

Exercise walk-through

24.2.1

**Intensity against hardness,
attenuation through layers, and
sharpness against contrast**

A-Level Physics

What we will cover

- 1 Ultrasound: piezo and pulse-echo
- 2 Acoustic impedance and attenuation
- 3 X-rays and CT
- 4 PET scanning
- 5 Recap

Impedance, reflection and attenuation

acoustic impedance

$Z = \rho c$ – **density** times the speed of sound. Soft tissue: $1060 \times 1540 \approx 1.6 \times 10^6 \text{ kg m}^{-2}\text{s}^{-1}$

reflection at a boundary

the fraction reflected depends on the **difference** in Z between the two media

best for imaging

different **enough** to give an echo, but not so different that nothing passes on

why gel is used

air's Z is so low that almost everything would reflect at the skin

attenuation

$I = I_0 e^{-\mu x}$; higher-energy X-rays penetrate further, so their μ is **smaller**

contrast

bone has a much larger μ than soft tissue, which is why it shows white

To find a thickness, rearrange to $x = \frac{1}{\mu} \ln \frac{I_0}{I}$. Half-value thickness is the special case $I = I_0/2$.

Minimum wavelength and imaging

The most energy one photon can have is eV:

$$hf_{\max} = eV, \quad \lambda_{\min} = \frac{hc}{eV}$$

Bone (high Z) attenuates more and looks lighter; soft tissue looks darker. A **contrast medium** (barium meal) makes soft tissues stand out. Half-value thickness

$x_{1/2} = \ln 2 / \mu$ halves the intensity.

$$x = 3.0 \text{ cm}, \mu = 40 \text{ m}^{-1}$$

$$I/I_0 = e^{-40 \times 0.030} = e^{-1.2} \approx 0.30$$

Exercises 2.1·2.2

2.1 State which control of an X-ray tube sets the **intensity** of the beam, and which sets its **hardness**. [2]

2.2 Calculate the minimum wavelength of the X-rays from a tube operating at 80 kV.[3]

Exercises 2.1-2.2

2.1 State which control of an X-ray tube sets the **intensity** of the beam, and which sets its **hardness**. [2]

2.1

- the **filament current** sets the intensity: a hotter filament releases more electrons per second
- the **accelerating p.d.** sets the hardness: a larger p.d. gives each electron more energy and so produces higher-energy, more penetrating photons

2.2 Calculate the minimum wavelength of the X-rays from a tube operating at 80 kV. [3]

2.2

- the shortest wavelength takes all the energy eV of one electron, $\lambda_{\min} = hc/(eV)$

$$\lambda_{\min} = \frac{hc}{eV} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{(1.60 \times 10^{-19})(80 \times 10^3)} = \mathbf{1.6 \times 10^{-11} \text{ m}}$$

Exercises 2.3·2.4

2.3 State what is meant by the **half-value thickness**, and write it in terms of μ . [2]

2.4 State the difference between the **sharpness** and the **contrast** of an X-ray image. [2]

Exercises 2.3·2.4

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2.3

- the **thickness** of a material that reduces the intensity of the beam to **half** its incident value
- $x_{1/2} = \ln 2 / \mu$, or $0.693 / \mu$

2.4 State the difference between the **sharpness** and the **contrast** of an X-ray image. [2]

2.4

- **sharpness** is how well **defined the edges** of structures are in the image
- **contrast** is the difference in **degree of blackening** between neighbouring regions, which comes from a difference in attenuation

Exercise 2.5

2.5 State why a metal filter is placed in the beam leaving an X-ray tube.

[2]

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2.5

- it absorbs the **softest** (lowest-energy, longest-wavelength) X-rays
- those would be absorbed in the patient's skin without reaching the detector, so they add to the radiation dose without contributing to the image

Exercise 3.1

3.1 An X-ray beam passes through 2.8 cm of tissue for which $\mu = 1.4 \text{ cm}^{-1}$.

(a) Calculate the percentage of the incident intensity that is **absorbed**. [3]

(b) Calculate the half-value thickness of this tissue, and use it to check your answer to (a). [3]

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3.1

(a) $I/I_0 = e^{-\mu x} = e^{-1.4 \times 2.8} = e^{-3.92}$

Full solution on next slide

Full solution 3.1

3.1

(a) $I/I_0 = e^{-\mu x} = e^{-1.4 \times 2.8} = e^{-3.92}$

- $= 0.020$, so 2.0% is transmitted
- the percentage **absorbed** is $100 - 2.0 = \mathbf{98\%}$
- quoting 2.0% answers the opposite question

(b) $x_{1/2} = \ln 2 / \mu = 0.693 / 1.4 = \mathbf{0.50 \text{ cm}}$

- 2.8 cm is $2.8 / 0.50 = 5.6$ half-value thicknesses, and $(1/2)^{5.6} = 0.020$, which agrees with (a)

Exercise 3.2

3.2 A beam passes through 1.5 cm of bone ($\mu = 2.9 \text{ cm}^{-1}$) and then 4.0 cm of soft tissue ($\mu = 0.20 \text{ cm}^{-1}$).

Calculate the fraction of the incident intensity that emerges.

[4]

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[4]

3.2

- the exponents add, because each layer attenuates what the one before it transmitted
- bone: $\mu x = 2.9 \times 1.5 = 4.35$
- tissue: $\mu x = 0.20 \times 4.0 = 0.80$, giving a total of 5.15
- $I/I_0 = e^{-5.15} = \mathbf{5.8 \times 10^{-3}}$
- multiplying the two thicknesses, or averaging the two μ values, is the error this question is set to catch

Exercise 3.3

3.3 Explain why bone appears white and soft tissue grey on an X-ray image, and state what is meant by the **contrast** of the image. [3]

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- bone has a much **larger attenuation coefficient** than soft tissue, so far less of the beam passes through it
- less radiation reaches the detector behind bone, so that region is **less blackened** and appears white, while more passes through soft tissue and blackens the image there
- the **contrast** is the difference in degree of blackening between neighbouring regions of the image

Exercise 3.4

3.4 State **two** changes to the apparatus that would improve the **sharpness** of an X-ray image, and explain why each works. [4]

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3.4 any two, each with its reason:

- use a **smaller** X-ray source – a smaller spot on the target – so that the edge of a structure casts a narrower penumbra
- keep the **patient still**, since any movement during the exposure blurs every edge
- place the **detector close to the patient**, which reduces the geometric spreading of the beam between the structure and the image
- a **contrast medium** is not accepted here: it improves contrast, not sharpness

Exercise 3.5 (a)

3.5 Computed tomography (CT) scanning produces a 3D image of an **internal** structure: **multiple** X-ray images of the same section are taken from different angles and combined to **obtain** a 2D image of that section, and the process is repeated along an axis so that the 2D images of many sections can be combined.

(a) State **two** advantages of a CT image over a single X-ray image of the same body part. [2]

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(a) State **two** advantages of a CT image over a single X-ray image of the same body part. [2]

(a) any two: a CT image shows a **cross-section**, so structures are not superimposed on one another as they are in a single X-ray

- it has much better **contrast between soft tissues**, because each voxel's attenuation is computed from many views rather than from one; and the image can be rotated and viewed as a three-dimensional structure or as any chosen slice

Exercise 3.5 (b)

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(b) State **one** disadvantage, and explain it.

[2]

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(b) State **one** disadvantage, and explain it.

[2]

(b) the **radiation dose** is much higher

- because many exposures are taken from many angles for each section, and the process is repeated for every section along the axis; the scan also takes longer and the equipment is far more expensive

Exercise 3.6

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State the effect on the minimum wavelength, on the penetrating power of the beam, and on the intensity. [3]

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- the minimum wavelength **decreases**, since $\lambda_{\min} = hc/(eV)$ and V has increased
- the beam becomes **more penetrating**, because the photons have higher energy: it is harder
- the **intensity** also rises somewhat, because each electron delivers more energy, but the intensity is **controlled** by the filament current, not by the p.d.