

Exercise walk-through

4.3

Density and pressure

A-Level Physics

4.3 Density, pressure and hydrostatic pressure

Name: _____ Class: _____ Date: _____

Total: 36 marks

KEY WORDS density

- atmospheric pressure

- pressure

- percentage uncertainty

- hydrostatic pressure

Objective

Answer exam questions on **density** and **pressure**, including the **hydrostatic pressure** equation $\Delta p = \rho g \Delta h$ and its derivation. Syllabus 4.3, learning outcomes 1 to 4.

You must be able to:

- define density and pressure, with their units
- calculate the pressure under a solid object, and a density from measurements with its percentage uncertainty
- derive $\Delta p = \rho g \Delta h$ from the definitions of pressure and density
- use it for the pressure at a depth, the total pressure with the atmosphere, and the

difference between two depths

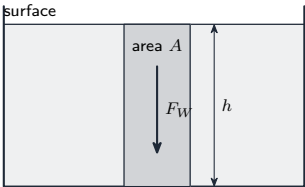
1 Key knowledge and worked examples

■ Key knowledge: the definitions

Quantity	Definition	Equation	Unit
density	mass per unit volume	$\rho = \frac{m}{V}$	kg m ⁻³
pressure	normal force per unit area	$p = \frac{F}{A}$	Pa = N m ⁻²
hydrostatic pressure	the pressure due to a depth Δh of a fluid	$\Delta p = \rho g \Delta h$	Pa

- conversions: $1 \text{ g cm}^{-3} = 1000 \text{ kg m}^{-3}$ (water); $1 \text{ cm}^2 = 10^{-4} \text{ m}^2$; $1 \text{ cm}^3 = 10^{-6} \text{ m}^3$
- under a solid: $p = \frac{mg}{A}$; the same weight on a smaller area gives a larger pressure
- in SI base units a pascal is $\text{kg m}^{-1} \text{ s}^{-2}$
- a density from measurements: add the **percentage** uncertainties of the mass and of every length multiplied together

■ **Key knowledge: deriving $\Delta p = \rho g \Delta h$**



$$\Delta p = \rho g h$$

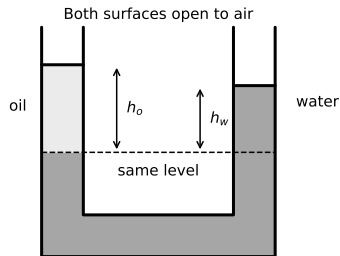
The four lines the mark scheme looks for, for a column of liquid of cross-sectional area A and height Δh :

1. volume $V = A \Delta h$
2. mass $m = \rho V = \rho A \Delta h$
3. weight $F_W = mg = \rho A \Delta h g$
4. pressure = $\frac{\text{force}}{\text{area}} = \frac{\rho A \Delta h g}{A}$, so $\Delta p = \rho g \Delta h$

Fact	Consequence
Δp depends only on the depth and the density	not on the shape of the container or the area
total pressure = $p_{\text{atm}} + \rho g h$, with $p_{\text{atm}} \approx 1.0 \times 10^5 \text{ Pa}$	"the pressure at a depth" may mean either: read what the question asks
same depth in the same liquid at rest	the same pressure, whatever lies between
a U-tube with two liquids	at the level of their boundary the pressures are equal: $\rho_1 h_1 = \rho_2 h_2$

■ **Key knowledge: connected liquids**

In a stationary connected liquid, pressures at the **same horizontal level in the same liquid** are equal. Start at a free surface, add ρgh when moving down, and subtract it when moving up. Both open surfaces have atmospheric pressure, which cancels.



Worked example. Oil of density 800 kg/m^3 has a 0.10 m column above the oil–water boundary in an open U-tube. The water surface is h_w above that level. $p_{\text{atm}} + \rho_o gh_o = p_{\text{atm}} + \rho_w gh_w$, hence $h_w = \rho_o h_o / \rho_w = 800 \times 0.10 / 1000 = 0.080 \text{ m}$. Compare heights from the same level, not from the tube's bottom.

■ **Worked example: a density with its uncertainty**

A block has $m = (0.300 \pm 0.001) \text{ kg}$ and sides $x = (5.00 \pm 0.01) \text{ cm}$, $y = (10.00 \pm 0.01) \text{ cm}$ and $z = (2.00 \pm 0.01) \text{ cm}$.

1. $V = xyz = 5.00 \times 10.00 \times 2.00 = 100 \text{ cm}^3 = 1.00 \times 10^{-4} \text{ m}^3$.
2. $\rho = m/V = 0.300 / (1.00 \times 10^{-4}) = 3.00 \times 10^3 \text{ kg m}^{-3}$.
3. For products/quotients add fractional uncertainties:

$$\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

$$\frac{\Delta \rho}{\rho} = \frac{0.001}{0.300} + \frac{0.01}{5.00} + \frac{0.01}{10.00} + \frac{0.01}{2.00} = 0.0113$$

4. The percentage uncertainty is 1.1%; the smallest side has the largest fractional uncertainty, so improving its measurement has the largest effect.

■ **Worked example: pressure under a solid**

A crate of mass 12 kg has a base 0.20 m by 0.15 m.

1. **Force** – the weight, $F_W = mg = 12 \times 9.81 = 118 \text{ N}$.
2. **Pressure** – from the definition, $p = \frac{F}{A} = \frac{118 \text{ N}}{0.20 \text{ m} \times 0.15 \text{ m}} = 3.9 \times 10^3 \text{ Pa}$.

■ **Worked example: pressure at a depth**

A diver is 12 m below the surface of sea water of density 1030 kg m^{-3} . Atmospheric pressure is $1.0 \times 10^5 \text{ Pa}$.

1. **Pressure due to the water** – hydrostatic pressure, $\Delta p = \rho g \Delta h = 1030 \times 9.81 \times 12 = 1.21 \times 10^5 \text{ Pa}$.
2. **Total pressure** – add the atmosphere: $p = p_{\text{atm}} + \rho g h = 1.0 \times 10^5 + 1.21 \times 10^5 = 2.2 \times 10^5 \text{ Pa}$.
3. **Difference between two depths** – only Δh matters: between 12 m and 12.5 m, $\Delta p = \rho g \Delta h = 1030 \times 9.81 \times 0.50 = 5.1 \times 10^3 \text{ Pa}$. This difference, over the area of an object, is where upthrust comes from (sheet 4.3.1).

■ **Worked example: a jet of water on a wall**

A horizontal jet of water, density 1000 kg m^{-3} and cross-sectional area $2.0 \times 10^{-4} \text{ m}^2$, hits a wall at $6.0 \frac{\text{m}}{\text{s}}$ and runs down it.

1. **Mass per second** – density $\times$ the volume arriving each second: $\frac{m}{t} = \rho A v = 1000 \times 2.0 \times 10^{-4} \times 6.0 = 1.2 \text{ kg s}^{-1}$.
2. **Force** – rate of change of momentum, the water losing all its horizontal speed: $F = \frac{m}{t} v = 1.2 \times 6.0 = 7.2 \text{ N}$.
3. **Pressure** – force per unit area: $p = \frac{F}{A} = \frac{7.2}{2.0 \times 10^{-4}} = 3.6 \times 10^4 \text{ Pa}$.

Exercises 2.1-2.2

2.1 Define **pressure**.

[1]

pressure

2.2 A block has mass 2400 kg and volume 0.30 m^3 . Calculate its **density**.

[2]



2400 kg

0.30 m^3

Exercises 2.1·2.2

2.1 Define **pressure**.

[1]

pressure

2.1

- pressure is the **normal force per unit area**
-

2.2 A block has mass 2400 kg and volume 0.30 m³. Calculate its **density**.

[2]



2400 kg

0.30 m³

2.2

- density is mass per unit volume, $\rho = m/V$

$$\rho = \frac{m}{V} = \frac{2400 \text{ kg}}{0.30 \text{ m}^3} = 8000 \text{ kg m}^{-3}$$

Exercise 2.3

2.3 A person of mass 60 kg stands on both feet. Each foot has a contact area of 0.020 m^2 . Calculate the pressure on the floor. [2]



60 kg

0.020 m^2

Exercise 2.3

2.3 A person of mass 60 kg stands on both feet. Each foot has a contact area of 0.020 m². Calculate the pressure on the floor. [2]



60 kg

0.020 m²

2.3

- the force is the weight, $F_W = mg = 60 \times 9.81 = 589 \text{ N}$
- it acts on two feet, a total area $A = 2 \times 0.020 = 0.040 \text{ m}^2$

$$p = \frac{F}{A} = \frac{589 \text{ N}}{0.040 \text{ m}^2} = \mathbf{1.5 \times 10^4 \text{ Pa}}$$

Exercise 2.4 (a)

2.4 A swimming pool is 2.5 m deep. Atmospheric pressure is 1.0×10^5 Pa. Calculate, at the bottom of the pool:

(a) the pressure due to the water

[2]



2.5 m

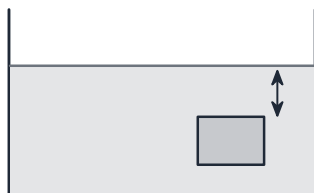
1.0×10^5 Pa

Exercise 2.4 (a)

2.4 A swimming pool is 2.5 m deep. Atmospheric pressure is 1.0×10^5 Pa. Calculate, at the bottom of the pool:

(a) the pressure due to the water

[2]



2.5 m

1.0×10^5 Pa

(a) hydrostatic pressure, $\Delta p = \rho g \Delta h$

$$\Delta p = \rho g \Delta h = 1000 \text{ kg m}^{-3} \times 9.81 \text{ m s}^{-2} \times 2.5 \text{ m} = \mathbf{2.5 \times 10^4 \text{ Pa}}$$

Exercise 2.4 (b)

2.4 A swimming pool is 2.5 m deep. Atmospheric pressure is 1.0×10^5 Pa. Calculate, at the bottom of the pool:

(b) the total pressure.

[1]

Exercise 2.4 (b)

2.4 A swimming pool is 2.5 m deep. Atmospheric pressure is 1.0×10^5 Pa. Calculate, at the bottom of the pool:

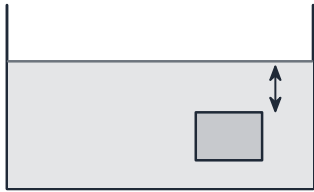
(b) the total pressure.

[1]

(b) total pressure $p = p_{\text{atm}} + \rho gh = 1.0 \times 10^5 + 2.5 \times 10^4 = \mathbf{1.25 \times 10^5}$ Pa

Exercise 2.5

2.5 Two points in an oil of density 850 kg m^{-3} are 0.40 m apart vertically. Calculate the difference in pressure between them. [2]

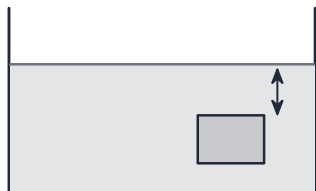


850 kg m^{-3}

0.40 m

Exercise 2.5

2.5 Two points in an oil of density 850 kg m^{-3} are 0.40 m apart vertically. Calculate the difference in pressure between them. [2]



850 kg m^{-3}

0.40 m

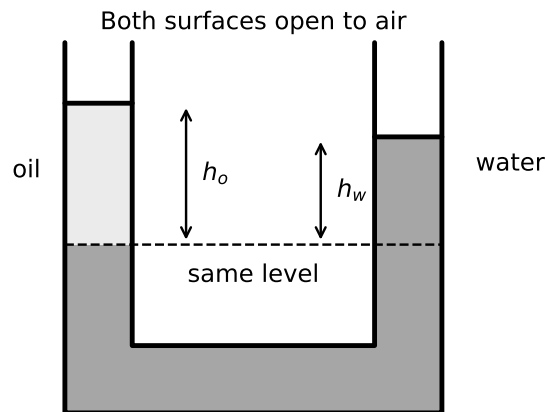
2.5

- hydrostatic pressure, only the difference in depth matters: $\Delta p = \rho g \Delta h$

$$\Delta p = \rho g \Delta h = 850 \times 9.81 \times 0.40 = \mathbf{3.3 \times 10^3 \text{ Pa}}$$

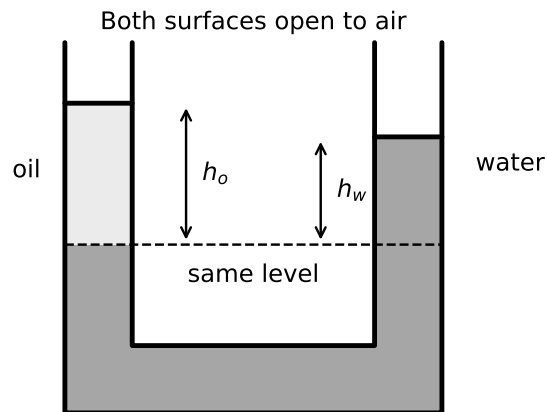
Exercise 2.6

2.6 Both arms of a U-tube are open to air. A 0.15 m column of oil of density 820 kg/m^3 sits above water. The water density is 1000 kg/m^3 . Find the water-surface height above the oil-water boundary and the difference between the free-surface heights. [4]



Exercise 2.6

2.6 Both arms of a U-tube are open to air. A 0.15 m column of oil of density 820 kg/m^3 sits above water. The water density is 1000 kg/m^3 . Find the water-surface height above the oil-water boundary and the difference between the free-surface heights. [4]



2.6

- At the common level, $\rho_o g h_o = \rho_w g h_w$.

Full solution 2.6

2.6

- At the common level, $\rho_o g h_o = \rho_w g h_w$.
- $h_w = \rho_o h_o / \rho_w = 820 \times 0.15 / 1000 = 0.123$ m.
- Surface difference $\Delta h = h_o - h_w = 0.15 - 0.123 = 0.027$ m.
- The oil surface is higher: the less dense liquid needs a taller column for the same pressure.

Exercise 3.1

3.1 The pressure due to a liquid at the bottom of a container depends on [1]

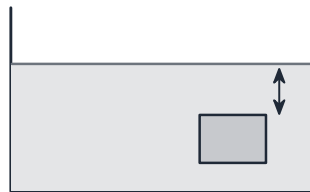
☐

A the cross-sectional area of the container only

B the depth and the density of the liquid

C the shape of the container

D the total volume of liquid



Exercise 3.1

3.1 The pressure due to a liquid at the bottom of a container depends on [1]

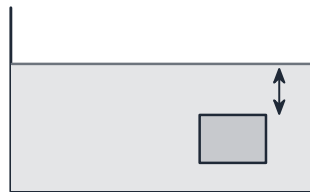
☐

A the cross-sectional area of the container only

B the depth and the density of the liquid

C the shape of the container

D the total volume of liquid



3.1 B

- $\Delta p = \rho g \Delta h$: only the depth and the density of the liquid appear
- the area cancels in the derivation, so **A** and **C** are wrong; **D** confuses pressure with the weight of all the liquid

Exercise 3.2

3.2 What is the pascal expressed in SI base units?

[1]

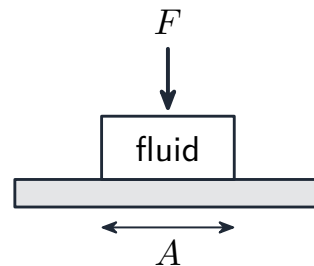
☐

A kg m s^{-2}

B $\text{kg m}^{-1} \text{s}^{-2}$

C $\text{kg m}^{-2} \text{s}^{-2}$

D $\text{kg m}^2 \text{s}^{-2}$



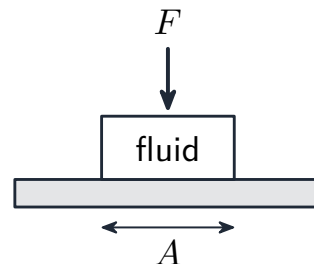
Exercise 3.2

3.2 What is the pascal expressed in SI base units?

[1]

☐

- A** kg m s^{-2}
B $\text{kg m}^{-1} \text{s}^{-2}$
C $\text{kg m}^{-2} \text{s}^{-2}$
D $\text{kg m}^2 \text{s}^{-2}$



3.2 B

- $p = F/A$: $\text{N/m}^2 = \text{kg m s}^{-2}/\text{m}^2 = \text{kg m}^{-1} \text{s}^{-2}$
- **A** is the newton; **D** is the joule

Exercise 3.3 (a)

3.3 A tank contains water of density 1000 kg m^{-3} to a depth of 3.2 m . A square hatch of side 0.40 m is set in the flat bottom of the tank.

(a) **Show that** the pressure due to a depth Δh of a liquid of density ρ is $\Delta p = \rho g \Delta h$. [3]



1000 kg m^{-3}

3.2 m

0.40 m

Exercise 3.3 (a)

3.3 A tank contains water of density 1000 kg m^{-3} to a depth of 3.2 m. A square hatch of side 0.40 m is set in the flat bottom of the tank.

(a) **Show that** the pressure due to a depth Δh of a liquid of density ρ is $\Delta p = \rho g \Delta h$. [3]



1000 kg m^{-3}

3.2 m

0.40 m

(a) a column of area A and height Δh has volume $A\Delta h$ and mass $m = \rho A\Delta h$

- its weight is $F_W = mg = \rho A\Delta h g$
- pressure = $\frac{F_W}{A} = \frac{\rho A\Delta h g}{A} = \rho g \Delta h$

Exercise 3.3 (b)

3.3 A tank contains water of density 1000 kg m^{-3} to a depth of 3.2 m. A square hatch of side 0.40 m is set in the flat bottom of the tank.

(b) Calculate the pressure due to the water at the bottom of the tank. [2]

Exercise 3.3 (b)

3.3 A tank contains water of density 1000 kg m^{-3} to a depth of 3.2 m. A square hatch of side 0.40 m is set in the flat bottom of the tank.

(b) Calculate the pressure due to the water at the bottom of the tank. [2]

(b) hydrostatic pressure, $\Delta p = \rho g \Delta h$

$$\Delta p = \rho g \Delta h = 1000 \times 9.81 \times 3.2 = \mathbf{3.1 \times 10^4 \text{ Pa}}$$

Exercise 3.3 (c)

3.3 A tank contains water of density 1000 kg m^{-3} to a depth of 3.2 m. A square hatch of side 0.40 m is set in the flat bottom of the tank.

(c) Calculate the force that the water exerts on the hatch.

[2]

Exercise 3.3 (c)

3.3 A tank contains water of density 1000 kg m^{-3} to a depth of 3.2 m. A square hatch of side 0.40 m is set in the flat bottom of the tank.

(c) Calculate the force that the water exerts on the hatch. [2]

(c) from the definition of pressure, $F = pA$

$$F = pA = 3.14 \times 10^4 \text{ Pa} \times (0.40 \text{ m})^2 = \mathbf{5.0 \times 10^3 \text{ N}}$$

Exercise 3.3 (d)

3.3 A tank contains water of density 1000 kg m^{-3} to a depth of 3.2 m. A square hatch of side 0.40 m is set in the flat bottom of the tank.

(d) The tank is replaced by a narrower one holding water to the same depth. State and explain what happens to the pressure on the hatch. [2]

Exercise 3.3 (d)

3.3 A tank contains water of density 1000 kg m^{-3} to a depth of 3.2 m. A square hatch of side 0.40 m is set in the flat bottom of the tank.

(d) The tank is replaced by a narrower one holding water to the same depth. State and explain what happens to the pressure on the hatch. [2]

(d) the pressure is **unchanged**

- it depends only on the depth and the density of the water, not on the width of the tank

Exercise 3.4 (a)

3.4 A student measures a rectangular metal block to find the density of the metal.

quantity	measurement
mass m	$(0.512 \pm 0.001) \text{ kg}$
length x	$(8.02 \pm 0.01) \text{ cm}$
width y	$(4.15 \pm 0.01) \text{ cm}$
height z	$(2.05 \pm 0.01) \text{ cm}$

(a) **Define** density.

[1]



$(0.512 \pm 0.001) \text{ kg}$

$(8.02 \pm 0.01) \text{ cm}$

Exercise 3.4 (a)

3.4 A student measures a rectangular metal block to find the density of the metal.

quantity	measurement
mass m	$(0.512 \pm 0.001) \text{ kg}$
length x	$(8.02 \pm 0.01) \text{ cm}$
width y	$(4.15 \pm 0.01) \text{ cm}$
height z	$(2.05 \pm 0.01) \text{ cm}$

(a) **Define** density.

[1]



$(0.512 \pm 0.001) \text{ kg}$

$(8.02 \pm 0.01) \text{ cm}$

(a) density is **mass per unit volume**

Exercise 3.4 (b)

3.4 A student measures a rectangular metal block to find the density of the metal.

quantity	measurement
mass m	$(0.512 \pm 0.001) \text{ kg}$
length x	$(8.02 \pm 0.01) \text{ cm}$
width y	$(4.15 \pm 0.01) \text{ cm}$
height z	$(2.05 \pm 0.01) \text{ cm}$

(b) **Determine** the density of the metal.

[3]

Exercise 3.4 (b)

3.4 A student measures a rectangular metal block to find the density of the metal.

quantity	measurement
mass m	$(0.512 \pm 0.001) \text{ kg}$
length x	$(8.02 \pm 0.01) \text{ cm}$
width y	$(4.15 \pm 0.01) \text{ cm}$
height z	$(2.05 \pm 0.01) \text{ cm}$

(b) **Determine** the density of the metal.

[3]

$$(b) V = xyz = 8.02 \times 4.15 \times 2.05 = 68.2 \text{ cm}^3 = 6.82 \times 10^{-5} \text{ m}^3$$

- density, $\rho = m/V$

$$\rho = \frac{m}{V} = \frac{0.512 \text{ kg}}{6.82 \times 10^{-5} \text{ m}^3} = \mathbf{7.5 \times 10^3 \text{ kg m}^{-3}}$$

Exercise 3.4 (c)

3.4 A student measures a rectangular metal block to find the density of the metal.

quantity	measurement
mass m	$(0.512 \pm 0.001) \text{ kg}$
length x	$(8.02 \pm 0.01) \text{ cm}$
width y	$(4.15 \pm 0.01) \text{ cm}$
height z	$(2.05 \pm 0.01) \text{ cm}$

(c) **Determine** the percentage uncertainty in the density.

[3]

Exercise 3.4 (c)

3.4 A student measures a rectangular metal block to find the density of the metal.

quantity	measurement
mass m	$(0.512 \pm 0.001) \text{ kg}$
length x	$(8.02 \pm 0.01) \text{ cm}$
width y	$(4.15 \pm 0.01) \text{ cm}$
height z	$(2.05 \pm 0.01) \text{ cm}$

(c) **Determine** the percentage uncertainty in the density.

[3]

(c) quantities multiplied or divided: add the percentage uncertainties

$$\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z} = \frac{0.001}{0.512} + \frac{0.01}{8.02} + \frac{0.01}{4.15} + \frac{0.01}{2.05}$$

$$\frac{\Delta \rho}{\rho} = 0.20\% + 0.12\% + 0.24\% + 0.49\%$$

- total = **1.1%**

Exercise 3.4 (d)

3.4 A student measures a rectangular metal block to find the density of the metal.

quantity	measurement
mass m	$(0.512 \pm 0.001) \text{ kg}$
length x	$(8.02 \pm 0.01) \text{ cm}$
width y	$(4.15 \pm 0.01) \text{ cm}$
height z	$(2.05 \pm 0.01) \text{ cm}$

(d) **State**, with a reason, which single measurement the student should improve first. [2]

Exercise 3.4 (d)

3.4 A student measures a rectangular metal block to find the density of the metal.

quantity	measurement
mass m	$(0.512 \pm 0.001) \text{ kg}$
length x	$(8.02 \pm 0.01) \text{ cm}$
width y	$(4.15 \pm 0.01) \text{ cm}$
height z	$(2.05 \pm 0.01) \text{ cm}$

(d) **State**, with a reason, which single measurement the student should improve first. [2]

(d) the height z

- it has the largest percentage uncertainty, 0.49%, because it is the smallest length measured to the same $\pm 0.01 \text{ cm}$

Exercise 3.4 (e)

3.4 A student measures a rectangular metal block to find the density of the metal.

quantity	measurement
mass m	$(0.512 \pm 0.001) \text{ kg}$
length x	$(8.02 \pm 0.01) \text{ cm}$
width y	$(4.15 \pm 0.01) \text{ cm}$
height z	$(2.05 \pm 0.01) \text{ cm}$

(e) The block rests on the bench on its largest face. Calculate the pressure it exerts on the bench. [2]

Exercise 3.4 (e)

3.4 A student measures a rectangular metal block to find the density of the metal.

quantity	measurement
mass m	$(0.512 \pm 0.001) \text{ kg}$
length x	$(8.02 \pm 0.01) \text{ cm}$
width y	$(4.15 \pm 0.01) \text{ cm}$
height z	$(2.05 \pm 0.01) \text{ cm}$

(e) The block rests on the bench on its largest face. Calculate the pressure it exerts on the bench. [2]

(e) the force is the weight, $F_W = mg = 0.512 \times 9.81 = 5.02 \text{ N}$

- the largest face has area $A = xy = 8.02 \times 4.15 = 33.3 \text{ cm}^2 = 3.33 \times 10^{-3} \text{ m}^2$

$$p = \frac{F}{A} = \frac{5.02 \text{ N}}{3.33 \times 10^{-3} \text{ m}^2} = \mathbf{1.5 \times 10^3 \text{ Pa}}$$