

Exercise walk-through

24.1.1

Ultrasound scanning: pulse, echo and image

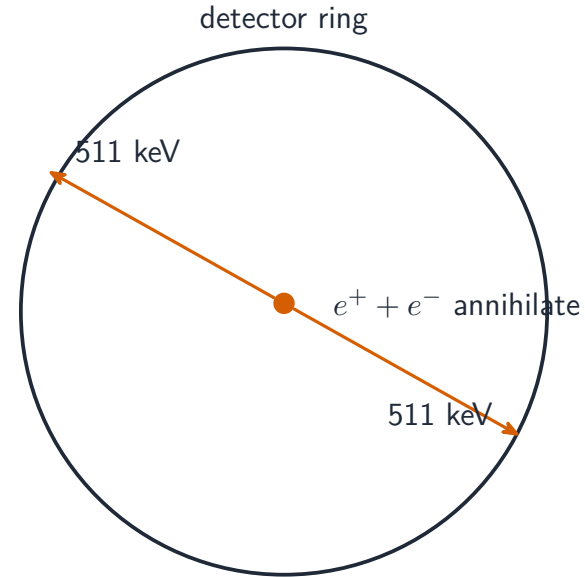
A-Level Physics

A β^+ tracer, then annihilation

A **tracer** (often FDG, fluorine-18 on a glucose analogue) is taken up more by active tissue. In PET it is a β^+ emitter – it gives out a **positron**. The positron

meets an electron and they **annihilate**: two photons, opposite directions, each 511 keV ($= m_e c^2$).

$$2hf = 2m_e c^2$$



two photons, opposite directions — the coincidence fixes the line

PET: annihilation gives two 511 keV photons in opposite directions; a coincidence fixes the line

A coincidence fixes the line

The two photons hit detector rings around the patient. Two simultaneous arrivals (a coincidence) fix the line the annihilation lay on. Many coincidences from many angles build a 3-D map of the tracer – tissues with high metabolic activity.

Comparing arrival times can refine the position along that line (time-of-flight PET).

Tracers and annihilation

a tracer

a substance containing radioactive nuclei, introduced into the body

how it works

it is **taken up more** by the tissue being studied, so that tissue emits more strongly

the choice

a γ or β^+ emitter with a **short half-life**, so the dose falls quickly

an antiparticle

the same mass, opposite charge – the positron is the electron's

annihilation

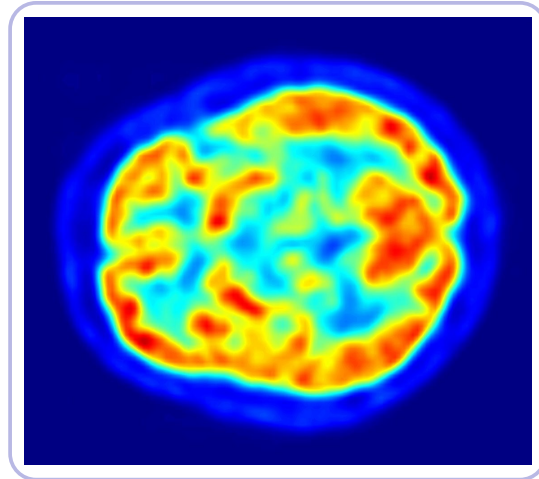
a particle meets its antiparticle and their **mass turns into electromagnetic energy**

the signature

two γ photons of 0.51 MeV leave in **opposite directions**, and detecting both fixes the line the event lay on

PET works because momentum conservation forces the two photons back to back. That geometry is what locates the tracer.

A finished PET image



Positron emission tomography

(PET) maps function, not shape: warm colours mark where the tracer collected – the tissue with the fastest metabolism.

Exam tips

- **Ultrasound** reflects at boundaries (acoustic impedance); a coupling gel reduces reflection at the skin.
- **X-rays** are attenuated as $I = I_0 e^{-\mu x}$; contrast media (barium, iodine) absorb strongly.
- **PET** uses a positron emitter; annihilation gives two γ -rays in opposite directions, located by the detector ring.

Exercises 2.1·2.2

2.1 Explain how a piezo-electric transducer generates ultrasound, and how the **same** crystal detects it. [3]

2.2 Explain why ultrasound is sent into the body as short **pulses** rather than continuously. [2]

Exercises 2.1·2.2

2.1 Explain how a piezo-electric transducer generates ultrasound, and how the **same** crystal detects it. [3]

2.1

- an **alternating p.d.** is applied across the crystal, which makes it change shape in step with the p.d. and so **vibrate**
- the vibration sends out an ultrasound wave; driving the crystal at its natural frequency gives the largest amplitude
- a returning wave **changes the crystal's shape**, and a crystal whose shape changes generates an **e.m.f.**, which is the detected signal

2.2 Explain why ultrasound is sent into the body as short **pulses** rather than continuously. [2]

2.2

- the same crystal both transmits and receives, so it must be **silent** while it listens
- with a continuous wave the returning echo could not be separated from the outgoing wave, and no time of flight – and therefore no depth – could be measured

Exercise 2.3

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- the pulse travels to the boundary **and back**, so the distance travelled is $2d$

$$d = \frac{ct}{2} = \frac{1540 \frac{\text{m}}{\text{s}} \times 52 \times 10^{-6} \text{ s}}{2}$$

$$d = 4.0 \times 10^{-2} \text{ m (4.0 cm)}$$

Exercise 2.4

2.4 Explain why almost no ultrasound would enter the body if there were a thin layer of air between the transducer and the skin. [3]

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- the specific acoustic impedance of air is about $4 \times 10^2 \text{ kg m}^{-2} \text{ s}^{-1}$, thousands of times smaller than that of tissue
- the reflection coefficient $\frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$ is then almost exactly 1
- so almost all the intensity is **reflected at the air gap** and essentially none enters the body; the coupling gel excludes the air for this reason

Exercise 3.1

3.1 For a **strong** reflection of ultrasound at a boundary between two media, their specific acoustic impedances must be [1]

☐

A equal to each other.

B very different from each other.

C both large.

D both small.

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3.1 B

- the reflection coefficient depends on $(Z_1 - Z_2)^2$, so it is large only when the two impedances are very **different**
- equal impedances give zero reflection; the absolute sizes do not matter on their own

Exercise 3.2 (a)

3.2 The specific acoustic impedances of three media, in $\text{kg m}^{-2} \text{s}^{-1}$, are: soft tissue 1.63×10^6 , bone 6.40×10^6 , air 4.29×10^2 .

(a) Define the specific acoustic impedance of a medium.

[1]

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3.2 The specific acoustic impedances of three media, in $\text{kg m}^{-2} \text{s}^{-1}$, are: soft tissue 1.63×10^6 , bone 6.40×10^6 , air 4.29×10^2 .

(a) Define the specific acoustic impedance of a medium. [1]

(a)

- the **density** of the medium multiplied by the **speed of sound** in it, $Z = \rho c$

Exercise 3.2 (b)

3.2 The specific acoustic impedances of three media, in $\text{kg m}^{-2} \text{s}^{-1}$, are: soft tissue 1.63×10^6 , bone 6.40×10^6 , air 4.29×10^2 .

(b) Calculate the intensity reflection coefficient at a soft tissue-bone boundary. [2]

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(b) Calculate the intensity reflection coefficient at a soft tissue-bone boundary. [2]

(b)

- substitute into the reflection coefficient

$$\frac{I_{\text{R}}}{I_0} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2} = \frac{(6.40 - 1.63)^2}{(6.40 + 1.63)^2} = \mathbf{0.35}$$

- the powers of ten cancel, so they need not be written out

Exercise 3.2 (c)

3.2 The specific acoustic impedances of three media, in $\text{kg m}^{-2} \text{s}^{-1}$, are: soft tissue 1.63×10^6 , bone 6.40×10^6 , air 4.29×10^2 .

(c) Calculate the intensity reflection coefficient at a soft tissue-air boundary, and state what your answer means for ultrasound scanning. [3]

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(c)

- substitute again, this time keeping the powers of ten

$$\frac{I_R}{I_0} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2} = \frac{(1.63 \times 10^6 - 4.29 \times 10^2)^2}{(1.63 \times 10^6 + 4.29 \times 10^2)^2} = \mathbf{0.999}$$

- more than 99.9% of the intensity is reflected at a tissue-air boundary, so a scan is impossible through even a thin layer of air – hence the coupling gel

Exercise 3.3 (a)

3.3 Ultrasound of initial intensity I_0 travels through soft tissue for which the attenuation coefficient is 20 m^{-1} .

(a) Calculate the fraction of the intensity remaining after the pulse has travelled 6.0 cm into the tissue. [3]

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- use $I = I_0 e^{-\mu x}$ with x in metres

$$\frac{I}{I_0} = e^{-\mu x} = e^{-20 \text{ m}^{-1} \times 0.060 \text{ m}} = e^{-1.2}$$

$$\frac{I}{I_0} = \mathbf{0.30}$$

Exercise 3.3 (b)

3.3 Ultrasound of initial intensity I_0 travels through soft tissue for which the attenuation coefficient is 20 m^{-1} .

(b) The pulse reflects from a boundary at that depth and returns to the transducer. Calculate the fraction of the original intensity that arrives back, ignoring the loss at the boundary itself. [2]

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(b)

- the total path is **there and back**, $2d = 2 \times 6.0 \text{ cm} = 12 \text{ cm}$

$$\frac{I}{I_0} = e^{-\mu x} = e^{-20 \text{ m}^{-1} \times 0.120 \text{ m}} = e^{-2.4} = \mathbf{0.091}$$

Exercise 3.3 (c)

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(c) Explain why using a higher frequency improves the resolution of a scan but reduces the depth that can be imaged. [2]

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(c)

- a higher frequency has a **shorter wavelength**, so smaller structures can be distinguished and the **resolution** is better
- but the attenuation coefficient **increases** with frequency, so the pulse is absorbed within a shorter distance and less of it returns from deep boundaries

Exercise 3.3 (d)

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(d) State and explain one advantage of using ultrasound rather than X-rays to image a fetus. [2]

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(d)

- ultrasound is **non-ionising**, whereas X-rays are **ionising radiation**
- ionising radiation can damage or mutate the rapidly dividing cells of a developing fetus, so ultrasound is far safer for this use