

Exercise walk-through

5.2.1

Potential energy, kinetic energy and energy transfers

A-Level Physics

Sheet 5.2 – what you need to know

You must be able to:

- derive $\Delta E_P = mg\Delta h$ from $W = Fs$
- derive $E_K = \frac{1}{2}mv^2$ from the equations of motion
- solve problems where potential and kinetic energy exchange
- follow the energy through a chain of transfers

What the exam asks

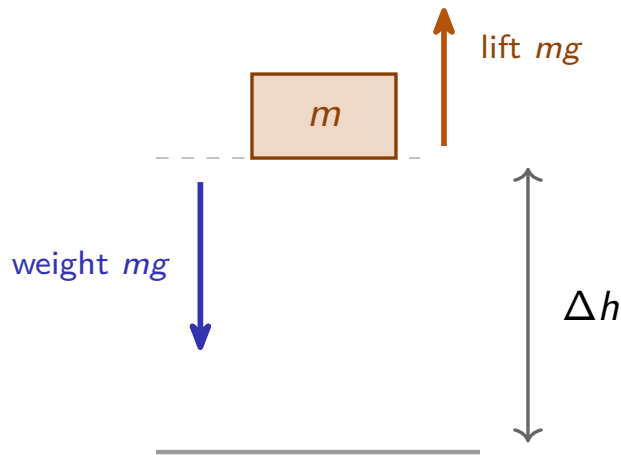
“Use the concept of work done to show that $\Delta E_P = mg\Delta h$.” [2]

“Calculate the kinetic energy of the block as it leaves the spring.” [2]

“Determine the gain in kinetic energy of the truck between A and B.” [3]

Practise on exercise sheet 5.2, then try the real exam questions on past-paper sheet 5.2.

Where $mg\Delta h$ comes from



Raising a **mass** m stores
gravitational potential energy

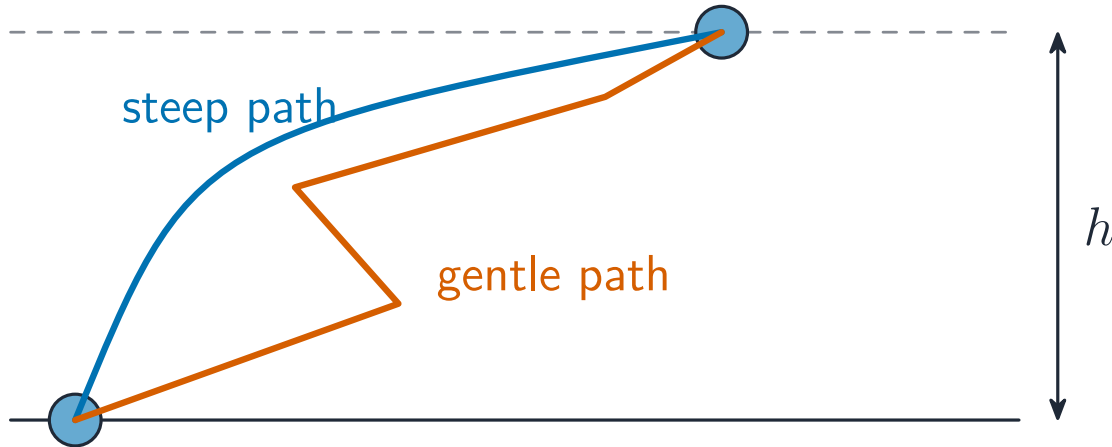
$$\Delta E_P = mg\Delta h$$

Lift it slowly, so no kinetic energy is gained. The upward force is mg , the work it does is $mg \cdot \Delta h$, and that work *is* the stored energy.

You need m , Δh and g – **never** speed or time.

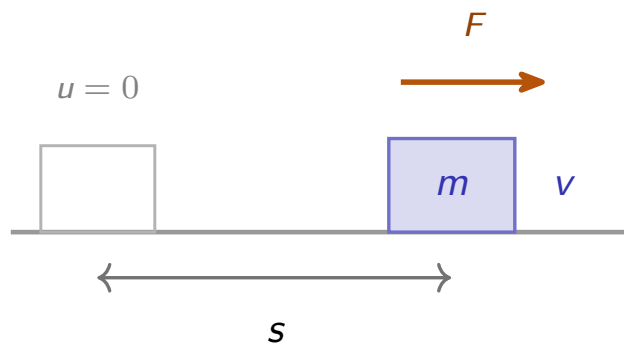
Only the height matters, not the path

same $\Delta E_P = mgh$ – only the height h matters



Steep or zig-zag, the gain is mgh either way: gravitational PE counts only the height risen.

Kinetic energy: where $\frac{1}{2}mv^2$ comes from



A push from rest, through a distance s , up to speed v .

$$v^2 = u^2 + 2as \text{ with } u = 0$$
$$\text{so } s = v^2 / (2a)$$

$$W = Fs = ma \cdot s$$

the a cancels

$$E_K = \frac{1}{2}mv^2$$

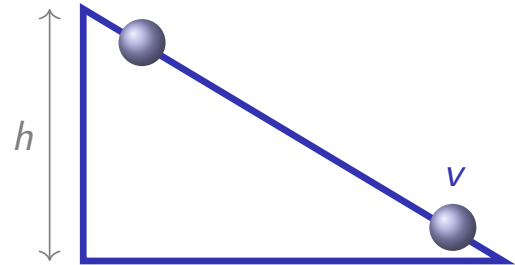
Worked example – speed down a ramp

From rest, smooth ramp, $h = 1.2 \text{ m}$

All the GPE becomes KE. Mass cancels.

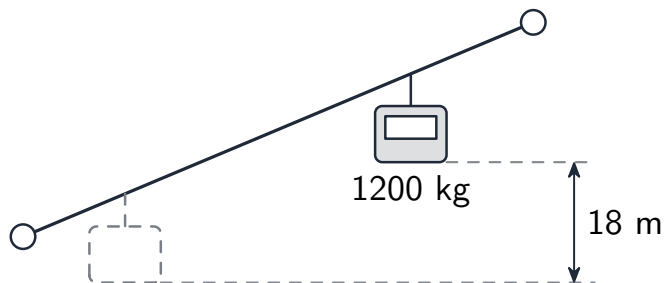
$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} = \sqrt{2(9.81 \text{ m s}^{-2})(1.2 \text{ m})} \approx 4.9 \text{ m s}^{-1}$$



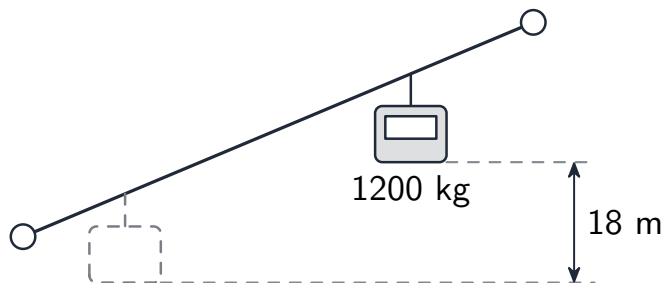
Exercise 2.1

2.1 A cable car of total mass 1200 kg climbs through a vertical height of 18 m. Calculate the gain in its gravitational potential energy. [2]



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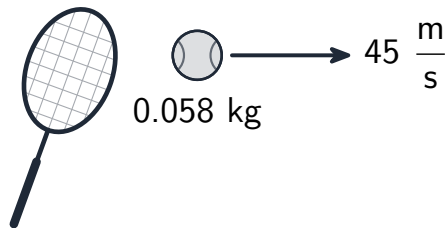
2.1

- $\Delta E_P = mg\Delta h$

$$\Delta E_P = mg\Delta h = 1200 \times 9.81 \times 18 = \mathbf{2.12 \times 10^5 \text{ J}}$$

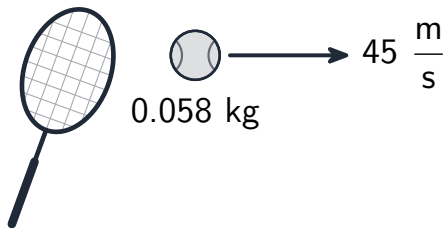
Exercise 2.2

2.2 A tennis ball of mass 0.058 kg is served at $45 \frac{\text{m}}{\text{s}}$. Calculate its kinetic energy. [2]



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2.2

- $E_K = \frac{1}{2}mv^2$

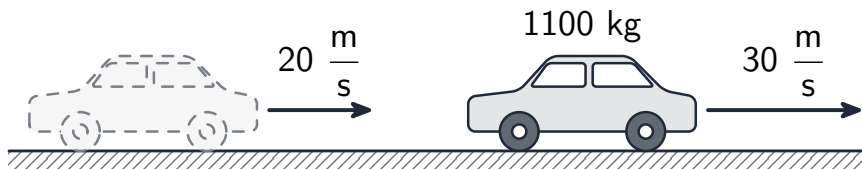
$$E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.058 \times 45^2 = \mathbf{59 \text{ J}}$$

Exercise 2.3 (a)

2.3 A car of mass 1100 kg speeds up from $20 \frac{\text{m}}{\text{s}}$ to $30 \frac{\text{m}}{\text{s}}$.

(a) State the factor by which its kinetic energy increases.

[1]

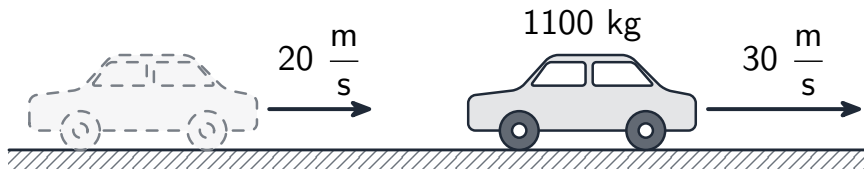


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(a) State the factor by which its kinetic energy increases.

[1]



(a) $E_K \propto v^2$, so the factor is $(30/20)^2 = \mathbf{2.25}$

Exercise 2.3 (b)

2.3 A car of mass 1100 kg speeds up from $20 \frac{\text{m}}{\text{s}}$ to $30 \frac{\text{m}}{\text{s}}$.

(b) Calculate the increase in its kinetic energy.

[2]

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(b) Calculate the increase in its kinetic energy.

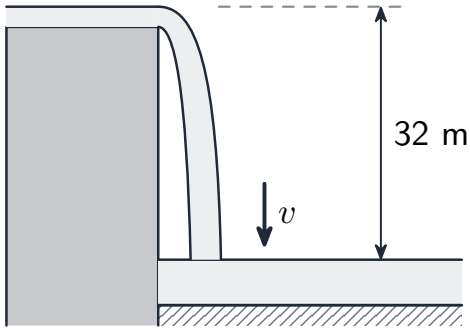
[2]

$$(b) \Delta E_K = \frac{1}{2}m(v^2 - u^2) = \frac{1}{2} \times 1100 \times (30^2 - 20^2)$$

- $\Delta E_K = \mathbf{2.75 \times 10^5 \text{ J}}$

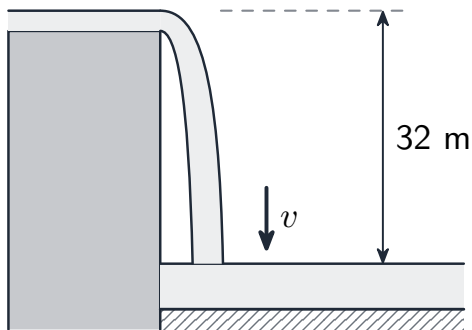
Exercise 2.4

2.4 Water moves very slowly over the top of a waterfall and falls through 32 m. Ignoring air resistance, calculate its speed at the bottom. [2]



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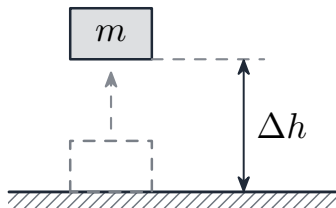
- $\frac{1}{2}mv^2 = mg\Delta h$

$$v = \sqrt{2g\Delta h} = \sqrt{2 \times 9.81 \times 32} = \mathbf{25} \frac{\text{m}}{\text{s}}$$

Exercise 2.5

2.5 State the condition for the formula $\Delta E_P = mg\Delta h$ to be used.

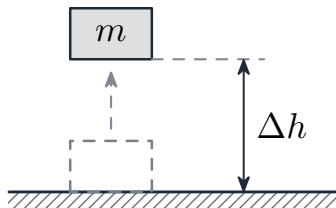
[1]



Exercise 2.5

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2.5

- the gravitational field must be uniform: g does not change with height, as near the Earth's surface for small changes in height

Exercise 3.1

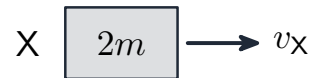
3.1 Two objects X and Y have the same momentum. The mass of X is twice the mass of Y. What is the ratio $\frac{\text{kinetic energy of X}}{\text{kinetic energy of Y}}$? [1]

A $\frac{1}{4}$

B $\frac{1}{2}$

C 2

D 4



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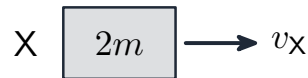


A $\frac{1}{4}$

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D 4



3.1 B

- $E_K = p^2/2m$, so with the same momentum $E_K \propto 1/m$ and X, with twice the mass, has half the kinetic energy
- C would be true for the same speed, not the same momentum

Exercise 3.2

3.2 A ball dropped from rest from a height h reaches a speed v . Air resistance is negligible. From what height must the ball be dropped to reach a speed $2v$? [1]

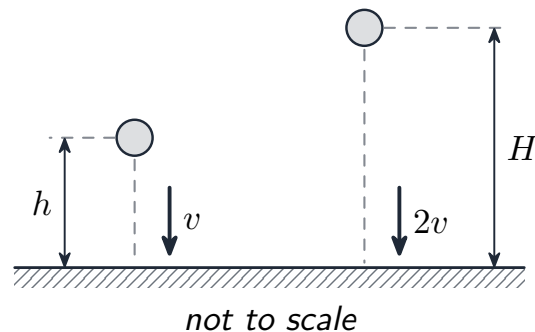


A $\sqrt{2}h$

B $2h$

C $4h$

D $8h$

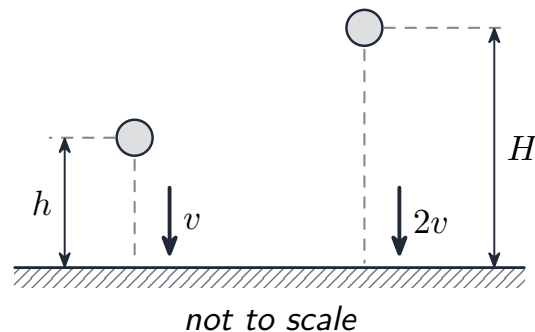


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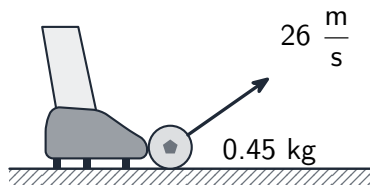
3.2 C

- $\frac{1}{2}mv^2 = mgh$, so $h \propto v^2$ and doubling v needs four times the height
- **B** assumes that h is proportional to v

Exercise 3.3 (a)

3.3 A footballer kicks a ball of mass 0.45 kg from rest. The ball leaves the foot at $26 \frac{\text{m}}{\text{s}}$.

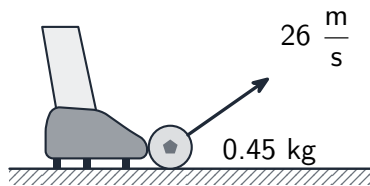
(a) Use the equations of motion to show that the kinetic energy of an object of mass m , accelerated from rest to a speed v , is $\frac{1}{2}mv^2$. [3]



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(a) the work done by the force is $W = Fs$ with $F = ma$

- from rest $v^2 = 2as$, so $s = v^2/2a$
- $W = ma \times \frac{v^2}{2a} = \frac{1}{2}mv^2$, and this work becomes the kinetic energy

Exercise 3.3 (b)

3.3 A footballer kicks a ball of mass 0.45 kg from rest. The ball leaves the foot at $26 \frac{\text{m}}{\text{s}}$.

(b) The foot pushes on the ball over a distance of 0.20 m. Calculate the average force on the ball. [2]

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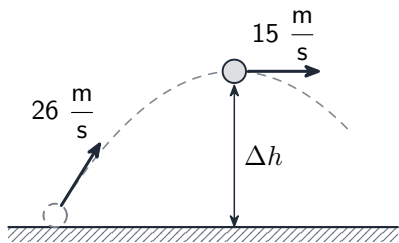
(b) $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.45 \times 26^2 = 152 \text{ J}$

- $F = \frac{E_K}{s} = \frac{152}{0.20} = \mathbf{760 \text{ N}}$

Exercise 3.3 (c)

3.3 A footballer kicks a ball of mass 0.45 kg from rest. The ball leaves the foot at $26 \frac{\text{m}}{\text{s}}$.

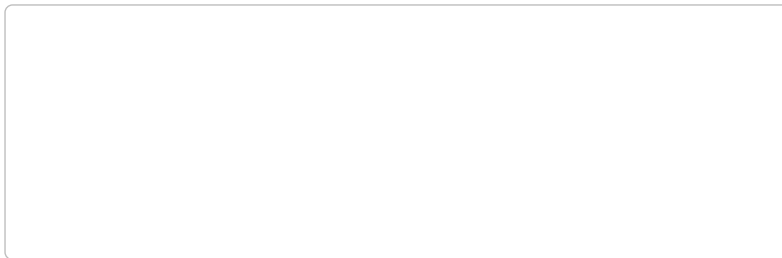
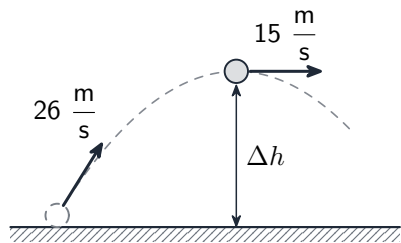
(c) At the highest point of its path the ball moves horizontally at $15 \frac{\text{m}}{\text{s}}$. Ignoring air resistance, calculate the height of this point above the point where the ball was kicked.[3]



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(c) kinetic energy at the top: $E_{K,\text{top}} = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.45 \times 15^2 = 50.6 \text{ J}$

- the gain in potential energy is the kinetic energy lost: $mg\Delta h = E_K - E_{K,\text{top}} = 152.1 - 50.6 = 101.5 \text{ J}$

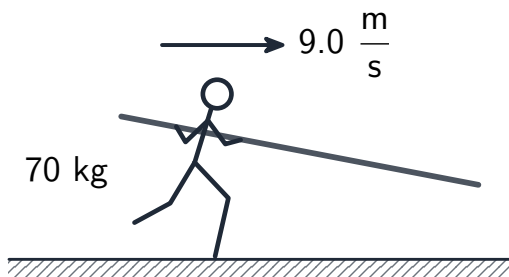
$$\Delta h = \frac{101.5}{mg} = \frac{101.5}{0.45 \times 9.81} = \mathbf{23 \text{ m}}$$

Exercise 3.4 (a)

3.4 A pole vaulter of mass 70 kg sprints along the runway at $9.0 \frac{\text{m}}{\text{s}}$, then uses the pole to change her kinetic energy into gravitational potential energy.

(a) Calculate her kinetic energy just before she plants the pole.

[2]

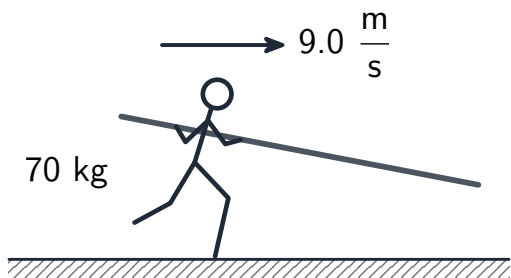


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(a) $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 70 \times 9.0^2$

- $E_K = 2.8 \times 10^3 \text{ J}$

Exercise 3.4 (b)

3.4 A pole vaulter of mass 70 kg sprints along the runway at $9.0 \frac{\text{m}}{\text{s}}$, then uses the pole to change her kinetic energy into gravitational potential energy.

(b) Assuming that all of this energy becomes gravitational potential energy, calculate how far her centre of gravity could rise. [2]

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(b) $mg\Delta h = 2835 \text{ J}$

$$\Delta h = \frac{2835}{mg} = \frac{2835}{70 \times 9.81} = \mathbf{4.1 \text{ m}}$$

Exercise 3.4 (c)

3.4 A pole vaulter of mass 70 kg sprints along the runway at $9.0 \frac{\text{m}}{\text{s}}$, then uses the pole to change her kinetic energy into gravitational potential energy.

(c) Her centre of gravity starts 1.0 m above the ground, and she only just clears a bar at 4.8 m. Suggest two reasons why she does not reach the height predicted by (b). [2]

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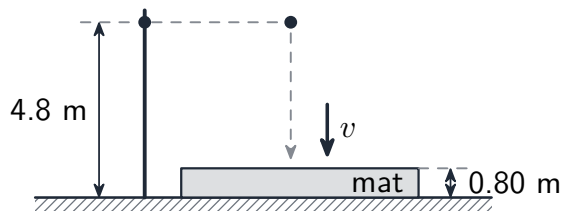
(c) she still has kinetic energy at the top, because she must keep moving horizontally over the bar

- some energy is transferred to thermal energy and sound in the pole and in her body, and by air resistance

Exercise 3.4 (d)

3.4 A pole vaulter of mass 70 kg sprints along the runway at $9.0 \frac{\text{m}}{\text{s}}$, then uses the pole to change her kinetic energy into gravitational potential energy.

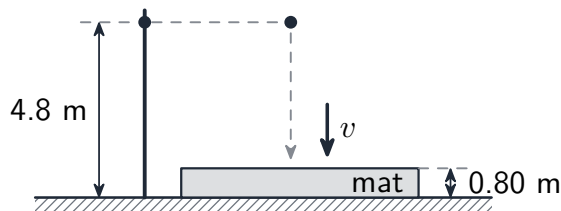
(d) After clearing the bar, she falls from rest at 4.8 m onto a landing mat whose top surface is 0.80 m above the ground. Treating her as a point mass, calculate her speed as she reaches the mat. [2]



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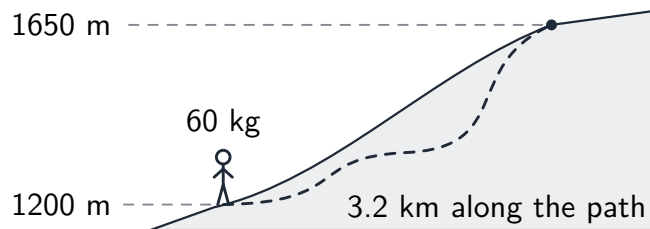
(d) $\frac{1}{2}mv^2 = mg\Delta h$ with $\Delta h = 4.8 - 0.80 = 4.0 \text{ m}$

$$v = \sqrt{2g\Delta h} = \sqrt{2 \times 9.81 \times 4.0} = \mathbf{8.9 \frac{m}{s}}$$

Exercise 3.5 (a)

3.5 A hiker of mass 60 kg walks 3.2 km along a mountain path, from a height of 1200 m to a height of 1650 m.

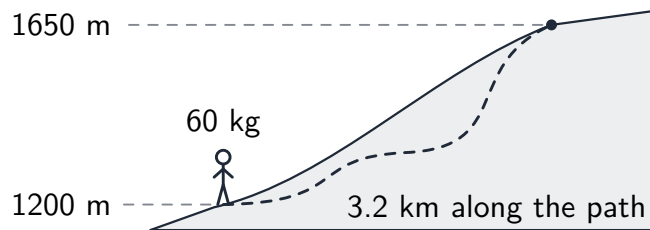
(a) Starting from the work done by a force, derive the formula for the change in gravitational potential energy near the Earth's surface. [2]



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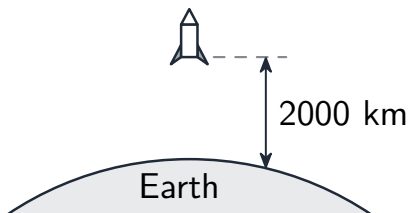
(a) to raise the mass at a steady speed, a force equal to its weight mg acts through the vertical distance Δh

- the work done $W = Fs = mg\Delta h$ is stored as gravitational potential energy, so $\Delta E_P = mg\Delta h$

Exercise 3.5 (b)

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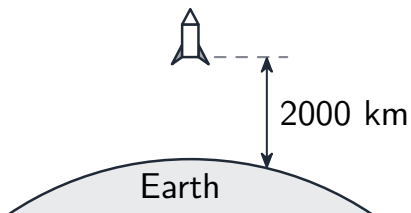
(b) Explain why this formula cannot be used for a rocket that rises 2000 km above the Earth. [1]



Exercise 3.5 (b)

3.5 A hiker of mass 60 kg walks 3.2 km along a mountain path, from a height of 1200 m to a height of 1650 m.

(b) Explain why this formula cannot be used for a rocket that rises 2000 km above the Earth. [1]



(b) g decreases as the height increases, so the field is not uniform over 2000 km

Exercise 3.5 (c)

3.5 A hiker of mass 60 kg walks 3.2 km along a mountain path, from a height of 1200 m to a height of 1650 m.

(c) Calculate the gain in gravitational potential energy of the hiker. [2]

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3.5 A hiker of mass 60 kg walks 3.2 km along a mountain path, from a height of 1200 m to a height of 1650 m.

(c) Calculate the gain in gravitational potential energy of the hiker. [2]

(c) $\Delta h = h_2 - h_1 = 1650 - 1200 = 450 \text{ m}$

$$\Delta E_P = mg\Delta h = 60 \times 9.81 \times 450 = \mathbf{2.6 \times 10^5 \text{ J}}$$

Exercise 3.5 (d)

3.5 A hiker of mass 60 kg walks 3.2 km along a mountain path, from a height of 1200 m to a height of 1650 m.

(d) The hiker's muscles transfer food energy into useful work with an efficiency of 25%. Calculate the food energy used to gain this potential energy. [2]

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(d) The hiker's muscles transfer food energy into useful work with an efficiency of 25%. Calculate the food energy used to gain this potential energy. [2]

(d) food energy = useful work $\div$ efficiency = $2.65 \times 10^5 / 0.25$

- food energy = 1.1×10^6 J