

Exercise walk-through

5.1.1

Efficiency, power and $P = Fv$

A-Level Physics

Sheet 5.1.1 – what you need to know

You must be able to:

- calculate an **efficiency** from energies or from powers
- define power and use $P = \frac{W}{t}$
- derive $P = Fv$ from the definitions of power and work
- use $P = Fv$ on the level, up a slope, and when drag grows with speed

What the exam asks

“Use the definition of power to show that the power P transferred by the force is given by $P = Fv$.”

[2]

“Calculate the percentage efficiency of the solar panel.” **[1]**

“Show that the resistive force opposing the forward motion of the lorry is 5200 N.” **[1]**

Practise on exercise sheet 5.1.1, then try the real exam questions on past-paper sheet 5.1.1.

Power – the rate of energy transfer

$$P = \frac{W}{t} = \frac{\Delta E}{\Delta t}, \quad [P] = W = \text{J s}^{-1}.$$

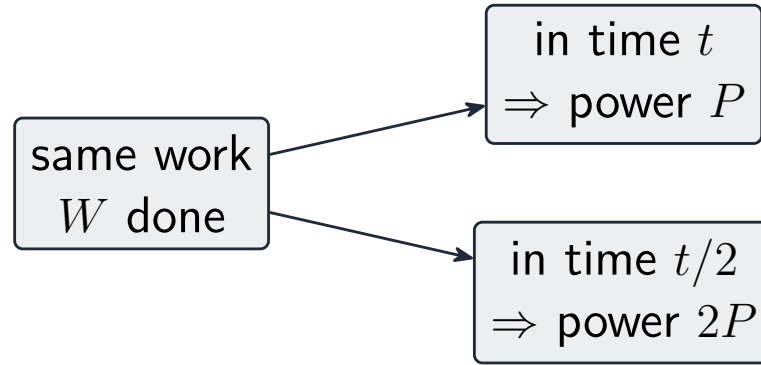
Power is a **scalar** .

same work
in less time



more power

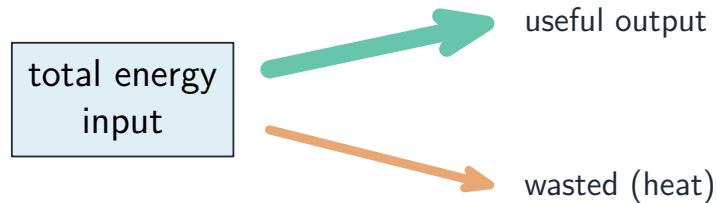
Same work, different power



$$P = \frac{W}{t} \text{ — less time, more power}$$

Power is **work per second**: climbing the stairs twice as fast does the *same* work against gravity, at **twice** the power.

Efficiency



$$\text{efficiency} = \frac{\text{useful output}}{\text{total input}} \times 100\%$$

$$\text{efficiency} = \frac{\text{useful output}}{\text{total input}}$$

A pump lifts 600 kg of water 12 m in 30 s, drawing 3.0 kW

useful power, the gain in potential energy per second:

$$P = \frac{mg\Delta h}{t} = \frac{(600 \text{ kg})(9.81 \text{ m s}^{-2})(12 \text{ m})}{30 \text{ s}} = 2350 \text{ W}$$

$$\text{efficiency} = \frac{2350 \text{ W}}{3000 \text{ W}} = 0.78, \text{ or } 78\%$$

The rest is wasted, usually as thermal energy. Real machines stay under 100%.

Power at a steady speed is force times velocity

$$P = Fv$$

In a time Δt the force moves through $v\Delta t$,
doing $Fv\Delta t$ of work. Divide by Δt .

Car at 25 m s^{-1} against 600 N

At constant speed the driving force
equals the resistive force, so
 $P = Fv = (600 \text{ N})(25 \text{ m s}^{-1}) =$
15 kW.

flat road

$$P = F_{\text{resist}} v$$

drag grows like v^2

double speed: force $\times 4$

power $P = Fv \propto v^3$, so $\times 8$

steady lift

$$P = mgv$$

Exercises 2.1-2.2

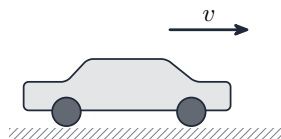
2.1 Define **power**.

[1]

power

2.2 Show that the power of a force F moving an object at a constant velocity v in the direction of the force is $P = Fv$.

[2]



Exercises 2.1·2.2

2.1 Define **power**.

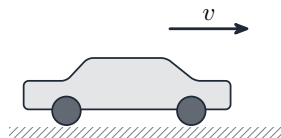
[1]

power

2.1

- power is the work done per unit time (the rate of transfer of energy)
-

2.2 Show that the power of a force F moving an object at a constant velocity v in the direction of the force is $P = Fv$. [2]



2.2

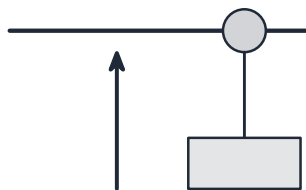
- $P = \frac{W}{t}$ and $W = Fs$, so $P = \frac{Fs}{t}$
- $\frac{s}{t} = v$, so $P = Fv$

Exercise 2.3 (a)

2.3 A motor lifts a load of mass 50 kg through 8.0 m in 10 s.

(a) Calculate the useful power output of the motor.

[2]



50 kg

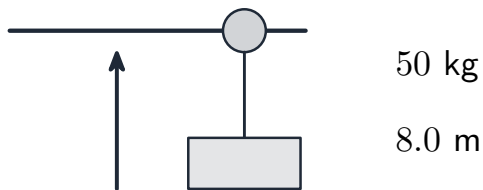
8.0 m

Exercise 2.3 (a)

2.3 A motor lifts a load of mass 50 kg through 8.0 m in 10 s.

(a) Calculate the useful power output of the motor.

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(a) power is work done per unit time, $P = \frac{mg\Delta h}{t}$

$$P = \frac{mg\Delta h}{t} = \frac{50 \times 9.81 \times 8.0}{10} = \mathbf{390 \text{ W}}$$

Exercise 2.3 (b)

2.3 A motor lifts a load of mass 50 kg through 8.0 m in 10 s.

(b) The input power is 500 W. Calculate the efficiency.

[1]

Exercise 2.3 (b)

2.3 A motor lifts a load of mass 50 kg through 8.0 m in 10 s.

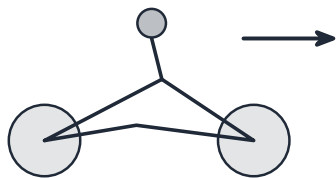
(b) The input power is 500 W. Calculate the efficiency.

[1]

$$(b) \text{ efficiency} = \frac{\text{useful output}}{\text{total input}} = \frac{392}{500} = \mathbf{0.78}$$

Exercise 2.4

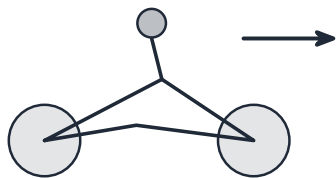
2.4 A cyclist rides at a steady $8.0 \frac{\text{m}}{\text{s}}$ along a level road against a total resistive force of 25 N. Calculate the useful power output of the cyclist. [2]

 8.0 m s^{-1}

25 N

Exercise 2.4

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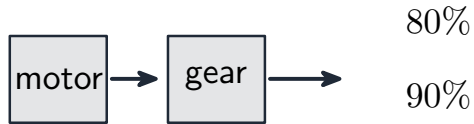
2.4

- steady speed, so the driving force equals the resistive force; $P = Fv$

$$P = Fv = 25 \text{ N} \times 8.0 \frac{\text{m}}{\text{s}} = \mathbf{200 \text{ W}}$$

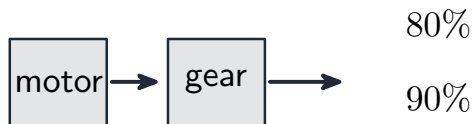
Exercise 2.5

2.5 An electric motor is 80% efficient and drives a gearbox that is 90% efficient. The motor takes 5.0 kW. Calculate the useful power from the gearbox. [2]



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2.5

- efficiencies in stages multiply: $\eta = \eta_1 \eta_2 = 0.80 \times 0.90 = 0.72$
- useful power = $\eta \times P_{\text{in}} = 0.72 \times 5.0 \text{ kW} = \mathbf{3.6 \text{ kW}}$

Exercise 3.1

3.1 What is the watt expressed in SI base units?

[1]

☐

A $\text{kg m}^2 \text{s}^{-2}$

B $\text{kg m}^2 \text{s}^{-3}$

C kg m s^{-3}

D $\text{kg m}^2 \text{s}^{-1}$

watt

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watt

3.1 B

- $W = J \text{ s}^{-1} = \text{kg m}^2 \text{s}^{-2} \times \text{s}^{-1} = \text{kg m}^2 \text{s}^{-3}$
- **A** is the joule

Exercise 3.2

3.2 The resistive force on a boat is proportional to the square of its speed. At $15 \frac{\text{m}}{\text{s}}$ the resistive force is 600 N. What power is needed to keep the boat moving at a steady $30 \frac{\text{m}}{\text{s}}$? [1]

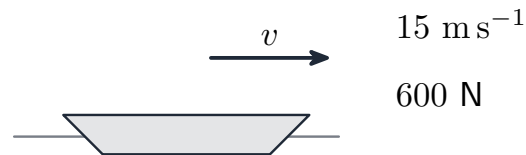
☐

A 18 kW

B 36 kW

C 72 kW

D 144 kW



Exercise 3.2

3.2 The resistive force on a boat is proportional to the square of its speed. At $15 \frac{\text{m}}{\text{s}}$ the resistive force is 600 N. What power is needed to keep the boat moving at a steady $30 \frac{\text{m}}{\text{s}}$? [1]

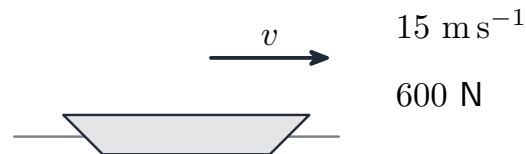
☐

A 18 kW

B 36 kW

C 72 kW

D 144 kW



3.2 C

- $F \propto v^2$, so at twice the speed $F = 2^2 \times 600 = 2400 \text{ N}$
- $P = Fv = 2400 \times 30 = 72 \text{ kW}$; **B** forgets that the force grows, **D** squares the power instead

Exercise 3.3 (a)

3.3 An electric car travels at a constant speed of $30 \frac{\text{m}}{\text{s}}$ along a straight level road. The total resistive force on the car is 1500 N.

(a) **Define** power.

[1]

Exercise 3.3 (a)

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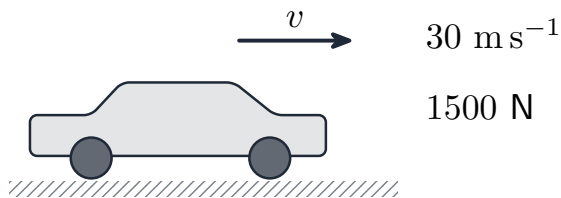
(a) the work done per unit time

Exercise 3.3 (b)

3.3 An electric car travels at a constant speed of $30 \frac{\text{m}}{\text{s}}$ along a straight level road. The total resistive force on the car is 1500 N .

(b) Calculate the power delivered to the wheels.

[2]

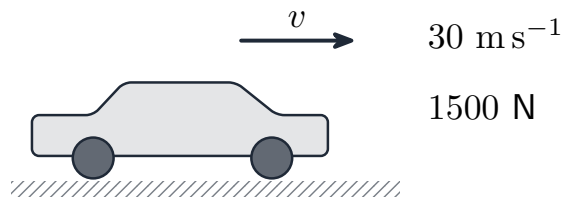


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(b) steady speed, so the driving force equals the resistive force; $P = Fv$

$$P = Fv = 1500 \text{ N} \times 30 \frac{\text{m}}{\text{s}} = \mathbf{4.5 \times 10^4 \text{ W}}$$

Exercise 3.3 (c)

3.3 An electric car travels at a constant speed of $30 \frac{\text{m}}{\text{s}}$ along a straight level road. The total resistive force on the car is 1500 N.

(c) Calculate the useful work done by the motor as the car travels 12 km. [2]

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(c) Calculate the useful work done by the motor as the car travels 12 km. [2]

(c) work done, $W = Fs$

$$W = Fs = 1500 \text{ N} \times 12 \times 10^3 \text{ m} = \mathbf{1.8 \times 10^7 \text{ J}}$$

Exercise 3.3 (d)

3.3 An electric car travels at a constant speed of $30 \frac{\text{m}}{\text{s}}$ along a straight level road. The total resistive force on the car is 1500 N.

(d) The motor is 85% efficient. Calculate the electrical power input to the motor. [2]

Exercise 3.3 (d)

3.3 An electric car travels at a constant speed of $30 \frac{\text{m}}{\text{s}}$ along a straight level road. The total resistive force on the car is 1500 N.

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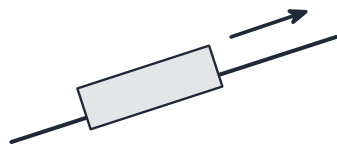
$$(d) \text{ efficiency} = \frac{\text{useful output}}{\text{total input}}, \text{ so input} = \frac{\text{output}}{\text{efficiency}}$$

$$P_{\text{in}} = \frac{P_{\text{out}}}{\text{efficiency}} = \frac{4.5 \times 10^4 \text{ W}}{0.85} = \mathbf{5.3 \times 10^4 \text{ W}}$$

Exercise 3.4 (a)

3.4 A lorry of mass 1.5×10^4 kg climbs a straight road inclined at 4.0° to the horizontal, at a steady speed of $12 \frac{\text{m}}{\text{s}}$. The total resistive force on the lorry is 3.0×10^3 N.

(a) **Show that** the driving force on the lorry is about 1.3×10^4 N. [2]



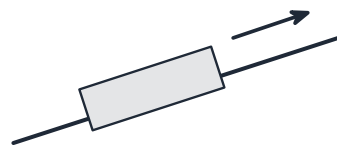
1.5×10^4 kg

4.0°

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(a) **Show that** the driving force on the lorry is about 1.3×10^4 N. [2]



1.5×10^4 kg

4.0°

(a) steady speed: driving force = resistive force + $mg \sin \theta$

$$mg \sin \theta = 1.5 \times 10^4 \times 9.81 \times \sin 4.0^\circ = 1.03 \times 10^4 \text{ N}$$

$$F = F_r + mg \sin \theta = 3.0 \times 10^3 + 1.03 \times 10^4 = \mathbf{1.3 \times 10^4 \text{ N}}$$

Exercise 3.4 (b)

3.4 A lorry of mass 1.5×10^4 kg climbs a straight road inclined at 4.0° to the horizontal, at a steady speed of $12 \frac{\text{m}}{\text{s}}$. The total resistive force on the lorry is 3.0×10^3 N.

(b) Calculate the useful output power of the engine.

[2]

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(b) $P = Fv$

$$P = Fv = 1.33 \times 10^4 \text{ N} \times 12 \frac{\text{m}}{\text{s}} = \mathbf{1.6 \times 10^5 \text{ W}}$$

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(c) The engine's efficiency is 35%. Calculate the rate at which the engine uses energy from its fuel. [2]

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$$(c) \text{ input} = \frac{\text{output}}{\text{efficiency}}$$

$$P_{\text{in}} = \frac{P_{\text{out}}}{\text{efficiency}} = \frac{1.6 \times 10^5 \text{ W}}{0.35} = \mathbf{4.6 \times 10^5 \text{ W}}$$

Exercise 3.4 (d)

3.4 A lorry of mass 1.5×10^4 kg climbs a straight road inclined at 4.0° to the horizontal, at a steady speed of $12 \frac{\text{m}}{\text{s}}$. The total resistive force on the lorry is 3.0×10^3 N.

(d) At the top of the hill the road becomes level. The engine keeps the same output power and the resistive force stays the same. **Explain** why the lorry speeds up, and calculate its greatest possible speed on the level. [3]

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(d) At the top of the hill the road becomes level. The engine keeps the same output power and the resistive force stays the same. **Explain** why the lorry speeds up, and calculate its greatest possible speed on the level. [3]

(d) on the level the driving force needed is only the resistive force, so the same power gives a driving force greater than the resistive force and a resultant force forwards

- top speed when the driving force equals the resistive force: $P = Fv$, so $v = \frac{P}{F}$

$$v = \frac{P}{F} = \frac{1.6 \times 10^5 \text{ W}}{3.0 \times 10^3 \text{ N}} = \mathbf{53} \frac{\text{m}}{\text{s}}$$