

Exercise walk-through

23.2

Radioactive decay

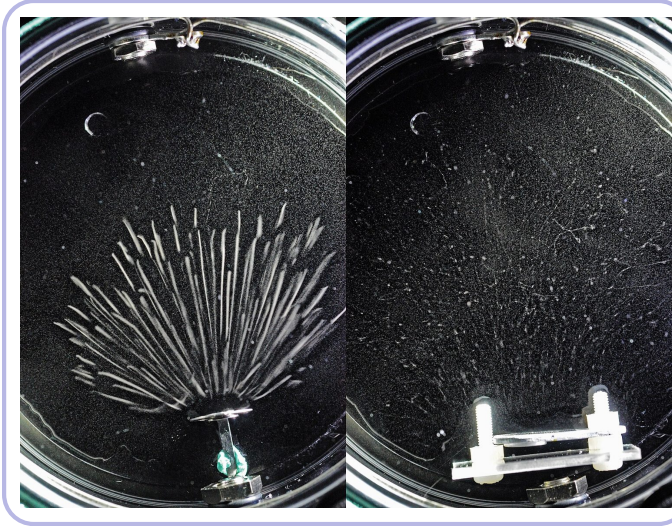
A-Level Physics

Random and spontaneous

Spontaneous – no outside trigger. Rate is not changed by temperature, pressure or chemical state.

Random – you cannot predict when a given nucleus will decay, only the probability. Evidence: the count rate **fluctuates**.

Seeing single decays



Each track is one decay – unpredictable individually, precisely regular in bulk. A **Geiger counter** beside a source clicks at uneven intervals, never a steady stream, which is the evidence that decay is random.

Activity, decay constant, exponential

$$A = \lambda N$$

A is **activity constant** (decays per unit time), in **becquerel** (Bq). λ is the **decay constant** – probability per unit time that a nucleus decays.

$$N = N_0 e^{-\lambda t}, \quad A = A_0 e^{-\lambda t}$$

The same **fraction** decays in each interval – that is what makes it **exponential decay**.

Half-life

Half-life $t_{1/2}$ is the time for N (or A , or the count rate) to fall to half:

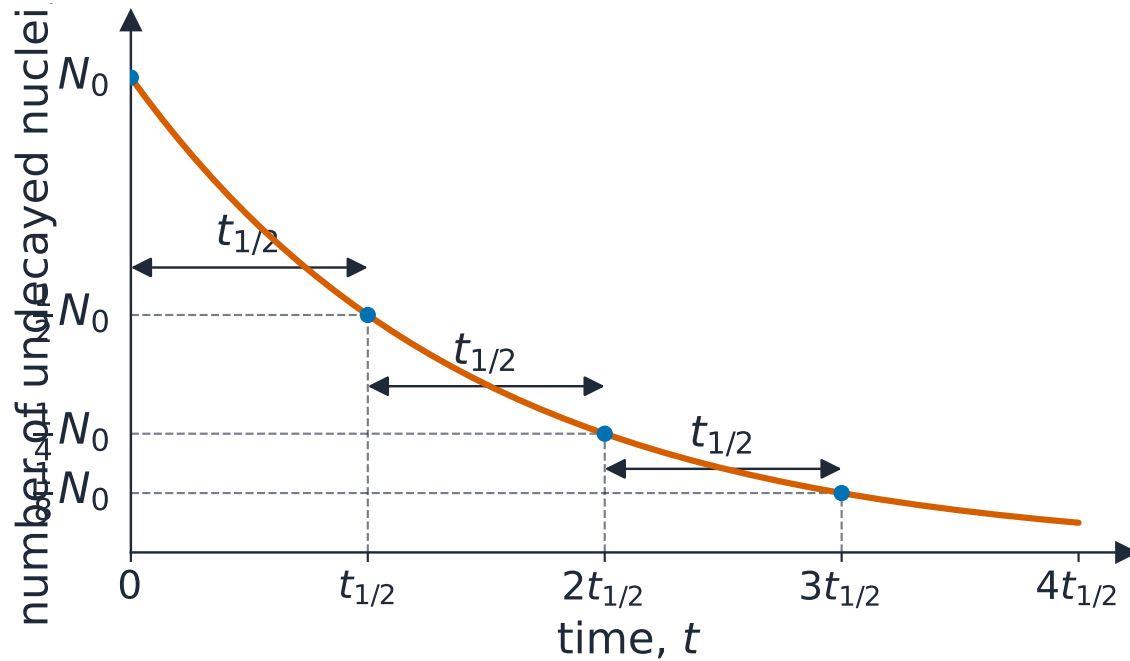
$$\lambda = \frac{\ln 2}{t_{1/2}} \approx \frac{0.693}{t_{1/2}}$$

After n half-lives the surviving fraction is $(1/2)^n$. After 5 half-lives, about 3% remains.

$$t_{1/2} = 6.0 \text{ hours}$$

$$6.0 \text{ h} = 21600 \text{ s, so } \lambda = 0.693/21600 \approx 3.2 \times 10^{-5} \text{ s}^{-1}$$

Half-life, in detail



$$N = N_0 e^{-\lambda t} \quad \cdot \quad T_{1/2} = \ln 2 / \lambda$$

The number of undecayed nuclei falls by half in each half-life

Exercises 2.1-2.3

2.1 State **two** features of radioactive decay. [2]

2.2 Define **activity** and the **decay constant**. [2]

2.3 Define **half-life**. [1]

Exercises 2.1-2.3

2.1 State **two** features of radioactive decay. [2]

2.1

- it is **spontaneous** (unaffected by external conditions)
 - and **random** (cannot predict which nucleus decays when)
-

2.2 Define **activity** and the **decay constant**. [2]

2.2

- **activity** – the number of decays per unit time
 - **decay constant** – the probability per unit time that a nucleus decays
-

2.3 Define **half-life**. [1]

2.3

- the time for **half** the undecayed nuclei (or the activity) to decay

Exercises 2.4·3.1

2.4 Write the equation linking **activity**, **decay constant** and **number** of nuclei. [1]

3.1 The **half-life** is the time for: [1]

☐

A all the nuclei to decay

B half the (undecayed) nuclei to decay

C the activity to reach exactly zero

D the decay constant to halve

Exercises 2.4·3.1

2.4 Write the equation linking **activity**, **decay constant** and **number** of nuclei. [1]

2.4

- $A = \lambda N$

3.1 The **half-life** is the time for: [1]

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A all the nuclei to decay

B half the (undecayed) nuclei to decay

C the activity to reach exactly zero

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3.1 B

- half-life is the time for **half the undecayed nuclei to decay**

Exercises 3.2-3.3

3.2 A source has a decay constant of 0.040 s^{-1} . Calculate its **half-life**. [2]

3.3 A sample has 2.0×10^{20} undecayed nuclei and a decay constant of $1.5 \times 10^{-9} \text{ s}^{-1}$. Calculate its **activity**. [2]

Exercises 3.2·3.3

3.2 A source has a decay constant of 0.040 s^{-1} . Calculate its **half-life**. [2]

3.2

- $t_{1/2} = 0.693/\lambda$

$$t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.040 \text{ s}^{-1}} = \mathbf{17 \text{ s}}$$

3.3 A sample has 2.0×10^{20} undecayed nuclei and a decay constant of $1.5 \times 10^{-9} \text{ s}^{-1}$. Calculate its **activity**. [2]

3.3

- $A = \lambda N$

$$A = \lambda N = (1.5 \times 10^{-9} \text{ s}^{-1})(2.0 \times 10^{20}) = \mathbf{3.0 \times 10^{11} \text{ Bq}}$$

Exercises 3.4·3.5

3.4 A radioactive isotope has a half-life of 6.0 hours. What **fraction** of the original nuclei remains after 18 hours? [2]

3.5 A radioactive source has a half-life of 8.0 days. Calculate the **time** for its activity to fall to 20% of its initial value. [4]

Exercises 3.4·3.5

3.4 A radioactive isotope has a half-life of 6.0 hours. What **fraction** of the original nuclei remains after 18 hours? [2]

3.4

- 18 h = 3 half-lives
- fraction = $(1/2)^3 = \mathbf{1/8}$

3.5 A radioactive source has a half-life of 8.0 days. Calculate the **time** for its activity to fall to 20% of its initial value. [4]

3.5

- $\lambda = 0.693/t_{1/2}$

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{8.0 \text{ day}} = 0.0866 \text{ day}^{-1}$$

- $t = -(1/\lambda) \ln(A/A_0)$

$$t = -\frac{1}{\lambda} \ln \frac{A}{A_0} = -\frac{1}{0.0866 \text{ day}^{-1}} \ln(0.20) = \mathbf{18.6 \text{ days}}$$

Exercise 3.6 (a)

3.6 Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(a) **State** what is meant by a **tracer**.

[2]

Exercise 3.6 (a)

3.6 Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(a) **State** what is meant by a **tracer**. [2]

(a) a radioactive substance introduced into a system

- whose progress can be followed from outside by detecting the radiation it emits

Exercise 3.6 (b)

3.6 Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(b) The decay is ${}^{15}_8\text{O} \rightarrow {}^P_Q\text{N} + {}^R_S\text{Z}$. **State** the values of P , Q , R and S , and **name** the particle Z. [3]

Exercise 3.6 (b)

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(b) The decay is ${}^{15}_8\text{O} \rightarrow {}^P_Q\text{N} + {}^R_S\text{Z}$. **State** the values of P , Q , R and S , and **name** the particle Z. [3]

(b) $P = 15$, $Q = 7$, $R = 0$, $S = +1$

- Z is a **positron**

Exercise 3.6 (c)

3.6 Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(c) **Define** the **activity** of a radioactive sample.

[1]

Exercise 3.6 (c)

3.6 Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(c) **Define** the **activity** of a radioactive sample. [1]

(c) the **rate of decay** – the number of nuclei decaying per unit time

Exercise 3.6 (d)

3.6 Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(d) **Determine** the decay constant of oxygen-15. Give a **unit** with your answer. [2]

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3.6 Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(d) **Determine** the decay constant of oxygen-15. Give a **unit** with your answer. [2]

(d) $\lambda = \ln 2 / t_{1/2}$

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{122 \text{ s}} = \mathbf{5.7 \times 10^{-3} \text{ s}^{-1}}$$

Exercise 3.6 (e)

3.6 Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(e) **Determine** the initial activity of the sample.

[2]

Exercise 3.6 (e)

3.6 Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(e) **Determine** the initial activity of the sample.

[2]

(e) $A = \lambda N$

$$A = \lambda N = (5.68 \times 10^{-3} \text{ s}^{-1})(4.8 \times 10^{14}) = \mathbf{2.7 \times 10^{12} \text{ Bq}}$$

Exercise 3.6 (f)

3.6 Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(f) **Determine** the activity 10 minutes after preparation.

[3]

Exercise 3.6 (f)

3.6 Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(f) **Determine** the activity 10 minutes after preparation. [3]

(f) 10 minutes is 600 s

$$A = A_0 e^{-\lambda t} = (2.73 \times 10^{12} \text{ Bq}) e^{-(5.68 \times 10^{-3} \text{ s}^{-1})(600 \text{ s})} = \mathbf{9.1 \times 10^{10} \text{ Bq}}$$

- about five half-lives, so roughly 1/32 of the start

Exercise 3.6 (g)

3.6 Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(g) **Suggest** why a tracer with a half-life of a few **days** would be unsuitable for this use. [2]

Exercise 3.6 (g)

3.6 Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(g) **Suggest** why a tracer with a half-life of a few **days** would be unsuitable for this use. [2]

(g) the patient would keep receiving **ionising radiation** long after the scan is over, raising the dose and the risk of cell damage

- the tracer should decay away soon after the measurement, which a half-life of minutes achieves and one of days does not