{ m}}$$

Exercise 3.6 (c)

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(c) All the energy lost by the block becomes elastic potential energy in the spring. **Determine** the maximum elastic potential energy stored. [1]

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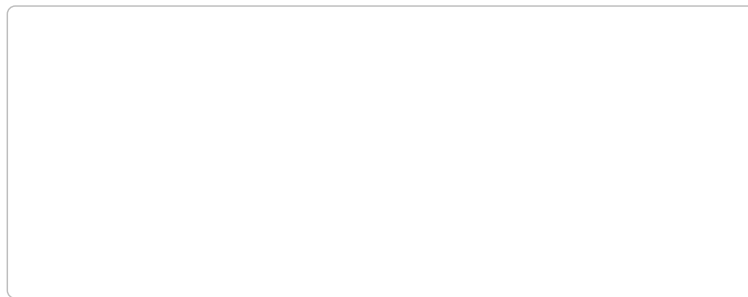
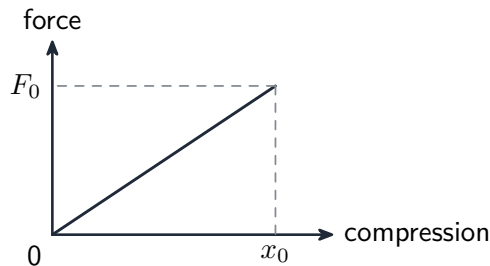
(c) All the energy lost by the block becomes elastic potential energy in the spring. **Determine** the maximum elastic potential energy stored. [1]

(c) all of it goes into the spring: $E = E_K + E_P = 72 \text{ J} + 12 \text{ J} = 84 \text{ J}$

Exercise 3.6 (d)

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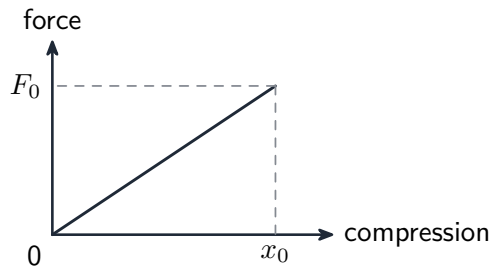
(d) The spring obeys Hooke's law, so the force it exerts is proportional to the compression. **Show that** the maximum force F_0 the spring exerts on the block is about 540 N . [2]



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(d) energy stored = area under the F - x graph = a triangle: $E = \frac{1}{2}F_0x_0$

$$F_0 = \frac{2W}{x} = \frac{2 \times 84 \text{ J}}{0.306 \text{ m}} = \mathbf{5.5 \times 10^2 \text{ N}}$$

Exercise 3.6 (e)(i)

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(e) At maximum compression, **determine**

(i) the resultant force on the block,

[2]

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(e)(i) spring **up** with F_0 , weight **down**: $W = mg = (4.0 \text{ kg})(9.81 \text{ m s}^{-2}) = 39 \text{ N}$

- resultant $F = F_0 - W = 549 \text{ N} - 39 \text{ N} = \mathbf{5.1 \times 10^2 \text{ N}}$ **upwards**

Exercise 3.6 (e)(ii)

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(e) At maximum compression, **determine**

(ii) the acceleration of the block.

[2]

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[2]

(e)(ii)

$$a = \frac{F}{m} = \frac{510 \text{ N}}{4.0 \text{ kg}} = \mathbf{1.3 \times 10^2 \text{ m s}^{-2}}$$

upwards

Exercise 3.6 (f)

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(f) **Sketch** a graph to show how the **kinetic energy** of the block varies with the compression of the spring, from the instant of contact to maximum compression. **Label** both axes.

[2]

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(f)

- axes labelled kinetic energy / J against compression / m
- starts at 72 J when $x = 0$, falls to zero at $x_0 = 0.31$ m, getting steeper as the spring force grows

Exercise 3.6 (g)

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(g) The same block is dropped from a greater height onto the same spring. **Explain** why the maximum compression is greater, and why the maximum acceleration is greater too.

[2]

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(g) The same block is dropped from a greater height onto the same spring. **Explain** why the maximum compression is greater, and why the maximum acceleration is greater too. [2]

(g) a greater drop height $\Rightarrow$ **more KE** at contact $\Rightarrow$ more energy stored; $E = \frac{1}{2}kx^2$ so x_0 is larger

- $F_0 \propto x_0$, so $a = (F_0 - mg)/m$ is larger too