

Exercise walk-through

3.3.1

Elastic collisions and momentum in two dimensions

A-Level Physics

Sheet 3.3.1 – what you need to know

You must be able to:

- use the **relative-speed rule** with momentum to solve an elastic collision
- resolve momenta, and conserve each component on its own
- find the missing fragment of an explosion
- test a two-dimensional collision using the full speeds

What the exam asks

“What is the speed and direction of ball X after the collision?” [1]

“By considering the components of the momenta along the line AB, calculate θ .” [3]

“By calculation of kinetic energies, state whether the collision is inelastic.” [2]

Practise on exercise sheet 3.3.1, then try the real exam questions on past-paper sheet 3.3.1.

An elastic collision gives you a second equation

Momentum alone leaves two unknowns. If the collision is **elastic**, kinetic energy is conserved as well:

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

or, the same thing in a form that is easier to solve:

$$u_1 - u_2 = -(v_1 - v_2)$$

Relative speed of approach = relative speed of separation. Now there are two equations for the two unknowns.

A head-on elastic collision with a stationary mass

Mass m at speed u hits a **stationary**

mass M head-on, elastically:

$$v_m = \frac{m - M}{m + M} u$$

$$v_M = \frac{2m}{m + M} u$$

equal masses, $m = M$

$v_m = 0$ and $v_M = u$:
they swap velocities

light on heavy, $m \ll M$

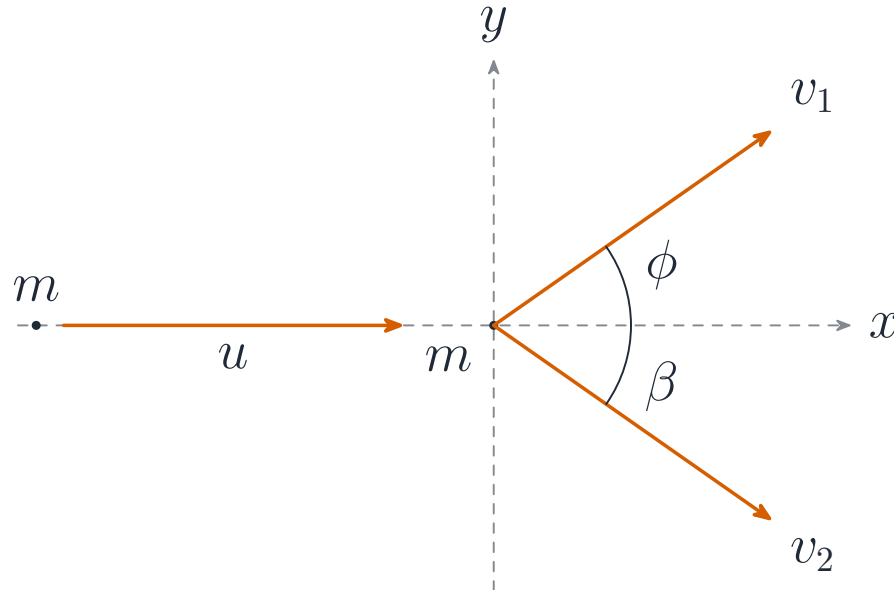
$v_m \approx -u$: it
bounces straight back

heavy on light, $m \gg M$

$v_m \approx u$ and $v_M \approx 2u$

Collisions in two dimensions

Split the velocities into **perpendicular components** and conserve momentum along **each direction** on its own. Choose one axis along the first object's motion and one across it.



Worked example – momentum in two dimensions

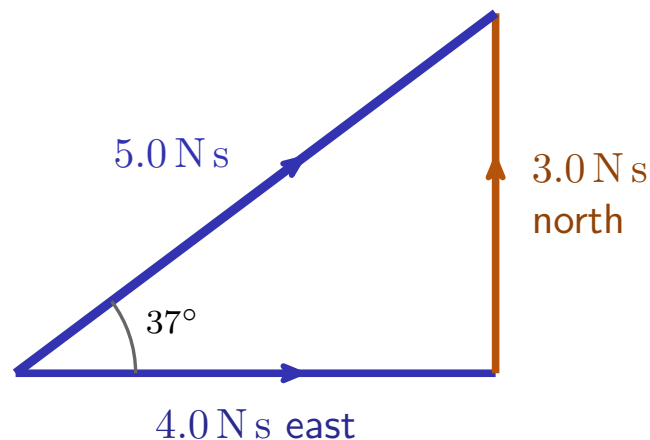
Two balls collide and stick

On a frictionless surface ball A has 4.0 N s due east and ball B 3.0 N s due north. Along each axis momentum is conserved, so after sticking it is still 4.0 east and 3.0 north:

$$p = \sqrt{p_E^2 + p_N^2} = \sqrt{(4.0 \text{ N s})^2 + (3.0 \text{ N s})^2} \\ = 5.0 \text{ N s}$$

$$\theta = \tan^{-1} \frac{p_N}{p_E} = \tan^{-1} \frac{3.0 \text{ N s}}{4.0 \text{ N s}} = 37^\circ$$

north of east



A multiple-choice diagram of two momentum arrows asks exactly this: add the arrows **tip to tail**.

Exercise 2.1

2.1 State **two** things that are true of an elastic collision, apart from the conservation of momentum. [2]

elastic collision

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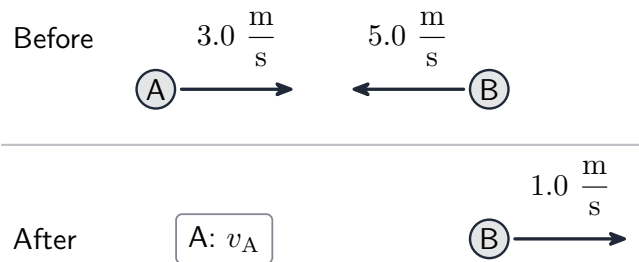
elastic collision

2.1

- the total **kinetic energy** is conserved
- the **relative speed of approach** equals the **relative speed of separation**

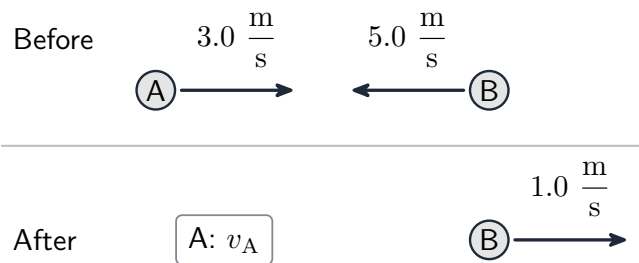
Exercise 2.2

2.2 Ball A moves right at $3.0 \frac{\text{m}}{\text{s}}$ and ball B moves left at $5.0 \frac{\text{m}}{\text{s}}$. They collide head-on and elastically. Afterwards B moves right at $1.0 \frac{\text{m}}{\text{s}}$. Use the relative-speed rule to find the velocity of A after the collision. [2]



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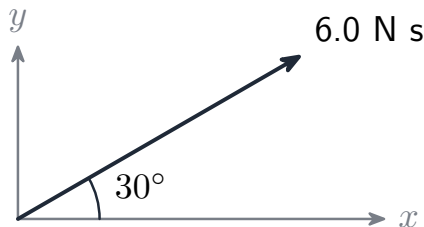


2.2

- right positive; elastic, so $u_A - u_B = v_B - v_A$: $3.0 - (-5.0) = 8.0 = 1.0 - v_A$
- $v_A = -7.0 \frac{\text{m}}{\text{s}}$: A moves **left** at **7.0 $\frac{\text{m}}{\text{s}}$** A

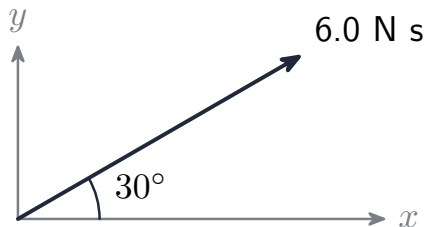
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2.3 A momentum of 6.0 kg m s^{-1} is directed at 30° above the horizontal. Calculate its horizontal and vertical components. [2]



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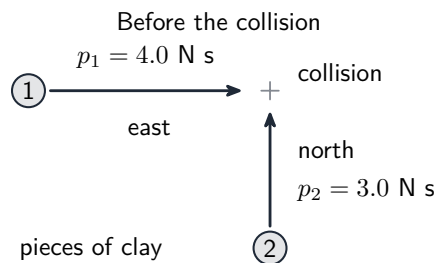
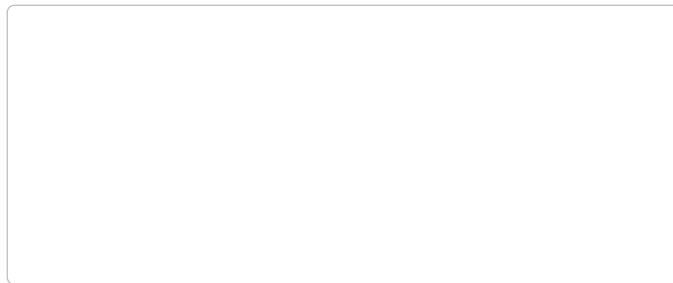


2.3

- horizontal: $p \cos \theta = 6.0 \cos 30^\circ = \mathbf{5.2 \text{ kg m s}^{-1}}$
- vertical: $p \sin \theta = 6.0 \sin 30^\circ = \mathbf{3.0 \text{ kg m s}^{-1}}$

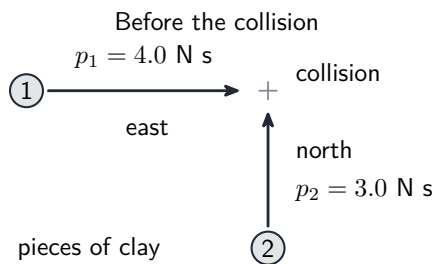
Exercise 2.4

2.4 Two pieces of clay collide and stick together. Just before the collision one has momentum 4.0 N s due east and the other 3.0 N s due north. Calculate the magnitude and the direction of the momentum of the combined lump. [3]



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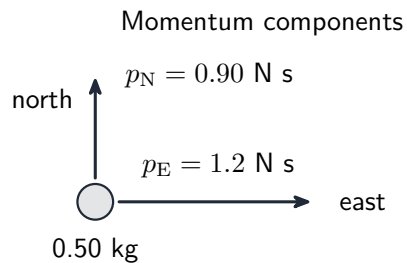


2.4

- momentum is conserved along each axis, so the lump has 4.0 N s east and 3.0 N s north
- Pythagoras: $p = \sqrt{p_{\text{east}}^2 + p_{\text{north}}^2} = \sqrt{4.0^2 + 3.0^2} = \mathbf{5.0 \text{ N s}}$
- $\theta = \tan^{-1} \frac{p_{\text{north}}}{p_{\text{east}}} = \tan^{-1} \frac{3.0}{4.0} = \mathbf{37^\circ}$ north of east

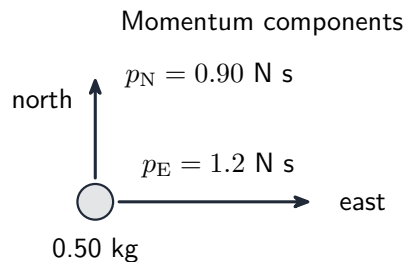
Exercise 2.5

2.5 After a collision, a 0.50 kg ball has momentum components 1.2 N s east and 0.90 N s north. Calculate its speed, and state why its kinetic energy must be found from this speed and not from a component. [3]



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2.5

- Pythagoras: $p = \sqrt{p_{\text{east}}^2 + p_{\text{north}}^2} = \sqrt{1.2^2 + 0.90^2} = 1.5 \text{ N s}$
- from $p = mv$: $v = p/m = 1.5/0.50 = \mathbf{3.0} \frac{\text{m}}{\text{s}}$
- kinetic energy is a **scalar** that depends on the full speed, $\frac{1}{2}mv^2$; a component gives a value that is too small

Exercise 3.1

3.1 Two balls of equal mass move towards each other along a straight line, one at speed $2v$ and the other at speed v . They collide head-on and elastically. What happens after the collision? [1]



A Both balls move at speed $1.5v$.

B Each ball moves back the way it came, the first at speed v and the second at speed $2v$.

C The first ball stops and the second moves back at speed $3v$.

D Each ball moves back the way it came, the first at speed $2v$ and the second at speed v .

before: equal masses



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before: equal masses



3.1 B

- equal masses in a head-on elastic collision **exchange velocities**: the first takes the second's velocity (v , backwards) and the second takes the first's ($2v$, backwards)
- check: approach $2v + v = 3v$, separation $v + 2v = 3v$; **D** has the right directions but breaks momentum, **C** breaks the relative-speed rule and **A** is a sticking-type answer

D

C

A

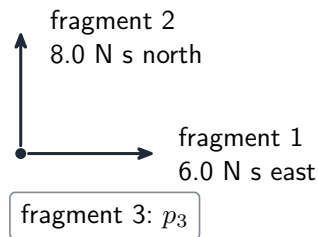
Exercise 3.2

3.2 A stationary object explodes into three fragments that move in a horizontal plane. One fragment has momentum 6.0 N s due east and another has momentum 8.0 N s due north. What is the momentum of the third fragment? [1]

☐

- A** 2.0 N s due south-west
- B** 10 N s , directed south of west
- C** 14 N s , directed south of west
- D** 10 N s , directed north of east

After the explosion

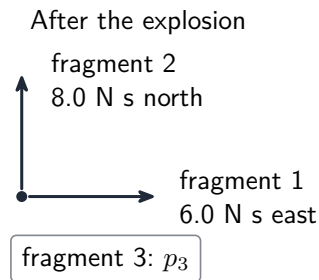


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- A** 2.0 N s due south-west
B 10 N s , directed south of west
C 14 N s , directed south of west
D 10 N s , directed north of east



3.2 B

- from rest, the three momenta add to zero, so the third cancels 6.0 N s east and 8.0 N s north
- $p = \sqrt{p_{\text{east}}^2 + p_{\text{north}}^2} = \sqrt{6.0^2 + 8.0^2} = 10 \text{ N s}$, directed south of west; **D** is the resultant of the other two, not its opposite, and **C** adds the magnitudes

D

C

Exercise 3.3 (a)

3.3 A ball P of mass 0.40 kg moving at $5.0 \frac{\text{m}}{\text{s}}$ collides head-on and elastically with a stationary ball Q of mass 0.60 kg.

(a) State the **two** conditions, other than the conservation of momentum, that an elastic collision satisfies. [2]

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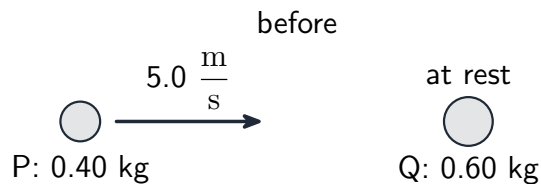
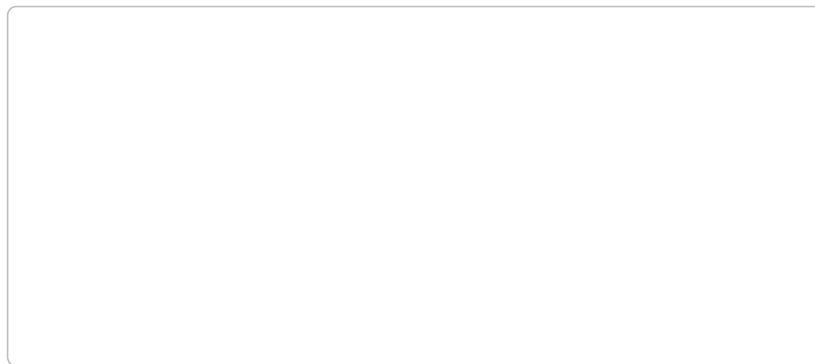
(a) the total kinetic energy is conserved

- the relative speed of approach equals the relative speed of separation

Exercise 3.3 (b)

3.3 A ball P of mass 0.40 kg moving at $5.0 \frac{\text{m}}{\text{s}}$ collides head-on and elastically with a stationary ball Q of mass 0.60 kg.

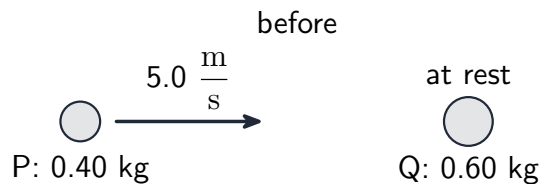
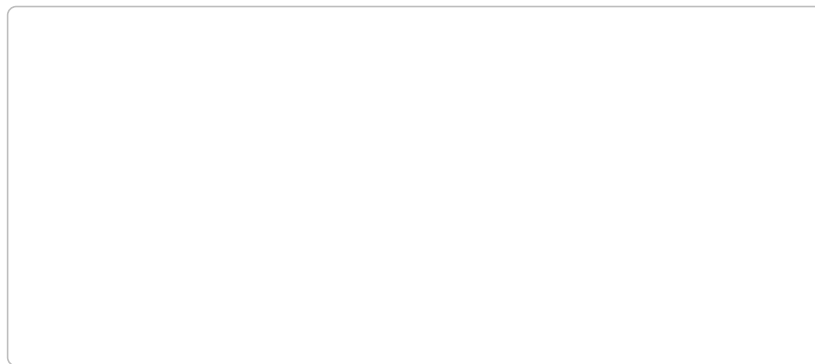
(b) Use conservation of momentum and the relative-speed rule to **determine** the velocities of P and Q after the collision. [4]



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(b) conservation of momentum, Q at rest: $m_P u_P = m_P v_P + m_Q v_Q$ Q
P

Full solution 3.3 (b)

(b) conservation of momentum, Q at rest: $m_P u_P = m_P v_P + m_Q v_Q$ Q

P

$$0.40 \times 5.0 = 0.40 v_P + 0.60 v_Q, \quad \text{so} \quad 2.0 = 0.40 v_P + 0.60 v_Q$$

- elastic, so $u_P - u_Q = v_Q - v_P$: $5.0 - 0 = v_Q - v_P$, so $v_Q = v_P + 5.0$
- $2.0 = 0.40 v_P + 0.60 v_P + 3.0$, so $v_P = -1.0 \frac{\text{m}}{\text{s}}$: P moves **back** at **1.0** $\frac{\text{m}}{\text{s}}$
- $v_Q = 4.0 \frac{\text{m}}{\text{s}}$ forwards Q

Exercise 3.3 (c)

3.3 A ball P of mass 0.40 kg moving at $5.0 \frac{\text{m}}{\text{s}}$ collides head-on and elastically with a stationary ball Q of mass 0.60 kg.

(c) Calculate the percentage of the initial kinetic energy of P that is transferred to Q. [2]

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(c) $E_k = \frac{1}{2}mv^2$ for P before and for Q after

	P	Q
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$$E_P = \frac{1}{2}m_P u_P^2 = \frac{1}{2} \times 0.40 \times 5.0^2 = 5.0 \text{ J}$$

$$E_Q = \frac{1}{2}m_Q v_Q^2 = \frac{1}{2} \times 0.60 \times 4.0^2 = 4.8 \text{ J}$$

- the share of P's energy that Q receives
- | | | |
|--|---|---|
| | Q | P |
|--|---|---|

$$\frac{E_Q}{E_P} \times 100\% = \frac{4.8}{5.0} \times 100\% = \mathbf{96\%}$$

Exercise 3.4 (a)

3.4 On a horizontal frictionless surface, a puck A of mass 0.25 kg moving due east at $4.0 \frac{\text{m}}{\text{s}}$ strikes a stationary puck B of mass 0.50 kg. After the collision, A moves at $2.0 \frac{\text{m}}{\text{s}}$ at 60° north of east.

(a) State the principle of conservation of momentum.

[2]

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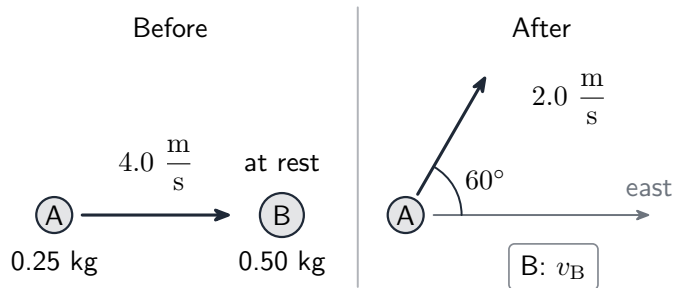
(a) the total momentum of a system of objects is constant

- provided no resultant external force acts on the system

Exercise 3.4 (b)

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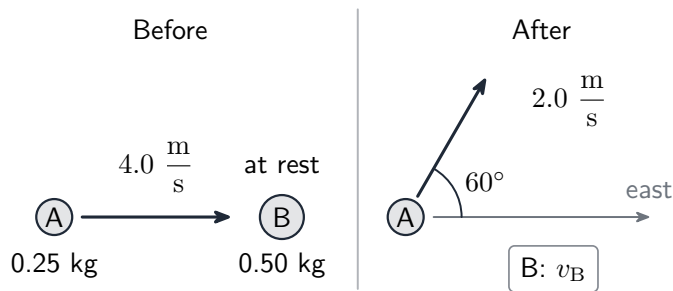
(b) Calculate the magnitude and the direction of the velocity of B after the collision. [5]



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(b) Calculate the magnitude and the direction of the velocity of B after the collision. [5]



(b) momentum along east is conserved, $m_A u_A = m_A v_A \cos 60^\circ + m_B v_x$
B

Full solution 3.4 (b)

(b) momentum along east is conserved, $m_A u_A = m_A v_A \cos 60^\circ + m_B v_x$
B

$$v_x = \frac{m_A u_A - m_A v_A \cos 60^\circ}{m_B} = \frac{0.25 \times 4.0 - 0.25 \times 2.0 \cos 60^\circ}{0.50} = 1.5 \frac{\text{m}}{\text{s}}$$

• along north the total before is zero, $0 = m_A v_A \sin 60^\circ + m_B v_y$
B

$$v_y = -\frac{m_A v_A \sin 60^\circ}{m_B} = -\frac{0.25 \times 2.0 \sin 60^\circ}{0.50} = -0.87 \frac{\text{m}}{\text{s}} \text{ (south)}$$

• Pythagoras: $v = \sqrt{v_x^2 + v_y^2} = \sqrt{1.5^2 + 0.87^2} = 1.7 \frac{\text{m}}{\text{s}}$

• $\theta = \tan^{-1} \frac{|v_y|}{v_x} = \tan^{-1} \frac{0.87}{1.5} = 30^\circ \text{ south of east}$

Exercise 3.4 (c)

3.4 On a horizontal frictionless surface, a puck A of mass 0.25 kg moving due east at $4.0 \frac{\text{m}}{\text{s}}$ strikes a stationary puck B of mass 0.50 kg. After the collision, A moves at $2.0 \frac{\text{m}}{\text{s}}$ at 60° north of east.

(c) **Determine** whether the collision is elastic. Show your working. [3]

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(c) the total $E_k = \frac{1}{2}mv^2$, with the full speeds

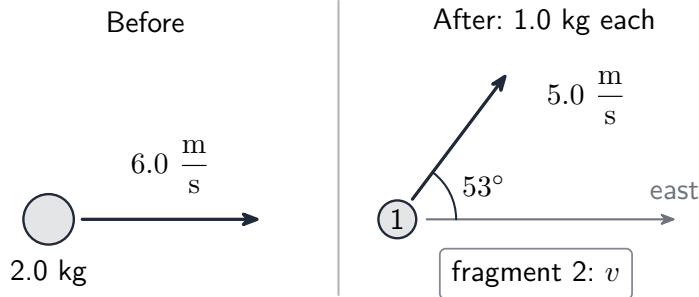
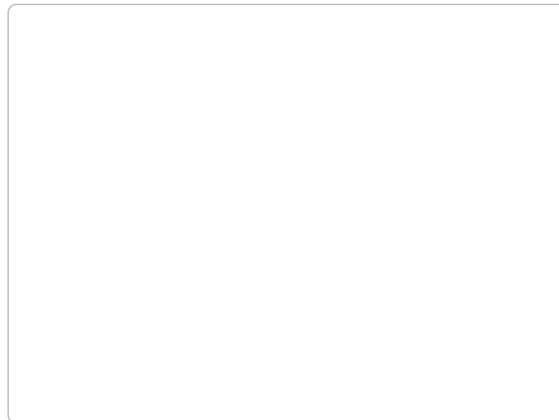
$$E_{\text{before}} = \frac{1}{2}m_A u_A^2 = \frac{1}{2}(0.25)(4.0)^2 = 2.0 \text{ J}$$

$$E_{\text{after}} = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}(0.25)(2.0)^2 + \frac{1}{2}(0.50)(1.73)^2 = 1.25 \text{ J}$$

- the kinetic energy decreases, so the collision is **inelastic**

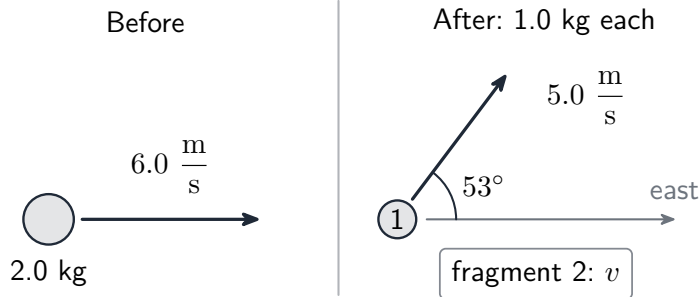
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3.5 An object of mass 2.0 kg moving due east at $6.0 \frac{\text{m}}{\text{s}}$ explodes into two fragments, each of mass 1.0 kg . One fragment moves at $5.0 \frac{\text{m}}{\text{s}}$ at 53° north of east. Calculate the magnitude and the direction of the velocity of the other fragment. [5]



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Full solution 3.5

3.5

- momentum along east is conserved, $Mu = m_1v_1 \cos 53^\circ + m_2v_x$

$$v_x = \frac{Mu - m_1v_1 \cos 53^\circ}{m_2} = \frac{2.0 \times 6.0 - 1.0 \times 5.0 \cos 53^\circ}{1.0} = 9.0 \frac{\text{m}}{\text{s}}$$

- along north the total before is zero, $0 = m_1v_1 \sin 53^\circ + m_2v_y$

$$v_y = -\frac{m_1v_1 \sin 53^\circ}{m_2} = -\frac{1.0 \times 5.0 \sin 53^\circ}{1.0} = -4.0 \frac{\text{m}}{\text{s}} \text{ (south)}$$

- Pythagoras: $v = \sqrt{v_x^2 + v_y^2} = \sqrt{9.0^2 + 4.0^2} = \mathbf{9.8} \frac{\text{m}}{\text{s}}$

- $\theta = \tan^{-1} \frac{|v_y|}{v_x} = \tan^{-1} \frac{4.0}{9.0} = \mathbf{24^\circ \text{ south of east}}$