

Exercise walk-through

3.3

Linear momentum and its conservation

A-Level Physics

Sheet 3.1 – what you need to know

You must be able to:

- define **mass**, **momentum** and **force**, and state Newton's three laws
- find a change in momentum with signs, and the average force $F = \Delta p / \Delta t$
- read a p - t graph (gradient = force) and an F - t graph (area = Δp)
- find the force from a stream of water or air

What the exam asks

“Define force.” [1]

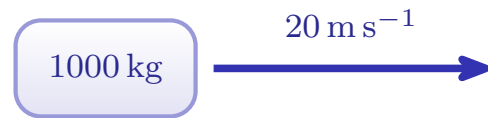
“Calculate the change in momentum of the ball during the collision.” [2]

“Sketch the variation with time of the force F_X exerted on ball X by ball Y.” [3]

Practise on exercise sheet 3.1, then try the real exam questions on past-paper sheet 3.1.

Mass and momentum

- **mass** : how hard it is to **change an object's motion**; a scalar, in kg
- **linear momentum** $p = mv$: a **vector**, pointing along the velocity
- unit of p : kg m s^{-1} , which is the same as N s



$$p = 1000 \times 20 = 2.0 \times 10^4 \text{ N s}$$



$$p = 10\,000 \times 2.0 = 2.0 \times 10^4 \text{ N s}$$

A light car moving fast and a heavy lorry moving slowly can carry the **same momentum**.

Force is the rate of change of momentum

Newton's second law in its general form:
the **resultant force** equals the rate of
change of momentum.

$$F = \frac{\Delta p}{\Delta t}$$

For a constant mass $\Delta p = m \Delta v$, so

$$F = m \frac{\Delta v}{\Delta t} = ma.$$

In a **collision** : average force =
change in momentum $\div$ **contact time**.

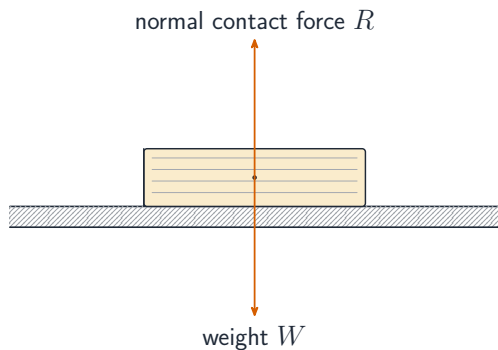


*A crash changes the momentum in a very short
time, so the force is very large.*

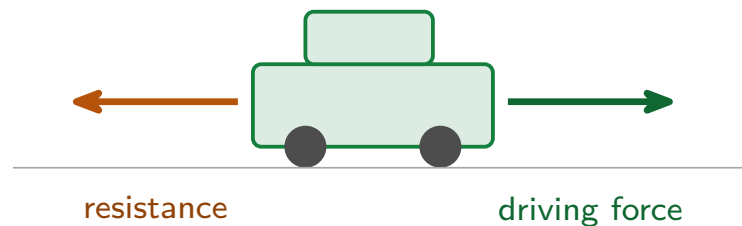
Newton's first law

An object stays **at rest**, or keeps moving at **constant velocity** in a straight line, unless a resultant **external force** acts on it.

Zero resultant force $\Leftrightarrow$ zero acceleration: the object's **inertia** keeps its motion unchanged.

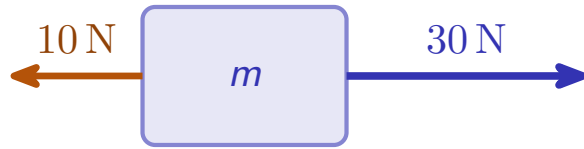


at rest: R balances W



constant velocity: the two forces balance

Newton's second law



resultant 20 N 

acceleration a  same direction

The resultant force equals the **rate of change of momentum**, and acts in the direction of that change:

$$F = \frac{\Delta p}{\Delta t} = ma \quad (\text{constant mass})$$

Acceleration and resultant force always point the **same way**. Add the forces as vectors first.

Exercise 2.1

2.1 State the principle of conservation of momentum.

[2]

conservation of momentum

Exercise 2.1

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[2]

conservation of momentum

2.1

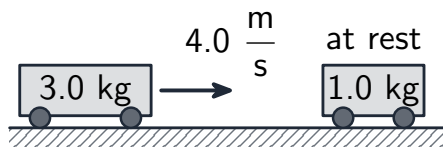
- the total momentum of a system of objects is constant (total momentum before = total momentum after)
- provided no resultant external force acts on the system

Exercise 2.2 (a)

2.2 A trolley of mass 3.0 kg moving at $4.0 \frac{\text{m}}{\text{s}}$ collides with a stationary trolley of mass 1.0 kg . The trolleys stick together.

(a) Calculate their common velocity.

[2]

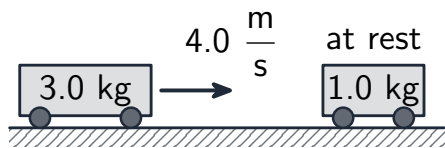


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(a) Calculate their common velocity.

[2]



(a) conservation of momentum, objects stick together: $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{3.0 \times 4.0 + 0}{3.0 + 1.0} = \mathbf{3.0} \frac{\text{m}}{\text{s}}$$

Exercise 2.2 (b)

2.2 A trolley of mass 3.0 kg moving at $4.0 \frac{\text{m}}{\text{s}}$ collides with a stationary trolley of mass 1.0 kg. The trolleys stick together.

(b) Calculate the percentage of the kinetic energy that is transferred to other forms in the collision. [2]

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(b) Calculate the percentage of the kinetic energy that is transferred to other forms in the collision. [2]

(b) $E_k = \frac{1}{2}mv^2$, before and after

$$E_{\text{before}} = \frac{1}{2}m_1u_1^2 = \frac{1}{2} \times 3.0 \times 4.0^2 = 24 \text{ J}$$

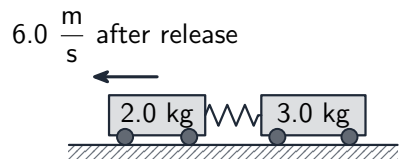
$$E_{\text{after}} = \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2} \times 4.0 \times 3.0^2 = 18 \text{ J}$$

- the energy lost as a percentage of the energy before

$$\frac{E_{\text{before}} - E_{\text{after}}}{E_{\text{before}}} \times 100\% = \frac{24 - 18}{24} \times 100\% = \mathbf{25\%}$$

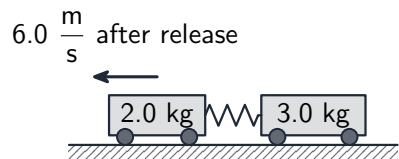
Exercise 2.3

2.3 Two trolleys, of mass 2.0 kg and 3.0 kg, are held at rest with a compressed spring between them. When they are released, the 2.0 kg trolley moves off at $6.0 \frac{\text{m}}{\text{s}}$. Calculate the velocity of the 3.0 kg trolley. [2]



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2.3

- conservation of momentum from rest: $0 = m_1v_1 + m_2v_2$

$$v_2 = -\frac{m_1v_1}{m_2} = -\frac{2.0 \times 6.0}{3.0} = -4.0 \frac{\text{m}}{\text{s}}$$

- 4.0 $\frac{\text{m}}{\text{s}}$** , in the **opposite** direction to the 2.0 kg trolley

Exercise 2.4 (a)

2.4 A ball of mass 0.50 kg moving at $4.0 \frac{\text{m}}{\text{s}}$ hits a stationary ball of the same mass head-on. Afterwards the first ball is at rest and the second moves at $4.0 \frac{\text{m}}{\text{s}}$.

(a) Show that momentum is conserved.

[1]

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(a) Show that momentum is conserved. [1]

(a) ball 1 moves and ball 2 is at rest; total momentum $p = mv$ before and after, the same m for both

$$p_{\text{before}} = mu_1 + mu_2 = 0.50 \times 4.0 + 0 = 2.0 \text{ N s}$$

$$p_{\text{after}} = mv_1 + mv_2 = 0 + 0.50 \times 4.0 = 2.0 \text{ N s}$$

Exercise 2.4 (b)

2.4 A ball of mass 0.50 kg moving at $4.0 \frac{\text{m}}{\text{s}}$ hits a stationary ball of the same mass head-on. Afterwards the first ball is at rest and the second moves at $4.0 \frac{\text{m}}{\text{s}}$.

(b) Show that the collision is elastic.

[2]

before  $\rightarrow 4.0 \frac{\text{m}}{\text{s}}$  at rest

after  at rest  $\rightarrow 4.0 \frac{\text{m}}{\text{s}}$
both 0.50 kg

Exercise 2.4 (b)

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before  $\rightarrow 4.0 \frac{\text{m}}{\text{s}}$  at rest

after  at rest  $\rightarrow 4.0 \frac{\text{m}}{\text{s}}$
both 0.50 kg

(b) total $E_k = \frac{1}{2}mv^2$, before and after

$$E_{\text{before}} = \frac{1}{2}mu_1^2 + 0 = \frac{1}{2} \times 0.50 \times 4.0^2 = 4.0 \text{ J}$$

$$E_{\text{after}} = 0 + \frac{1}{2}mv_2^2 = \frac{1}{2} \times 0.50 \times 4.0^2 = 4.0 \text{ J}$$

- the total kinetic energy is conserved, so the collision is **elastic**

Exercise 2.4 (c)

2.4 A ball of mass 0.50 kg moving at $4.0 \frac{\text{m}}{\text{s}}$ hits a stationary ball of the same mass head-on. Afterwards the first ball is at rest and the second moves at $4.0 \frac{\text{m}}{\text{s}}$.

(c) Show that the relative speed of approach equals the relative speed of separation. [1]

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(c) Show that the relative speed of approach equals the relative speed of separation. [1]

(c) relative speed of approach = $u_1 - u_2 = 4.0 - 0 = 4.0 \frac{\text{m}}{\text{s}}$

- relative speed of separation = $v_2 - v_1 = 4.0 - 0 = 4.0 \frac{\text{m}}{\text{s}}$, the same

Exercise 2.5 (a)

2.5 A stationary nucleus of mass 212 u emits an α -particle of mass 4 u at a speed of $1.6 \times 10^7 \frac{\text{m}}{\text{s}}$.

(a) Explain why the new nucleus and the α -particle move off in opposite directions. [2]

Exercise 2.5 (a)

2.5 A stationary nucleus of mass 212 u emits an α -particle of mass 4 u at a speed of $1.6 \times 10^7 \frac{\text{m}}{\text{s}}$.

(a) Explain why the new nucleus and the α -particle move off in opposite directions. [2]

(a) the nucleus was at rest, so the total momentum before is **zero**

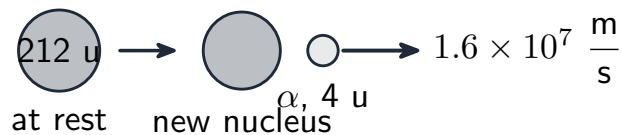
- momentum is conserved, so the two momenta after are equal in size and **opposite** in direction

Exercise 2.5 (b)

2.5 A stationary nucleus of mass 212 u emits an α -particle of mass 4 u at a speed of $1.6 \times 10^7 \frac{\text{m}}{\text{s}}$.

(b) Calculate the speed of the new nucleus.

[2]

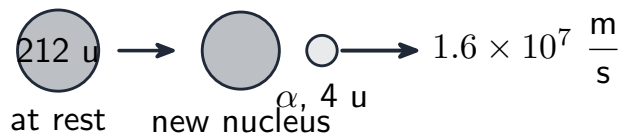


Exercise 2.5 (b)

2.5 A stationary nucleus of mass 212 u emits an α -particle of mass 4 u at a speed of $1.6 \times 10^7 \frac{\text{m}}{\text{s}}$.

(b) Calculate the speed of the new nucleus.

[2]



(b) conservation of momentum, $m_{\text{new}} v_{\text{new}} = m_{\alpha} v_{\alpha}$, with the new mass $m_{\text{new}} = 212 - 4 = 208 \text{ u}$

$$v_{\text{new}} = \frac{m_{\alpha} v_{\alpha}}{m_{\text{new}}} = \frac{4 \times 1.6 \times 10^7}{208} = \mathbf{3.1 \times 10^5} \frac{\text{m}}{\text{s}}$$

Exercise 3.1

3.1 A stationary nucleus of mass 220 u emits an α -particle of mass 4 u. What is the ratio $\frac{\text{speed of the } \alpha\text{-particle}}{\text{speed of the new nucleus}}$? [1]

☐

A $\frac{4}{216}$

B $\frac{4}{220}$

C $\frac{216}{4}$

D $\frac{220}{4}$



 α : 4 u

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- ☐ **A** $\frac{4}{216}$
- B** $\frac{4}{220}$
- C** $\frac{216}{4}$
- D** $\frac{220}{4}$



3.1 C

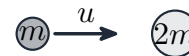
- the momenta are equal in size, so speed $\propto 1/\text{mass}$: $\frac{v_\alpha}{v_{\text{new}}} = \frac{m_{\text{new}}}{m_\alpha} = \frac{216}{4}$, with the new nucleus of mass $m_{\text{new}} = 220 - 4 = 216$ u
- D** uses the mass of the nucleus before the decay; **A** and **B** are upside down **D**

Exercise 3.2

3.2 A ball of mass m moving at speed u hits a stationary ball of mass $2m$, and the two stick together. What fraction of the initial kinetic energy is transferred to other forms? [1]

☐

- A $\frac{1}{3}$
B $\frac{1}{2}$
C $\frac{2}{3}$
D $\frac{3}{4}$

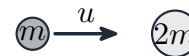


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3.2 C

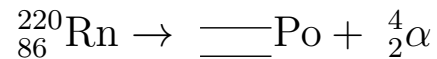
- momentum: $mu = 3mv$, so $v = u/3$
- E_k after = $\frac{1}{2}(3m)(u/3)^2 = \frac{1}{3} \times \frac{1}{2}mu^2$, so one third remains and **two thirds** are transferred
- **A** is the fraction that remains A

Exercise 3.3 (a)

3.3 A radon-220 nucleus at rest breaks apart into a polonium nucleus and an α -particle. The polonium nucleus moves off at $2.2 \times 10^5 \frac{\text{m}}{\text{s}}$.

(a) **Complete** the equation for the decay.

[1]

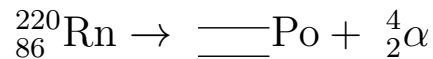


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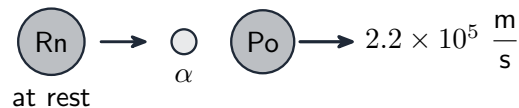


(a) nucleon numbers and proton numbers both balance: ${}_{86}^{220}\text{Rn} \rightarrow {}_{84}^{216}\text{Po} + {}_2^4\alpha$

Exercise 3.3 (b)

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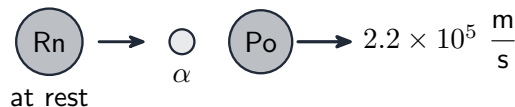
(b) By considering momentum, **calculate** the speed of the α -particle. [3]



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(b) By considering momentum, **calculate** the speed of the α -particle. [3]



(b) the nucleus was at rest, so the total momentum stays **zero**

- conservation of momentum, $m_{\alpha}v_{\alpha} = m_{\text{Po}}v_{\text{Po}}$, masses in u

$$v_{\alpha} = \frac{m_{\text{Po}}v_{\text{Po}}}{m_{\alpha}} = \frac{216 \times 2.2 \times 10^5}{4} = \mathbf{1.2 \times 10^7} \frac{\text{m}}{\text{s}}, \text{ opposite to the polonium}$$

Exercise 3.3 (c)

3.3 A radon-220 nucleus at rest breaks apart into a polonium nucleus and an α -particle. The polonium nucleus moves off at $2.2 \times 10^5 \frac{\text{m}}{\text{s}}$.

(c) **Determine** the ratio $\frac{\text{kinetic energy of the } \alpha\text{-particle}}{\text{kinetic energy of the polonium nucleus}}$. [3]

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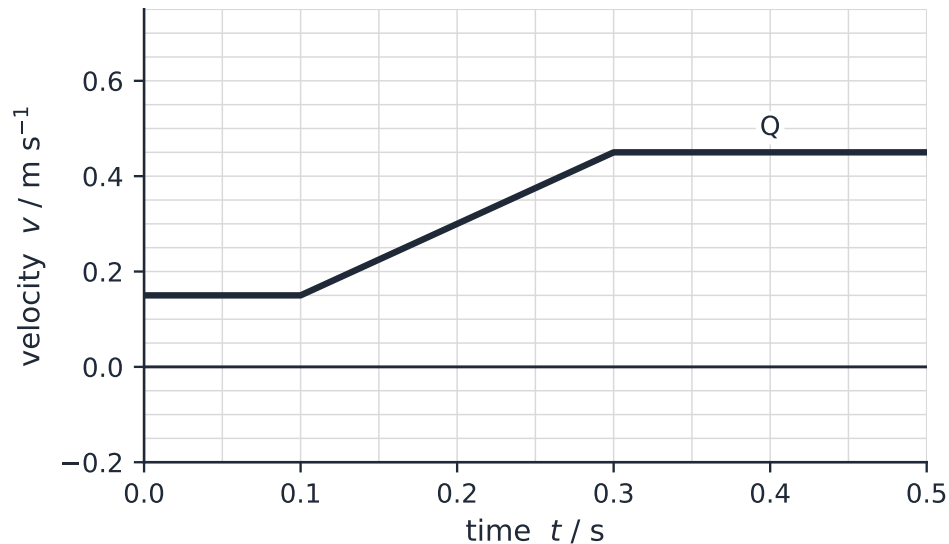
(c) $E_k = \frac{1}{2}mv^2$, so $\frac{E_\alpha}{E_{\text{Po}}} = \frac{m_\alpha v_\alpha^2}{m_{\text{Po}} v_{\text{Po}}^2}$

- the momenta are equal in size, so $v_\alpha/v_{\text{Po}} = m_{\text{Po}}/m_\alpha$ and the ratio becomes $\frac{m_{\text{Po}}}{m_\alpha}$

$$\frac{E_\alpha}{E_{\text{Po}}} = \frac{m_{\text{Po}}}{m_\alpha} = \frac{216}{4} = \mathbf{54}$$

Setup for exercise 3.4

3.4 Trolley P, of mass 0.40 kg , moves at $0.60 \frac{\text{m}}{\text{s}}$ along a horizontal track and catches trolley Q, of mass 0.80 kg , which is moving in the same direction. The graph shows how the velocity of Q varies with time t . The trolleys are in contact from $t = 0.10 \text{ s}$ to $t = 0.30 \text{ s}$.



Exercise 3.4 (a)

(a) **Determine** the acceleration of Q during the collision.

[2]

Exercise 3.4 (a)

(a) **Determine** the acceleration of Q during the collision.

[2]

(a) acceleration is the gradient of the v - t line

—

$$a = \frac{\Delta v}{\Delta t} = \frac{0.45 - 0.15}{0.30 - 0.10} = \mathbf{1.5 \text{ m s}^{-2}}$$

Exercise 3.4 (b)

(b) Use conservation of momentum to **determine** the velocity of P after the collision. [2]

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(b) conservation of momentum, bounce apart

$$m_P u_P + m_Q u_Q = m_P v_P + m_Q v_Q : \quad 0.40 \times 0.60 + 0.80 \times 0.15 = 0.40 v_P + 0.80 \times 0.45$$

- so $0.36 = 0.40 v_P + 0.36$ and $v_P = \mathbf{0}$: P stops P P

Exercise 3.4 (c)

(c) On the graph, **sketch** the variation with time t of the velocity of P from $t = 0$ to $t = 0.50$ s. [3]

Exercise 3.4 (c)

(c) On the graph, **sketch** the variation with time t of the velocity of P from $t = 0$ to $t = 0.50$ s. [3]

(c) a horizontal line at $0.60 \frac{\text{m}}{\text{s}}$ from $t = 0$ to 0.10 s P

- a straight line from $(0.10, 0.60)$ down to $(0.30, 0)$: the change happens over the same contact time as for Q P Q
- a horizontal line along $v = 0$ from 0.30 to 0.50 s P

Exercise 3.4 (d)

(d) **Show that** the collision is elastic.

[2]

Exercise 3.4 (d)

(d) **Show that** the collision is elastic.

[2]

(d) add $E_k = \frac{1}{2}mv^2$ for the two trolleys, before and after

$$E_{\text{before}} = \frac{1}{2}m_P u_P^2 + \frac{1}{2}m_Q u_Q^2 = \frac{1}{2}(0.40)(0.60)^2 + \frac{1}{2}(0.80)(0.15)^2 = 0.081 \text{ J}$$

$$E_{\text{after}} = \frac{1}{2}m_P v_P^2 + \frac{1}{2}m_Q v_Q^2 = 0 + \frac{1}{2}(0.80)(0.45)^2 = 0.081 \text{ J}$$

- the total kinetic energy is conserved, so the collision is elastic