

Exercise walk-through

# 4.1.1

## Centre of gravity, moments at an angle, non-uniform objects

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A-Level Physics

# 4.1.1 Centre of gravity, moments at an angle, non-uniform objects

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Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ **Total: 33 marks**

- KEY WORDS** centre of gravity • moment • uniform
- perpendicular distance • torque

# Objective

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Answer the harder moment questions: the **centre of gravity** and how to find it, a **moment of a force at an angle**, the moment of the weight of a tilted object, and objects that are **not uniform**. Syllabus 4.1, learning outcome 1, with outcomes 2 to 4 applied.

**You must be able to:**

- define the centre of gravity, and describe how to find it for an irregular flat sheet
- calculate the moment of a force at an angle, by the perpendicular distance or by resolving the force
- find the moment of the weight of a tilted object, such as a trapdoor, about its hinge

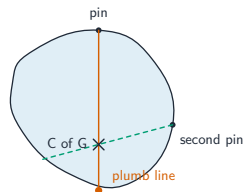
- find the position of the centre of gravity, or the weight, of a non-uniform object from its moments

## 1 Key knowledge and worked examples

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### ■ Key knowledge: the centre of gravity

- **centre of gravity**: the point at which the **whole weight** of an object may be taken to act
- a **uniform** object: at its geometric centre (the middle of a rod, the centre of a disc); a **non-uniform** object: nearer its heavier end
- an object hanging freely comes to rest with its centre of gravity **vertically below** the point of suspension; otherwise its weight has a moment about that point and it swings



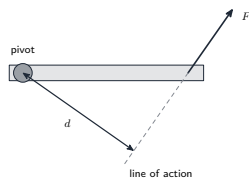
hanging freely, the sheet settles with its centre of gravity vertically below the pin

hang it from a second point: the two plumb lines cross at the centre of gravity

Finding it for an irregular sheet: (1) hang it from a pin near its edge so it swings freely; (2) hang a plumb line from the pin and draw the vertical line; (3) repeat from a second hole; (4) the centre of gravity is where the lines cross; a third hole checks it.

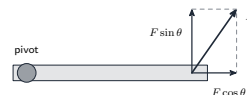
### ■ Key knowledge: a moment at an angle

(a) perpendicular distance



$$d = L \sin \theta, \text{ so the moment is } F \times L \sin \theta$$

(b) resolve the force



only the perpendicular component turns it:  $F \sin \theta \times L$

The two routes give the same answer, and either earns full marks. The component *along* the bar,  $F \cos \theta$ , has a line of action passing **through the pivot**, so its perpendicular distance is zero and it produces no turning effect at all.

A force  $F$  at an angle  $\theta$  to a bar, a distance  $L$  from the pivot:

| Route                  | What to do                                                                                                                          | Moment                   |
|------------------------|-------------------------------------------------------------------------------------------------------------------------------------|--------------------------|
| perpendicular distance | extend the line of action; the perpendicular from the pivot to it is $d = L \sin \theta$                                            | $FL \sin \theta$         |
| resolve the force      | only the component $F \sin \theta$ <b>perpendicular</b> to the bar turns it; $F \cos \theta$ along the bar passes through the pivot | $F \sin \theta \times L$ |

- a tilted uniform object: its weight acts **vertically** through its centre, so about

the hinge the perpendicular distance is  $\frac{L}{2} \cos \theta$  for an object at  $\theta$  to the horizontal; equivalently, use the component of the weight perpendicular to the object,  $F_W \cos \theta$ , at a distance  $\frac{L}{2}$

#### ■ Worked example: a spanner pulled at an angle

A force of 50 N acts at the end of a spanner 0.40 m long, at  $30^\circ$  to the spanner.

1. **By the perpendicular distance** –  $d = L \sin \theta = 0.40 \sin 30^\circ = 0.20$  m; moment of a force,  $M = Fd = 50 \times 0.20 = 10$  N m.
2. **By resolving** – perpendicular component  $F \sin \theta = 50 \sin 30^\circ = 25$  N;  $M = (F \sin \theta)L = 25 \times 0.40 = 10$  N m. Both routes give the same moment.

#### ■ Worked example: a trapdoor

A uniform trapdoor of length 0.80 m and weight 75 N is hinged at one end and held open at  $30^\circ$  to the horizontal by a force  $F$  at its other end, perpendicular to the door.

1. **Component of the weight perpendicular to the door** –  $F_W \cos \theta = 75 \cos 30^\circ = 65$  N, acting at the centre, 0.40 m from the hinge.
2. **Moment of the weight about the hinge** –  $M = Fd = 65 \times 0.40 = 26$  N m.
3. **The force** – the door is still, so the moments about the hinge balance:  $F \times 0.80 = 26$ , so  $F = 32$  N.

#### ■ Worked example: a non-uniform rod

A non-uniform rod AB, 1.00 m long, balances on a pivot 0.40 m from A. When a 2.0 N weight hangs from A, the rod balances on a pivot 0.30 m from A.

1. **Centre of gravity** – balanced with no load, so it is directly above the first pivot: 0.40 m from A.
2. **Moments about the new pivot** – moment of a force,  $M = Fd$ : the 2.0 N load is 0.30 m on one side,  $M = Fd = 2.0 \times 0.30 = 0.60$  N m; the rod's weight  $F_W$  acts 0.10 m on the other side,  $F_W \times 0.10$ .
3. **Weight of the rod** – balanced, so the moments are equal:  $F_W \times 0.10 = 0.60$ , so  $F_W = \frac{M}{d} = \frac{0.60}{0.10} = 6.0$  N.

#### ■ Worked example: holding a non-uniform plate still

A non-uniform plate of mass 2.0 kg is fixed to a horizontal rod through a point that is not its centre of gravity. A torque of 1.6 N m applied to the rod holds it still.

1. **Moment of the weight** –  $M = mgd$ , where  $d$  is the horizontal distance from the rod to the centre of gravity; it must equal the applied torque.
2. **Distance** –  $d = \frac{\tau}{mg} = \frac{1.6}{2.0 \times 9.81} = 0.082$  m.
3. **Direction** – if the centre of gravity is to the right of the rod, the weight turns the plate clockwise, so the applied torque is anticlockwise.

## Exercises 2.1·2.2

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2.1 Define the **centre of gravity** of an object.

[1]

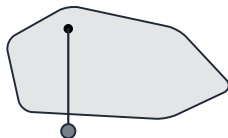
centre of gravity

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2.2 Describe how you would find the centre of gravity of an irregularly shaped flat sheet of card.

[3]



## Exercises 2.1·2.2

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2.1 Define the **centre of gravity** of an object.

[1]

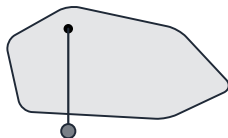
centre of gravity

2.1

- the point at which the **whole weight** of the object may be taken to act
- 

2.2 Describe how you would find the centre of gravity of an irregularly shaped flat sheet of card.

[3]



2.2

- hang the sheet from a pin through a hole near its edge so that it swings freely
- hang a plumb line from the pin and mark the vertical line on the sheet
- repeat from a second hole: the centre of gravity is where the two lines cross

## Exercises 2.3·2.4

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**2.3** A force of 40 N acts at the end of a spanner 0.25 m long, at  $60^\circ$  to the spanner. Calculate its moment about the nut. [2]



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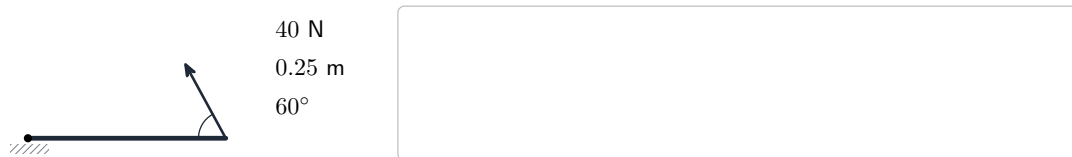
**2.4** Explain why the component of a force **along** a bar has no moment about a pivot at the end of the bar. [1]



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## Exercises 2.3·2.4

**2.3** A force of 40 N acts at the end of a spanner 0.25 m long, at  $60^\circ$  to the spanner. Calculate its moment about the nut. [2]



**2.3**

- moment of a force,  $M = Fd$  with  $d = L \sin \theta$

$$M = FL \sin \theta = 40 \text{ N} \times 0.25 \text{ m} \times \sin 60^\circ = \mathbf{8.7 \text{ N m}}$$

**2.4** Explain why the component of a force **along** a bar has no moment about a pivot at the end of the bar. [1]

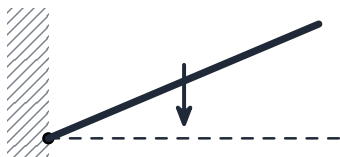


**2.4**

- its line of action passes through the pivot, so its perpendicular distance from the pivot, and its moment, are zero

## Exercise 2.5

**2.5** A uniform trapdoor of length 1.2 m and weight 90 N is hinged at one end and held at  $40^\circ$  to the horizontal. Calculate the moment of its weight about the hinge. [2]



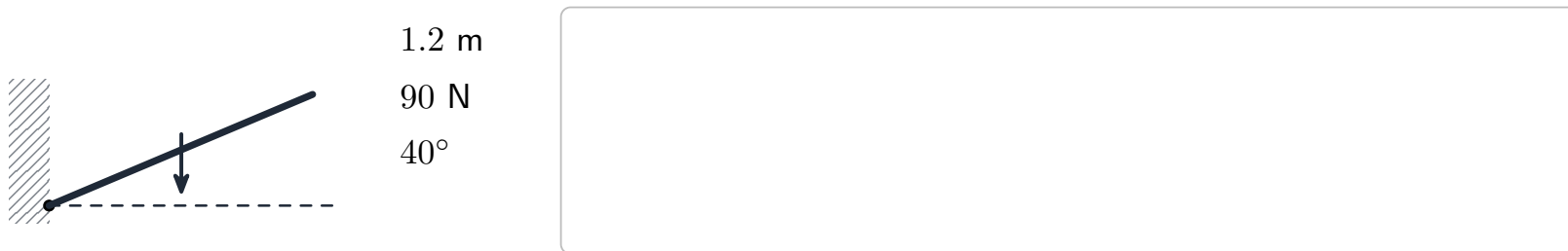
1.2 m

90 N

 $40^\circ$

## Exercise 2.5

**2.5** A uniform trapdoor of length 1.2 m and weight 90 N is hinged at one end and held at  $40^\circ$  to the horizontal. Calculate the moment of its weight about the hinge. [2]



**2.5**

- the weight acts vertically at the centre; perpendicular distance from the hinge  $d = \frac{L}{2} \cos \theta = 0.60 \cos 40^\circ = 0.46 \text{ m}$

$$M = F_W d = 90 \text{ N} \times 0.46 \text{ m} = \mathbf{41 \text{ N m}}$$

## Exercise 3.1

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**3.1** A force  $F$  acts at a point a distance  $L$  from a pivot, at an angle  $\theta$  to the line joining the pivot to that point. What is the moment of the force about the pivot? [1]

☐

**A**  $FL$

**B**  $FL \sin \theta$

**C**  $FL \cos \theta$

**D**  $FL \tan \theta$



## Exercise 3.1

---

**3.1** A force  $F$  acts at a point a distance  $L$  from a pivot, at an angle  $\theta$  to the line joining the pivot to that point. What is the moment of the force about the pivot? [1]

☐

**A**  $FL$

**B**  $FL \sin \theta$

**C**  $FL \cos \theta$

**D**  $FL \tan \theta$



**3.1 B**

- the perpendicular distance from the pivot to the line of action is  $L \sin \theta$
- **C** takes the component along the bar, which passes through the pivot

## Exercise 3.2

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**3.2** A non-uniform rod of weight  $8.0\text{ N}$  is balanced horizontally on a pivot at its midpoint when a  $2.0\text{ N}$  weight hangs  $0.30\text{ m}$  to one side of the pivot. How far is the centre of gravity of the rod from the pivot? [1]

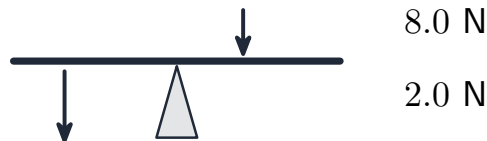
☐

**A**  $0.075\text{ m}$

**B**  $0.12\text{ m}$

**C**  $0.30\text{ m}$

**D**  $1.2\text{ m}$



## Exercise 3.2

**3.2** A non-uniform rod of weight 8.0 N is balanced horizontally on a pivot at its midpoint when a 2.0 N weight hangs 0.30 m to one side of the pivot. How far is the centre of gravity of the rod from the pivot? [1]

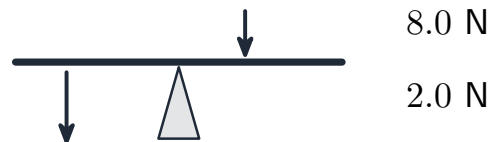
☐

**A** 0.075 m

**B** 0.12 m

**C** 0.30 m

**D** 1.2 m



**3.2 A**

- balance:  $8.0 \times d = 2.0 \times 0.30$ , so  $d = 0.075$  m, on the other side of the pivot from the load
- **D** divides the wrong way round; **C** is the distance of the load

## Exercise 3.3 (a)

---

**3.3** A uniform trapdoor of length 0.90 m and weight 120 N is hinged at end H. It is held open at  $35^\circ$  to the horizontal by a rope attached to its other end.

(a) **Define** the moment of a force.

[2]

## Exercise 3.3 (a)

---

**3.3** A uniform trapdoor of length 0.90 m and weight 120 N is hinged at end H. It is held open at  $35^\circ$  to the horizontal by a rope attached to its other end.

- (a) **Define** the moment of a force. [2]
- (a) the force multiplied by the perpendicular distance
- from the point to the line of action of the force

## Exercise 3.3 (b)

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**3.3** A uniform trapdoor of length 0.90 m and weight 120 N is hinged at end H. It is held open at  $35^\circ$  to the horizontal by a rope attached to its other end.

(b) Calculate the component of the weight of the trapdoor that is perpendicular to the trapdoor. [1]

## Exercise 3.3 (b)

---

**3.3** A uniform trapdoor of length 0.90 m and weight 120 N is hinged at end H. It is held open at  $35^\circ$  to the horizontal by a rope attached to its other end.

(b) Calculate the component of the weight of the trapdoor that is perpendicular to the trapdoor. [1]

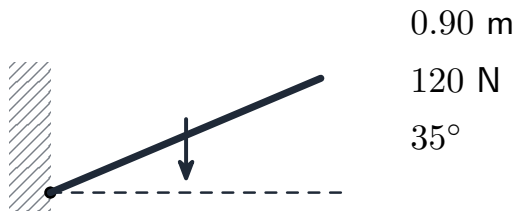
$$(b) F_W \cos \theta = 120 \cos 35^\circ = \mathbf{98 \text{ N}}$$

## Exercise 3.3 (c)

**3.3** A uniform trapdoor of length 0.90 m and weight 120 N is hinged at end H. It is held open at  $35^\circ$  to the horizontal by a rope attached to its other end.

(c) Calculate the moment of the weight of the trapdoor about H.

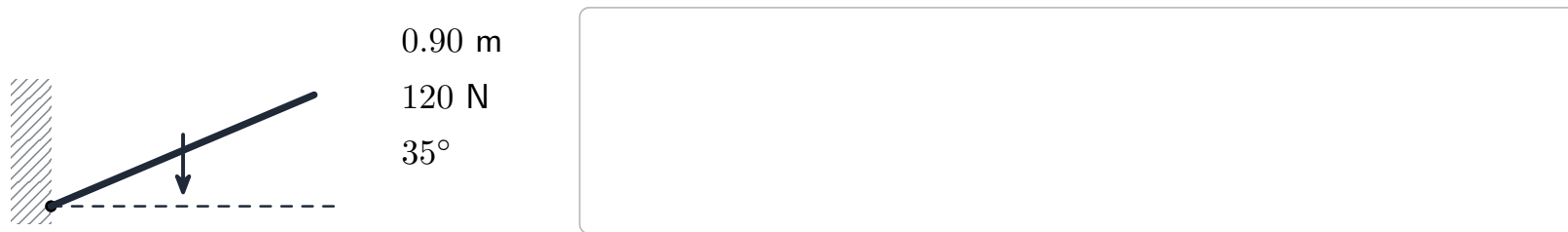
[2]



## Exercise 3.3 (c)

**3.3** A uniform trapdoor of length 0.90 m and weight 120 N is hinged at end H. It is held open at  $35^\circ$  to the horizontal by a rope attached to its other end.

(c) Calculate the moment of the weight of the trapdoor about H. [2]



(c) moment of a force,  $M = Fd$ , with the perpendicular component at the centre, 0.45 m from H

$$M = Fd = 98 \text{ N} \times 0.45 \text{ m} = \mathbf{44 \text{ N m}}$$

## Exercise 3.3 (d)

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**3.3** A uniform trapdoor of length 0.90 m and weight 120 N is hinged at end H. It is held open at  $35^\circ$  to the horizontal by a rope attached to its other end.

(d) The rope pulls perpendicular to the trapdoor. Calculate the tension in the rope. [2]

## Exercise 3.3 (d)

---

**3.3** A uniform trapdoor of length 0.90 m and weight 120 N is hinged at end H. It is held open at  $35^\circ$  to the horizontal by a rope attached to its other end.

(d) The rope pulls perpendicular to the trapdoor. Calculate the tension in the rope. [2]

(d) the trapdoor is still, so the moments about H balance:  $F_T \times 0.90 = 44$

$$F_T = \frac{M}{d} = \frac{44 \text{ N m}}{0.90 \text{ m}} = \mathbf{49 \text{ N}}$$

## Exercise 3.3 (e)

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**3.3** A uniform trapdoor of length 0.90 m and weight 120 N is hinged at end H. It is held open at  $35^\circ$  to the horizontal by a rope attached to its other end.

(e) The rope is now moved so that it pulls **vertically** upwards on the same end. State and explain whether the tension needed is greater than, equal to or less than in (d). [2]

## Exercise 3.3 (e)

---

**3.3** A uniform trapdoor of length 0.90 m and weight 120 N is hinged at end H. It is held open at  $35^\circ$  to the horizontal by a rope attached to its other end.

(e) The rope is now moved so that it pulls **vertically** upwards on the same end. State and explain whether the tension needed is greater than, equal to or less than in (d). [2]

(e) **greater**

- a vertical rope has a smaller perpendicular distance from H ( $0.90 \cos 35^\circ$ ), so it needs a larger force for the same moment

## Exercise 3.4 (a)

---

**3.4** A square sheet of **non-uniform** metal has a mass of 1.6 kg. A thin rod through the centre of the sheet holds it in a vertical plane. To stop the sheet turning, a torque of 1.9 N m must be applied to the rod.

(a) **Define** the centre of gravity.

[1]

## Exercise 3.4 (a)

---

**3.4** A square sheet of **non-uniform** metal has a mass of 1.6 kg. A thin rod through the centre of the sheet holds it in a vertical plane. To stop the sheet turning, a torque of 1.9 N m must be applied to the rod.

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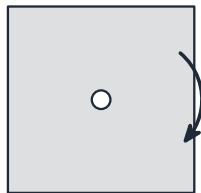
[1]

(a) the point at which the whole weight of the sheet may be taken to act

## Exercise 3.4 (b)

**3.4** A square sheet of **non-uniform** metal has a mass of 1.6 kg. A thin rod through the centre of the sheet holds it in a vertical plane. To stop the sheet turning, a torque of 1.9 N m must be applied to the rod.

(b) **Show that** the centre of gravity of the sheet is a horizontal distance of about 0.12 m from the rod. [2]



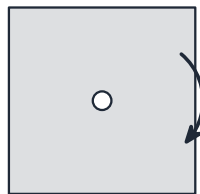
1.6 kg

1.9 N m

## Exercise 3.4 (b)

**3.4** A square sheet of **non-uniform** metal has a mass of 1.6 kg. A thin rod through the centre of the sheet holds it in a vertical plane. To stop the sheet turning, a torque of 1.9 N m must be applied to the rod.

(b) **Show that** the centre of gravity of the sheet is a horizontal distance of about 0.12 m from the rod. [2]



1.6 kg

1.9 N m

(b) the moment of the weight balances the applied torque:  $mgd = \tau$

$$d = \frac{\tau}{mg} = \frac{1.9 \text{ N m}}{1.6 \text{ kg} \times 9.81 \text{ m s}^{-2}} = \mathbf{0.12 \text{ m}}$$

## Exercise 3.4 (c)

---

**3.4** A square sheet of **non-uniform** metal has a mass of 1.6 kg. A thin rod through the centre of the sheet holds it in a vertical plane. To stop the sheet turning, a torque of 1.9 N m must be applied to the rod.

(c) The centre of gravity is to the left of the rod. **State and explain** whether the torque applied to the rod is clockwise or anticlockwise. [2]

## Exercise 3.4 (c)

---

**3.4** A square sheet of **non-uniform** metal has a mass of 1.6 kg. A thin rod through the centre of the sheet holds it in a vertical plane. To stop the sheet turning, a torque of 1.9 N m must be applied to the rod.

(c) The centre of gravity is to the left of the rod. **State and explain** whether the torque applied to the rod is clockwise or anticlockwise. [2]

(c) the weight, to the left of the rod, turns the sheet **anticlockwise**

- so the applied torque must be **clockwise** to balance it

## Exercise 3.4 (d)

---

**3.4** A square sheet of **non-uniform** metal has a mass of 1.6 kg. A thin rod through the centre of the sheet holds it in a vertical plane. To stop the sheet turning, a torque of 1.9 N m must be applied to the rod.

(d) The rod is removed and the sheet hangs freely from one corner. **State** where its centre of gravity lies relative to that corner, and explain why. [2]

## Exercise 3.4 (d)

---

**3.4** A square sheet of **non-uniform** metal has a mass of 1.6 kg. A thin rod through the centre of the sheet holds it in a vertical plane. To stop the sheet turning, a torque of 1.9 N m must be applied to the rod.

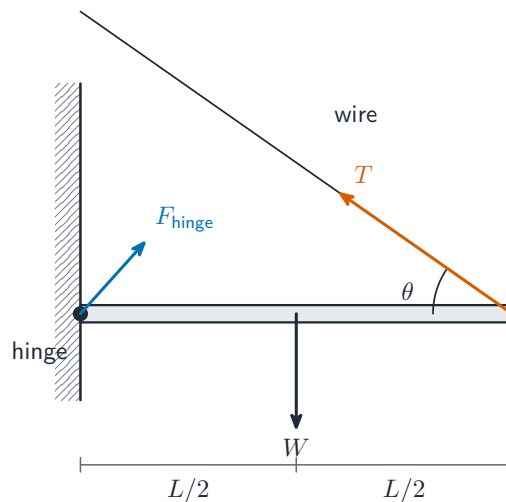
(d) The rod is removed and the sheet hangs freely from one corner. **State** where its centre of gravity lies relative to that corner, and explain why. [2]

(d) **vertically below** the corner

- in equilibrium the weight has no moment about the point of suspension, so its line of action passes through that point

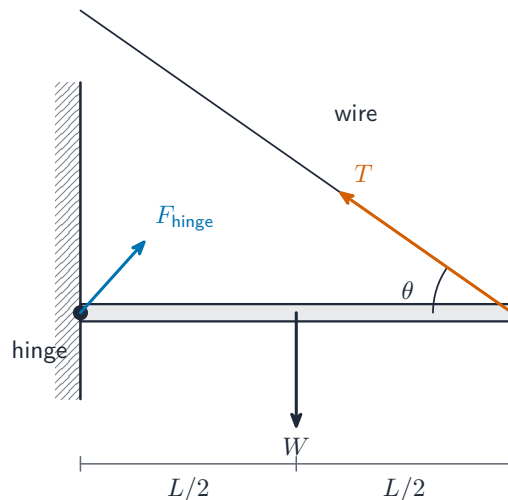
## Exercise 3.5

**3.5** A uniform horizontal beam is hinged at its left end. Its weight is 90 N and its length is 2.0 m. A wire from the right end pulls at  $30^\circ$  above the beam. Calculate the tension and the horizontal and vertical components of the hinge force. [6]



## Exercise 3.5

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**3.5**

- Take moments about the hinge: the hinge force has zero moment.  $\sum M_{\text{cw}} = \sum M_{\text{acw}}$ ,  
so  $F_W L/2 = F_T \sin \theta L$ .

Full solution on next slide

## Full solution 3.5

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### 3.5

- Take moments about the hinge: the hinge force has zero moment.  $\sum M_{\text{cw}} = \sum M_{\text{acw}}$ , so  $F_W L/2 = F_T \sin \theta L$ .
- $F_T = F_W / (2 \sin \theta) = 90 / (2 \sin 30^\circ) = 90 \text{ N}$ .
- Horizontal balance:  $F_H = F_T \cos \theta = 90 \cos 30^\circ = 78 \text{ N}$ , away from the wall.
- Vertical balance:  $F_V = F_W - F_T \sin \theta = 90 - 90 \sin 30^\circ = 45 \text{ N}$  upwards.