

Exercise walk-through

25.2

Stellar radii

A-Level Physics

Wien – hotter means bluer

A blackbody spectrum peaks at λ_{max} set by temperature:

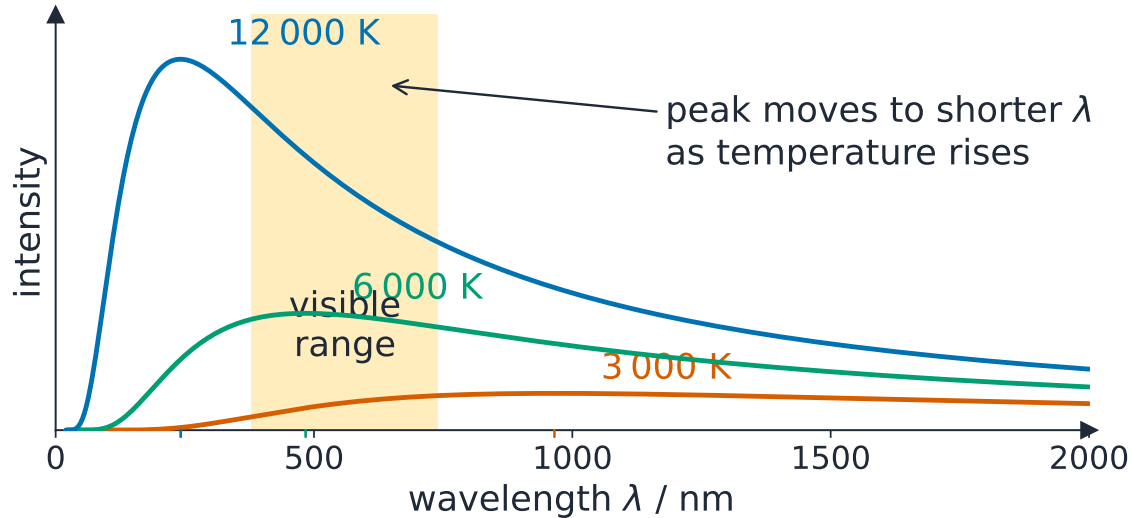
$$\lambda_{\text{max}} T = b \approx 2.90 \times 10^{-3} \text{ m K}$$

Cool red star $\sim 3000 \text{ K}$ peaks in the infrared; the Sun $\sim 5800 \text{ K}$ near 500 nm ; a hot blue-white star $\sim 20\,000 \text{ K}$ in the ultraviolet.

$$\lambda_{\text{max}} = 500 \text{ nm}$$

$$T = b/\lambda_{\text{max}} = 2.90 \times 10^{-3} / (500 \times 10^{-9}) \approx 5800 \text{ K}$$

Wien – hotter means bluer, in detail



Wien: $\lambda_{\max} T = \text{const}$ · hotter $\Rightarrow$ peak shifts left

A hotter black body radiates more, and its peak wavelength shifts towards the blue (Wien's law)

Wien's displacement law



The Pillars of Creation in the Eagle Nebula – clouds of gas and dust lit by hot, newly formed stars

Stefan–Boltzmann – $L \propto r^2 T^4$

A star as a blackbody sphere: The **Stefan–Boltzmann law** – , with the **Stefan–Boltzmann constant** – σ :

$$L = 4\pi\sigma r^2 T^4, \quad \sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

Twice the radius $\Rightarrow$ four times L . Twice the temperature $\Rightarrow$ sixteen times L . To estimate a radius: $\lambda_{\text{max}} \rightarrow T$ (Wien); F and $d \rightarrow L$; then

$$r = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

Estimating a star's radius

Wien, then Stefan–Boltzmann

- 1 $\lambda_{\text{max}} \Rightarrow T$
($\lambda_{\text{max}} T = 2.9 \times 10^{-3} \text{ m K}$)
- 2 $L = 4\pi d^2 F$ from flux and distance
- 3 $L = 4\pi r^2 \sigma T^4 \Rightarrow r = \sqrt{\frac{L}{4\pi \sigma T^4}}$



Two laws and a distance are enough to size a star nobody can resolve.

Exercises 2.1-2.3

2.1 Write **Wien's displacement law**.

[1]

2.2 A star's spectrum peaks at 500 nm. Find its surface **temperature**.

[2]

2.3 Write the **Stefan-Boltzmann law**.

[1]

Exercises 2.1-2.3

2.1 Write **Wien's displacement law**.

[1]

2.1

- $\lambda_{\max} T = \text{constant}$ (or $\lambda_{\max} \propto 1/T$)

2.2 A star's spectrum peaks at 500 nm. Find its surface **temperature**.

[2]

2.2

- $T = b/\lambda_{\max}$

$$T = \frac{b}{\lambda_{\max}} = \frac{2.90 \times 10^{-3} \text{ m K}}{500 \times 10^{-9} \text{ m}} = \mathbf{5800 \text{ K}}$$

2.3 Write the **Stefan-Boltzmann law**.

[1]

2.3

- $L = 4\pi\sigma r^2 T^4$

Exercises 2.4·3.1

2.4 State how λ_{max} changes for a **hotter** star.

[1]

3.1 A **hotter** star has its peak emission at a:

[1]

☐

A longer wavelength

B shorter wavelength

C the same wavelength

D no peak

Exercises 2.4·3.1

2.4 State how λ_{max} changes for a **hotter** star.

[1]

2.4

- it is **shorter** ($\lambda_{\text{max}} \propto 1/T$)

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☐

A longer wavelength

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3.1 B

- a hotter star has its peak at a **shorter wavelength**

Exercise 3.2

3.2 A star's black-body spectrum peaks at 290 nm. Calculate its surface **temperature**. [2]

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3.2

- $T = b/\lambda_{\text{max}}$

$$T = \frac{b}{\lambda_{\text{max}}} = \frac{2.90 \times 10^{-3} \text{ m K}}{290 \times 10^{-9} \text{ m}} = \mathbf{1.0 \times 10^4 \text{ K}}$$

Exercise 3.3 (a)

3.3 Star P has radius 7.0×10^8 m and surface temperature 5800 K. Star Q has the **same** surface temperature but **three times** the radius. ($\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$.)

(a) Calculate the luminosity of star P.

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(a) Calculate the luminosity of star P.

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(a) $L = 4\pi r^2 \sigma T^4$

$$L = 4\pi r^2 \sigma T^4 = 4\pi(7.0 \times 10^8)^2(5.67 \times 10^{-8})(5800)^4 = \mathbf{4.0 \times 10^{26} \text{ W}}$$

Exercise 3.3 (b)

3.3 Star P has radius 7.0×10^8 m and surface temperature 5800 K. Star Q has the **same** surface temperature but **three times** the radius. ($\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$.)

(b) Without further calculation, state and explain the luminosity of star Q. [2]

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(b) Without further calculation, state and explain the luminosity of star Q. [2]

(b) $L \propto r^2$ at fixed temperature

- so tripling the radius makes the luminosity $3^2 = 9$ times larger: **3.6×10^{27} W**

Exercise 3.4

3.4 Outline how a star's **radius** can be estimated from its spectrum.

[2]

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3.4

- find T from λ_{max} (Wien's law) and measure L
- then use the Stefan–Boltzmann law $L = 4\pi\sigma r^2 T^4$ to solve for r

Exercise 3.5 (a)

3.5 The light from a distant star is analysed. Its intensity is greatest at a wavelength of 580 nm, and its radiant flux intensity at the Earth is $3.6 \times 10^{-10} \text{ W m}^{-2}$. The star is $8.4 \times 10^{17} \text{ m}$ away. Take Wien's displacement constant as $2.90 \times 10^{-3} \text{ m K}$ and $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$.

(a) **State** Wien's displacement law.

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(a) **State** Wien's displacement law.

[2]

(a) the wavelength at which the emitted intensity is a **maximum** is **inversely proportional** to the thermodynamic temperature of the body

- $\lambda_{\text{max}} T = \text{constant}$

Exercise 3.5 (b)

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(b) **Determine** the surface temperature of the star.

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(b) $T = b/\lambda_{\text{max}}$

$$T = \frac{b}{\lambda_{\text{max}}} = \frac{2.90 \times 10^{-3} \text{ m K}}{580 \times 10^{-9} \text{ m}} = \mathbf{5.0 \times 10^3 \text{ K}}$$

Exercise 3.5 (c)

3.5 The light from a distant star is analysed. Its intensity is greatest at a wavelength of 580 nm, and its radiant flux intensity at the Earth is $3.6 \times 10^{-10} \text{ W m}^{-2}$. The star is $8.4 \times 10^{17} \text{ m}$ away. Take Wien's displacement constant as $2.90 \times 10^{-3} \text{ m K}$ and $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$.

(c) **Determine** the luminosity of the star.

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(c) **Determine** the luminosity of the star.

[2]

(c) $L = 4\pi d^2 F$

$$L = 4\pi(8.4 \times 10^{17} \text{ m})^2(3.6 \times 10^{-10} \text{ W m}^{-2}) = \mathbf{3.2 \times 10^{27} \text{ W}}$$

Exercise 3.5 (d)

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(d) **Determine** the radius of the star, and **compare** it with the Sun's radius of $7.0 \times 10^8 \text{ m}$. [3]

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$$(d) \ r = \sqrt{L/(4\pi\sigma T^4)}$$

$$r = \sqrt{\frac{L}{4\pi\sigma T^4}} = \sqrt{\frac{3.19 \times 10^{27}}{4\pi(5.67 \times 10^{-8})(5000)^4}} = \mathbf{2.7 \times 10^9 \text{ m}}$$

- a few solar radii, so a star of ordinary main-sequence size

Exercise 3.5 (e)

3.5 The light from a distant star is analysed. Its intensity is greatest at a wavelength of 580 nm, and its radiant flux intensity at the Earth is $3.6 \times 10^{-10} \text{ W m}^{-2}$. The star is $8.4 \times 10^{17} \text{ m}$ away. Take Wien's displacement constant as $2.90 \times 10^{-3} \text{ m K}$ and $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$.

(e) A second star has the **same** luminosity but a surface temperature twice as high. **Determine** the ratio $\frac{\text{radius of the second star}}{\text{radius of the first}}$, and **state** what kind of star that suggests.[3]

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(e) at fixed luminosity $r^2 T^4$ is constant, so $r \propto 1/T^2$

- doubling T gives a radius 1/4 as large, i.e. a ratio of 0.25
- a star of the same luminosity packed into a quarter of the radius and much hotter suggests a **white dwarf** rather than a main-sequence star