

Exercise walk-through

4.2

Equilibrium of forces

A-Level Physics

4.2 Equilibrium, the principle of moments and vector triangles

Name: _____ Class: _____ Date: _____

Total: 44 marks

- KEY WORDS**
- equilibrium
 - principle of moments
 - vector triangle
 - coplanar
 - resultant force

Objective

Answer exam questions on **equilibrium** : the **principle of moments**, the two conditions for equilibrium, and the **vector triangle** of three forces. Syllabus 4.2, learning outcomes 1, 2 and 3.

You must be able to:

- state the principle of moments and the two conditions for equilibrium
- choose the point to take moments about, so that an unknown force drops out
- find the support forces on a beam, and the force from a hinge, including its direction

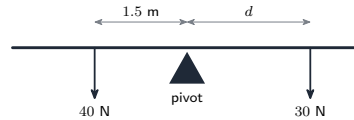
- solve three forces in equilibrium with a closed vector triangle or by resolving

1 Key knowledge and worked examples

■ Key knowledge: equilibrium

Statement	In the examiner's words
principle of moments (2 marks)	for an object in equilibrium, the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about the same point
equilibrium	the resultant force is zero and the resultant moment (torque) is zero
three coplanar forces in equilibrium	drawn tip to tail they form a closed triangle ; their lines of action meet at one point

Method step	Why
draw every force on the object, the weight at the centre of gravity	a missing weight is a missing moment
take moments about the point where an unknown force acts	that force has zero moment there and drops out
then resolve: total up = total down, total left = total right	this gives the last unknown force
a hinge force may point down	a support between the hinge and the load can only balance if the hinge pulls down



A beam balanced on its centre: its own weight has no moment about the pivot

■ Worked example: balancing a beam

In the figure above the beam is uniform, so its weight acts at the pivot. A 40 N weight hangs 1.5 m to the left; where must a 30 N weight hang?

1. **Principle of moments** about the pivot – clockwise moment = anticlockwise moment, each $M = Fd$: $F_{cw} d = F_{acw} d_{acw}$.
2. **Distance** – rearrange for d :

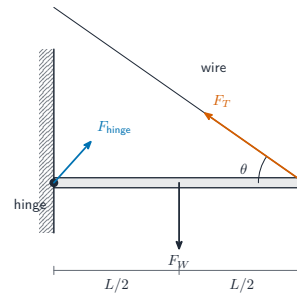
$$d = \frac{F_{acw} d_{acw}}{F_{cw}} = \frac{40 \text{ N} \times 1.5 \text{ m}}{30 \text{ N}} = 2.0 \text{ m to the right}$$

■ Worked example: a plank on two supports

A uniform plank of length 4.0 m and weight 200 N rests on supports at its ends A and B. A person of weight 500 N stands 1.0 m from A.

1. **Moments about A** – the force at A drops out; principle of moments, $\sum M_{cw} = \sum M_{acw}$ with $M = Fd$: $F_{N,B} \times 4.0 = 200 \times 2.0 + 500 \times 1.0 = 900 \text{ N m}$, so $F_{N,B} = 225 \text{ N}$.
2. **Resolve vertically** – no resultant force, $F_{N,A} + F_{N,B} = F_{W,plank} + F_{W,person}$, so $F_{N,A} = F_{W,plank} + F_{W,person} - F_{N,B} = 200 + 500 - 225 = 475 \text{ N}$.

■ **Worked example: a rod held by a wire**



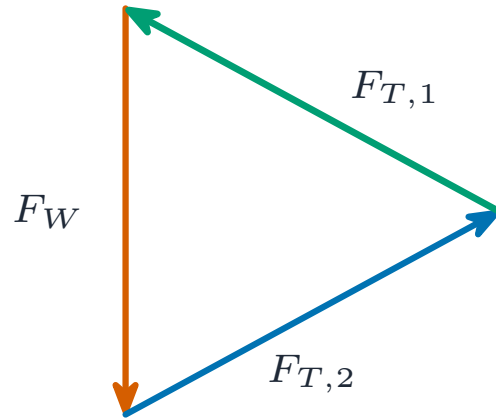
A uniform rod of length 1.2 m and weight 40 N is hinged to a wall and held horizontal by a wire from its far end, at 30° to the rod.

1. **Moments about the hinge** – only the component of the tension perpendicular to the rod, $F_T \sin 30^\circ$, has a moment: $F_T \sin 30^\circ \times 1.2 = 40 \times 0.60$, so $F_T = 40$ N.
2. **Hinge force, horizontal** – resolve horizontally: the wire pulls the rod towards the wall with $F_T \cos 30^\circ = 35$ N, so the hinge pushes it away from the wall with $F_H = 35$ N.
3. **Hinge force, vertical** – resolve vertically: $F_V + F_T \sin 30^\circ = F_W$, so $F_V = F_W - F_T \sin 30^\circ = 40 - 20 = 20$ N upwards.

■ **Worked example: a sign on two wires**

A sign of weight 250 N hangs from a horizontal wire and a second wire at 40° to the vertical.

1. **Vertical** – resolve first where there is one unknown: $F_T \cos 40^\circ = F_W$, so $F_T = \frac{F_W}{\cos 40^\circ} = \frac{250}{\cos 40^\circ} = 326$ N.
2. **Horizontal** – $F = F_T \sin 40^\circ = 326 \times 0.643 = 210$ N in the horizontal wire.
3. **Check** – a closed triangle of the three forces, drawn to scale, gives the same two answers.



the three forces
form a **closed**
triangle $\rightarrow$ equilibrium

Three forces in equilibrium: tip to tail, a closed triangle

Exercises 2.1·2.2

2.1 State the **principle of moments**.

[2]



2.2 State the **two** conditions for an object to be in equilibrium.

[2]

equilibrium

Exercises 2.1·2.2

2.1 State the **principle of moments**.

[2]



2.1

- for an object in equilibrium
- the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about the same point

2.2 State the **two** conditions for an object to be in equilibrium.

[2]

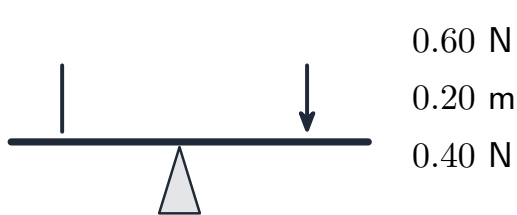
equilibrium

2.2

- the resultant force is zero
- the resultant moment (torque) is zero

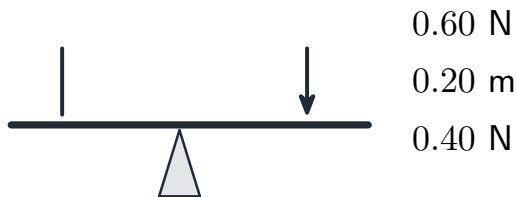
Exercise 2.3

2.3 A uniform metre rule balances at its centre. A 0.60 N weight hangs 0.20 m to the left of the centre. Calculate where a 0.40 N weight must hang to balance it. [2]



Exercise 2.3

2.3 A uniform metre rule balances at its centre. A 0.60 N weight hangs 0.20 m to the left of the centre. Calculate where a 0.40 N weight must hang to balance it. [2]



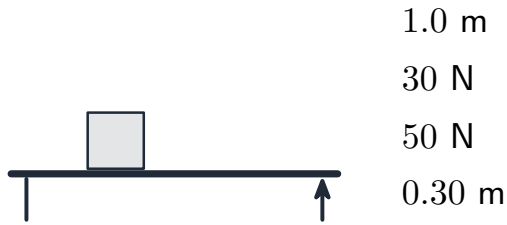
2.3

- principle of moments about the centre, $F_1 d_1 = F_2 d_2$: $0.40 \times d = 0.60 \times 0.20$

$$d = \frac{F_2 d_2}{F_1} = \frac{0.60 \text{ N} \times 0.20 \text{ m}}{0.40 \text{ N}} = \mathbf{0.30} \text{ m to the right of the centre}$$

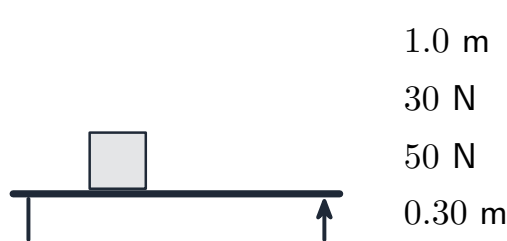
Exercise 2.4

2.4 A uniform shelf of length 1.0 m and weight 30 N rests on supports at its ends. A 50 N box stands 0.30 m from the left-hand end. Calculate the force from each support.[3]



Exercise 2.4

2.4 A uniform shelf of length 1.0 m and weight 30 N rests on supports at its ends. A 50 N box stands 0.30 m from the left-hand end. Calculate the force from each support.[3]



2.4

- moments about the left-hand end: $F_{N,\text{right}} \times 1.0 = 30 \times 0.50 + 50 \times 0.30 = 30 \text{ N m}$, so $F_{N,\text{right}} = \mathbf{30 \text{ N}}$
- resolving vertically, $F_{N,\text{left}} + F_{N,\text{right}} = F_{W,\text{shelf}} + F_{W,\text{box}}$: $F_{N,\text{left}} = 30 + 50 - 30 = \mathbf{50 \text{ N}}$

Exercise 2.5

2.5 A 20 N lamp hangs from two strings, each at 30° to the vertical. Calculate the tension in each string. [2]

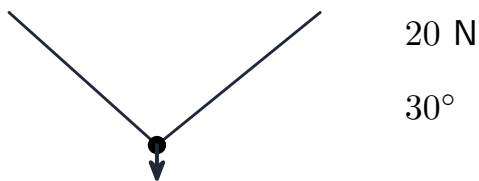


20 N

 30°

Exercise 2.5

2.5 A 20 N lamp hangs from two strings, each at 30° to the vertical. Calculate the tension in each string. [2]



2.5

- resolving vertically: $2F_T \cos 30^\circ = F_W$

$$F_T = \frac{F_W}{2 \cos 30^\circ} = \frac{20 \text{ N}}{2 \cos 30^\circ} = \mathbf{12 \text{ N}}$$

Exercise 3.1

3.1 Three coplanar forces act on an object that is in equilibrium. Which statement **must** be true? [1]

☐

A The three forces are equal in magnitude.

B Drawn tip to tail, the three forces form a closed triangle.

C The three forces are parallel.

D The resultant of two of the forces is equal in magnitude and direction to the third.



Exercise 3.1

3.1 Three coplanar forces act on an object that is in equilibrium. Which statement **must** be true? [1]

☐

A The three forces are equal in magnitude.

B Drawn tip to tail, the three forces form a closed triangle.

C The three forces are parallel.

D The resultant of two of the forces is equal in magnitude and direction to the third.



3.1 B

- no resultant force means the three force vectors, added tip to tail, return to the start: a closed triangle
- **D** has the direction wrong: the resultant of two is equal and **opposite** to the third

Exercise 3.2

3.2 A uniform rod of weight 12 N is supported at one end by a pivot and held horizontal by a vertical string attached to its other end. What is the tension in the string? [1]

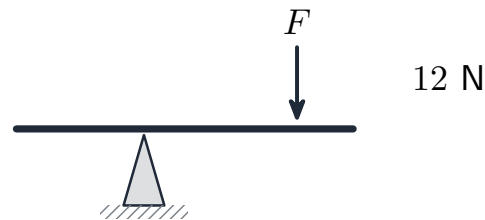
☐

A 3.0 N

B 6.0 N

C 12 N

D 24 N



Exercise 3.2

3.2 A uniform rod of weight 12 N is supported at one end by a pivot and held horizontal by a vertical string attached to its other end. What is the tension in the string? [1]

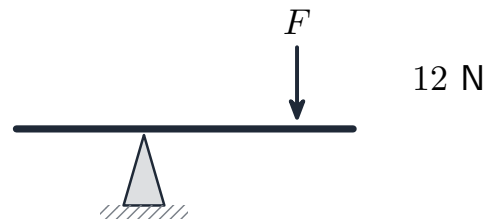
☐

A 3.0 N

B 6.0 N

C 12 N

D 24 N



3.2 B

- moments about the pivot: $F_T \times L = 12 \times \frac{L}{2}$, so $F_T = 6.0$ N
- the pivot supplies the other 6.0 N; **C** forgets the pivot takes a share

Exercise 3.3 (a)

3.3 A uniform beam of weight 60 N and length 4.0 m rests on supports A, at the left-hand end, and B, at the right-hand end. A 40 N load sits 1.0 m from A.

(a) **Show that** the upward force from support B is 40 N. [3]



Exercise 3.3 (a)

3.3 A uniform beam of weight 60 N and length 4.0 m rests on supports A, at the left-hand end, and B, at the right-hand end. A 40 N load sits 1.0 m from A.

(a) **Show that** the upward force from support B is 40 N. [3]



(a) take moments about A so that the force at A drops out

- principle of moments: $F_{N,B} \times 4.0 = 60 \times 2.0 + 40 \times 1.0 = 160 \text{ N m}$

$$F_{N,B} = \frac{M}{d} = \frac{160 \text{ N m}}{4.0 \text{ m}} = \mathbf{40 \text{ N}}$$

Exercise 3.3 (b)

3.3 A uniform beam of weight 60 N and length 4.0 m rests on supports A, at the left-hand end, and B, at the right-hand end. A 40 N load sits 1.0 m from A.

(b) Calculate the upward force from support A.

[1]

Exercise 3.3 (b)

3.3 A uniform beam of weight 60 N and length 4.0 m rests on supports A, at the left-hand end, and B, at the right-hand end. A 40 N load sits 1.0 m from A.

(b) Calculate the upward force from support A. [1]

(b) resolving vertically, $F_{N,A} + F_{N,B} = F_{W,\text{beam}} + F_{W,\text{load}}$: $F_{N,A} = 60 + 40 - 40 = \mathbf{60\text{ N}}$

Exercise 3.4 (a)

3.4 A sign of weight 180 N hangs from two cables. Cable 1 makes an angle of 30° with the vertical and cable 2 an angle of 60° with the vertical, on the other side.

(a) State the two conditions for the sign to be in equilibrium.

[2]

Exercise 3.4 (a)

3.4 A sign of weight 180 N hangs from two cables. Cable 1 makes an angle of 30° with the vertical and cable 2 an angle of 60° with the vertical, on the other side.

- (a) State the two conditions for the sign to be in equilibrium. [2]
- (a) the resultant force on the sign is zero
- the resultant moment on the sign is zero

Exercise 3.4 (b)

3.4 A sign of weight 180 N hangs from two cables. Cable 1 makes an angle of 30° with the vertical and cable 2 an angle of 60° with the vertical, on the other side.

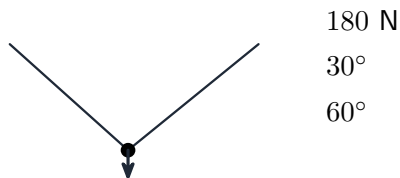
(b) By resolving horizontally and vertically, **determine** the tension in each cable. [4]



Exercise 3.4 (b)

3.4 A sign of weight 180 N hangs from two cables. Cable 1 makes an angle of 30° with the vertical and cable 2 an angle of 60° with the vertical, on the other side.

(b) By resolving horizontally and vertically, **determine** the tension in each cable. [4]

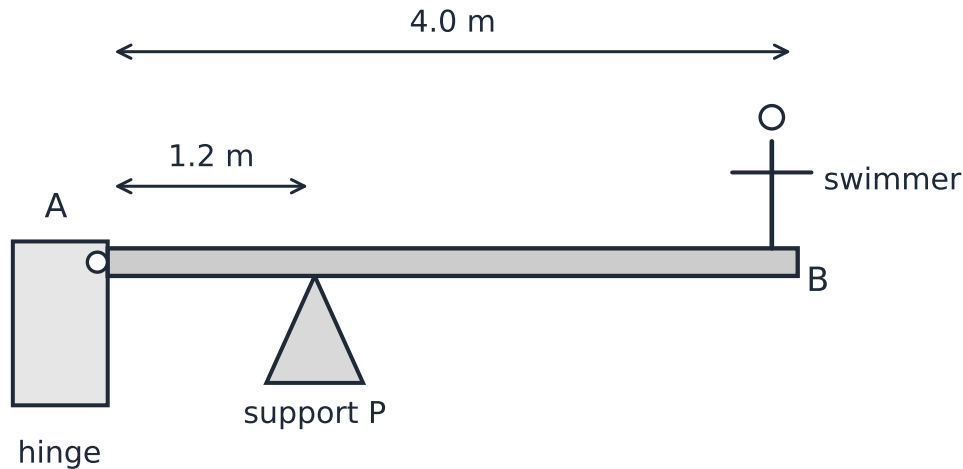


(b) horizontally: $F_{T,1} \sin 30^\circ = F_{T,2} \sin 60^\circ$, so $F_{T,1} = \sqrt{3} F_{T,2}$

- vertically: $F_{T,1} \cos 30^\circ + F_{T,2} \cos 60^\circ = 180$
- substituting: $\sqrt{3} F_{T,2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} F_{T,2} = 2F_{T,2} = 180$, so $F_{T,2} = \mathbf{90 \text{ N}}$
- $F_{T,1} = \sqrt{3} F_{T,2} = \sqrt{3} \times 90 = \mathbf{156 \text{ N}}$

Setup for exercise 3.5

3.5 A uniform diving board AB has length 4.0 m and weight 800 N. It is hinged to a wall at end A and rests on a support P a distance 1.2 m from A. A swimmer of weight 620 N stands at end B, and the board is in equilibrium.



Exercise 3.5 (a)

(a) **State** the principle of moments.

[2]

Exercise 3.5 (a)

(a) **State** the principle of moments.

[2]

(a) for a body in **equilibrium**

- sum of **clockwise moments** about any point = sum of **anticlockwise moments** about that same point

Exercise 3.5 (b)

(b) On the diagram above, **draw** arrows showing the **weight of the board**, the **force from the support P** and the **weight of the swimmer**. **Label** each arrow. [3]

Exercise 3.5 (b)

(b) On the diagram above, **draw** arrows showing the **weight of the board**, the **force from the support P** and the **weight of the swimmer**. **Label** each arrow. [3]

(b)

- arrow **down** at the middle of the board (2.0 m from A) labelled *weight of board / 800 N*
- arrow **up** at P labelled *support force*
- arrow **down** at B labelled *weight of swimmer / 620 N*

Exercise 3.5 (c)

(c) By taking moments about A, **determine** the force that the support P exerts on the board. [3]

Exercise 3.5 (c)

(c) By taking moments about A, **determine** the force that the support P exerts on the board. [3]

(c) moments about A (hinge force has no moment)

- clockwise $\sum M = F_{W,1}d_1 + F_{W,2}d_2 = (800 \text{ N})(2.0 \text{ m}) + (620 \text{ N})(4.0 \text{ m}) = 4080 \text{ N m}$
- anticlockwise $= F_P \times 1.2 \text{ m}$

$$F_P = \frac{\sum M}{d} = \frac{4080 \text{ N m}}{1.2 \text{ m}} = \mathbf{3400 \text{ N}}$$

Exercise 3.5 (d)

(d) **Determine** the magnitude and the direction of the force that the hinge at A exerts on the board. [3]

Exercise 3.5 (d)

(d) **Determine** the magnitude and the direction of the force that the hinge at A exerts on the board. [3]

(d) vertical equilibrium: up = down

- $\sum F_{\text{up}} = \sum F_{\text{down}}$: $3400 \text{ N} + F_{\text{hinge}} = 800 \text{ N} + 620 \text{ N}$
- $F_{\text{hinge}} = -1980 \text{ N}$, so **1980 N downwards**

Exercise 3.5 (e)

(e) **Explain** why taking moments about A is a better choice than taking them about P. [1]

Exercise 3.5 (e)

(e) **Explain** why taking moments about A is a better choice than taking them about P. [1]

(e) the hinge force is unknown; moments **about** A give it zero moment, so it drops out

Exercise 3.5 (f)

(f) The swimmer now walks slowly back towards A. **Give** the distance from A at which she must stand for the hinge to exert **no** force on the board. [3]

Exercise 3.5 (f)

(f) The swimmer now walks slowly back towards A. **Give** the distance from A at which she must stand for the hinge to exert **no** force on the board. [3]

(f) no hinge: take moments about **P**

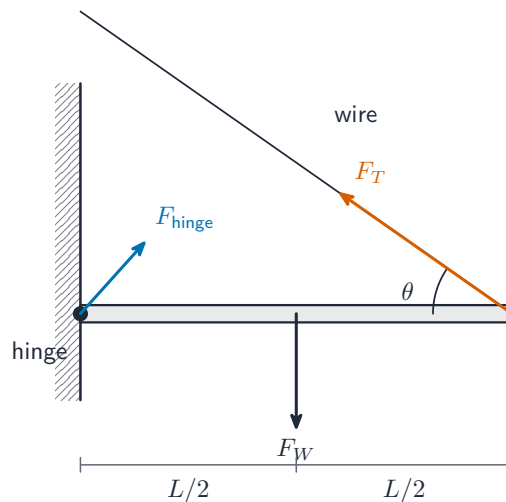
- board's weight is $d = 2.0 \text{ m} - 1.2 \text{ m} = 0.80 \text{ m}$ to the right of P
- clockwise: $M = Fd = (800 \text{ N})(0.80 \text{ m}) = 640 \text{ N m}$
- swimmer must be on the **other** side of P, a distance $(1.2 \text{ m} - x)$ from it

$$\sum M_{\text{anticlockwise}} = \sum M_{\text{clockwise}} : \quad (620 \text{ N})(1.2 \text{ m} - x) = 640 \text{ N m} \Rightarrow x = \mathbf{0.17 \text{ m}}$$

from A

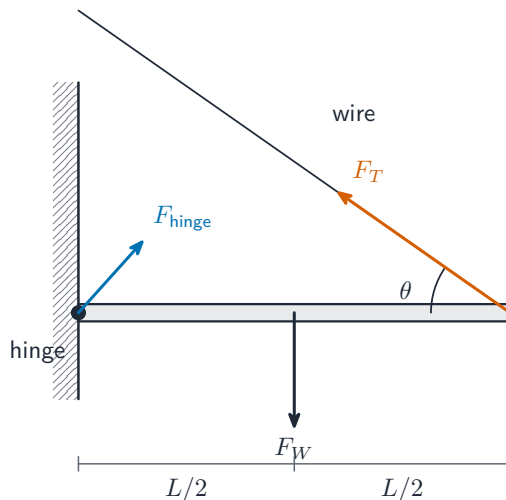
Exercise 3.6

3.6 A uniform horizontal beam is hinged at its left end. Its weight is 90 N and its length is 2.0 m. A wire from the right end pulls at 30° above the beam. Calculate the tension and the horizontal and vertical components of the hinge force. [6]



Exercise 3.6

3.6 A uniform horizontal beam is hinged at its left end. Its weight is 90 N and its length is 2.0 m. A wire from the right end pulls at 30° above the beam. Calculate the tension and the horizontal and vertical components of the hinge force. [6]



3.6

- Take moments about the hinge: the hinge force has zero moment. $\sum M_{\text{cw}} = \sum M_{\text{acw}}$,
so $F_W L/2 = F_T \sin \theta L$.

Full solution on next slide

Full solution 3.6

3.6

- Take moments about the hinge: the hinge force has zero moment. $\sum M_{\text{cw}} = \sum M_{\text{acw}}$, so $F_W L/2 = F_T \sin \theta L$.
- $F_T = F_W / (2 \sin \theta) = 90 / (2 \sin 30^\circ) = 90 \text{ N}$.
- Horizontal balance: $F_H = F_T \cos \theta = 90 \cos 30^\circ = 78 \text{ N}$, away from the wall.
- Vertical balance: $F_V = F_W - F_T \sin \theta = 90 - 90 \sin 30^\circ = 45 \text{ N}$ upwards.