

Exercise walk-through

23.2.1

Random decay, the exponential law and the log graph

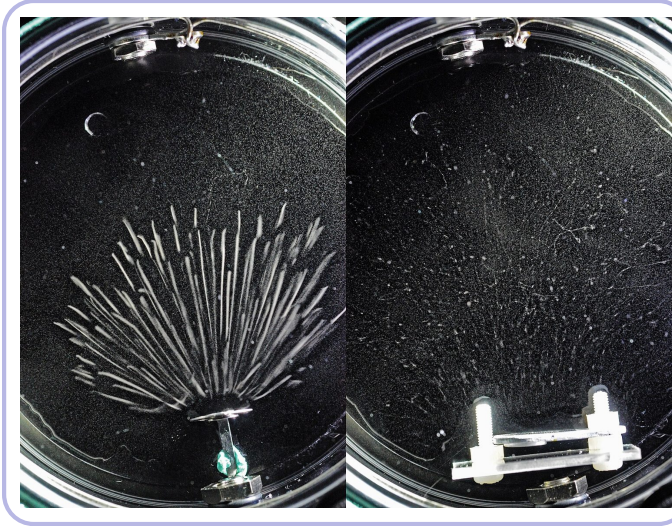
A-Level Physics

Random and spontaneous

Spontaneous – no outside trigger. Rate is not changed by temperature, pressure or chemical state.

Random – you cannot predict when a given nucleus will decay, only the probability. Evidence: the count rate **fluctuates**.

Seeing single decays



Each track is one decay – unpredictable individually, precisely regular in bulk. A **Geiger counter** beside a source clicks at uneven intervals, never a steady stream, which is the evidence that decay is random.

Activity, decay constant, exponential

$$A = \lambda N$$

A is **activity constant** (decays per unit time), in **becquerel** (Bq). λ is the **decay constant** – probability per unit time that a nucleus decays.

$$N = N_0 e^{-\lambda t}, \quad A = A_0 e^{-\lambda t}$$

The same **fraction** decays in each interval – that is what makes it **exponential decay**.

Half-life

Half-life $t_{1/2}$ is the time for N (or A , or the count rate) to fall to half:

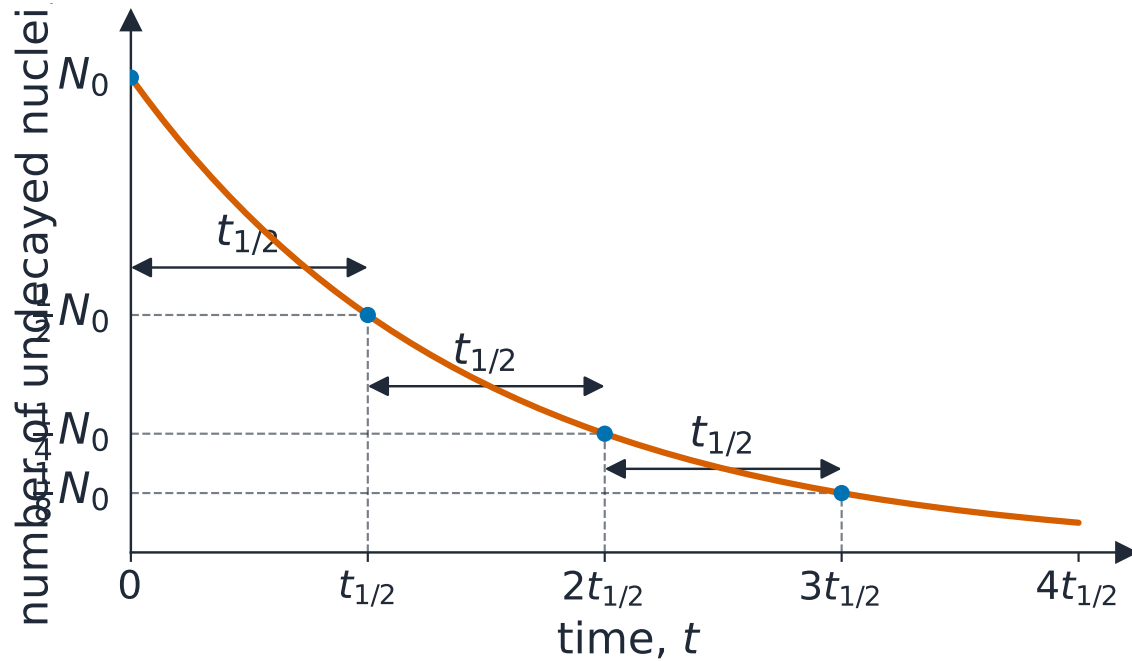
$$\lambda = \frac{\ln 2}{t_{1/2}} \approx \frac{0.693}{t_{1/2}}$$

After n half-lives the surviving fraction is $(1/2)^n$. After 5 half-lives, about 3% remains.

$$t_{1/2} = 6.0 \text{ hours}$$

$$6.0 \text{ h} = 21600 \text{ s, so } \lambda = 0.693/21600 \approx 3.2 \times 10^{-5} \text{ s}^{-1}$$

Half-life, in detail



$$N = N_0 e^{-\lambda t} \quad \cdot \quad T_{1/2} = \ln 2 / \lambda$$

The number of undecayed nuclei falls by half in each half-life

Exercises 2.1·2.2

2.1 Explain what is meant by saying that radioactive decay is **spontaneous** and **random**. [2]

2.2 A student records the count rate from a long-lived source over several equal intervals and finds that the readings differ. Explain how this supports the idea that decay is random. [2]

Exercises 2.1·2.2

2.1 Explain what is meant by saying that radioactive decay is **spontaneous** and **random**. [2]

2.1

- **spontaneous**: the decay is not affected by anything outside the nucleus, so changing the temperature, pressure or chemical state does not change the rate
- **random**: it is impossible to predict which nucleus will decay next or when, and every undecayed nucleus has the same probability of decaying in the next interval

2.2 A student records the count rate from a long-lived source over several equal intervals and finds that the readings differ. Explain how this supports the idea that decay is random. [2]

2.2

- the source's activity is effectively constant, so any difference between readings cannot be a change in the source
- the readings **fluctuate about a mean**, which is what a random process produces; a predictable process would give the same count every time

Exercises 2.3·2.4

2.3 A radioactive isotope has a half-life of 4.5 hours. Calculate its decay constant, in s^{-1} . [3]

2.4 A sample contains 6.0×10^{20} undecayed nuclei of an isotope whose decay constant is $2.2 \times 10^{-8} \text{ s}^{-1}$. Calculate its activity. [2]

Exercises 2.3·2.4

2.3 A radioactive isotope has a half-life of 4.5 hours. Calculate its decay constant, in s^{-1} . [3]

2.3

- convert the half-life to seconds first

$$t_{1/2} = 4.5 \text{ hours} \times 3600 \text{ s hour}^{-1} = 1.62 \times 10^4 \text{ s}$$

- then

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{1.62 \times 10^4 \text{ s}} = \mathbf{4.3 \times 10^{-5} \text{ s}^{-1}}$$

2.4 A sample contains 6.0×10^{20} undecayed nuclei of an isotope whose decay constant is $2.2 \times 10^{-8} \text{ s}^{-1}$. Calculate its activity. [2]

2.4

- use $A = \lambda N$

$$A = \lambda N = 2.2 \times 10^{-8} \text{ s}^{-1} \times 6.0 \times 10^{20} = \mathbf{1.3 \times 10^{13} \text{ Bq}}$$

Exercises 2.5·2.6

2.5 Sketch a graph of $\ln A$ against t for a radioactive source, and state what its gradient represents. [2]

2.6 The activity of a source falls to $\frac{1}{8}$ of its initial value. State how many half-lives have passed. [1]

Exercises 2.5·2.6

2.5 Sketch a graph of $\ln A$ against t for a radioactive source, and state what its gradient represents. [2]

2.5

- a **straight line** with a **negative** gradient, starting at $\ln A_0$ on the vertical axis
- the gradient is $-\lambda$, so the decay constant is minus the gradient

2.6 The activity of a source falls to $\frac{1}{8}$ of its initial value. State how many half-lives have passed. [1]

2.6

- $\frac{1}{8} = \left(\frac{1}{2}\right)^3$, so **3 half-lives**

Exercise 3.1

3.1 The activity of a source falls from 800 Bq to 100 Bq in 27 minutes. Its half-life is [1]

☐

A 3.0 minutes

B 9.0 minutes

C 13.5 minutes

D 27 minutes

Exercise 3.1

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☐

A 3.0 minutes

B 9.0 minutes

C 13.5 minutes

D 27 minutes

3.1 B

- $800 \rightarrow 400 \rightarrow 200 \rightarrow 100$ is **three** halvings
- $27 \text{ min} \div 3 = 9.0 \text{ min}$. **A** divides by the wrong number of halvings; **C** assumes two; **D** assumes one

Exercise 3.2 (a)

3.2 A student measures the count rate from a source whose half-life is many years, recording the number of counts in five successive 10 s intervals:

412 389 405 421 398

(a) The activity of this source is effectively constant over the measurement. Explain why the five readings are nevertheless different. [2]

Exercise 3.2 (a)

3.2 A student measures the count rate from a source whose half-life is many years, recording the number of counts in five successive 10 s intervals:

412 389 405 421 398

(a) The activity of this source is effectively constant over the measurement. Explain why the five readings are nevertheless different. [2]

(a)

- decay is a **random** process, so the number of nuclei decaying in any 10 s interval is not fixed
- the counts therefore **fluctuate about a mean** value; the variation is in the counting, not in the source

Exercise 3.2 (b)

3.2 A student measures the count rate from a source whose half-life is many years, recording the number of counts in five successive 10 s intervals:

412 389 405 421 398

(b) Calculate the mean count rate, in s^{-1} .

[2]

Exercise 3.2 (b)

3.2 A student measures the count rate from a source whose half-life is many years, recording the number of counts in five successive 10 s intervals:

412 389 405 421 398

(b) Calculate the mean count rate, in s^{-1} . [2]

(b)

- add the five readings and divide by five, then by the interval

$$\bar{C} = \frac{\sum C}{5} = \frac{412 + 389 + 405 + 421 + 398}{5} = 405 \text{ counts per } 10 \text{ s} = \mathbf{40.5 \text{ s}^{-1}}$$

Exercise 3.2 (c)

3.2 A student measures the count rate from a source whose half-life is many years, recording the number of counts in five successive 10 s intervals:

412 389 405 421 398

(c) The detector receives and records only 1.2% of the radiation emitted by the source. Calculate the activity of the source. [2]

Exercise 3.2 (c)

3.2 A student measures the count rate from a source whose half-life is many years, recording the number of counts in five successive 10 s intervals:

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(c) The detector receives and records only 1.2% of the radiation emitted by the source. Calculate the activity of the source. [2]

(c)

- the detector records a fixed fraction of the emitted radiation

$$A = \frac{\bar{C}}{\text{fraction detected}} = \frac{40.5 \text{ s}^{-1}}{0.012} = \mathbf{3.4 \times 10^3 \text{ Bq}}$$

Exercise 3.3 (a)

3.3 A student measured the count rate C from a radioactive source at intervals, subtracted the background count, and plotted $\ln(C/\text{s}^{-1})$ against time t . The points lay on a straight line passing through $(0, 5.60)$ and $(900 \text{ s}, 3.44)$.

(a) Explain how the straight line shows that the decay is exponential.

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(a) Explain how the straight line shows that the decay is exponential. [2]

(a)

- taking logs of $C = C_0 e^{-\lambda t}$ gives $\ln C = \ln C_0 - \lambda t$
- this has the form $y = c + mx$, so $\ln C$ against t is a straight line **only if** the decay is exponential; the points lying on a line is therefore the evidence

Exercise 3.3 (b)

3.3 A student measured the count rate C from a radioactive source at intervals, subtracted the background count, and plotted $\ln(C/\text{s}^{-1})$ against time t . The points lay on a straight line passing through $(0, 5.60)$ and $(900 \text{ s}, 3.44)$.

(b) Use the graph to calculate the decay constant of the source.

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(b) Use the graph to calculate the decay constant of the source. [2]

(b)

- the gradient of the line is $-\lambda$

$$\text{gradient} = \frac{\Delta(\ln C)}{\Delta t} = \frac{3.44 - 5.60}{900 \text{ s} - 0} = -2.4 \times 10^{-3} \text{ s}^{-1}$$

$$\lambda = \mathbf{2.4 \times 10^{-3} \text{ s}^{-1}}$$

Exercise 3.3 (c)

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(c) Calculate the half-life of the source.

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(c) Calculate the half-life of the source.

[2]

(c)

- use $\lambda = \frac{0.693}{t_{1/2}}$

$$t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{2.4 \times 10^{-3} \text{ s}^{-1}} = \mathbf{290 \text{ s}}$$

Exercise 3.3 (d)

3.3 A student measured the count rate C from a radioactive source at intervals, subtracted the background count, and plotted $\ln(C/\text{s}^{-1})$ against time t . The points lay on a straight line passing through $(0, 5.60)$ and $(900 \text{ s}, 3.44)$.

(d) Calculate the corrected count rate at $t = 0$.

[2]

Exercise 3.3 (d)

3.3 A student measured the count rate C from a radioactive source at intervals, subtracted the background count, and plotted $\ln(C/\text{s}^{-1})$ against time t . The points lay on a straight line passing through $(0, 5.60)$ and $(900 \text{ s}, 3.44)$.

(d) Calculate the corrected count rate at $t = 0$. [2]

(d)

- the intercept is $\ln C_0$, so take the exponential of it

$$C_0 = e^{5.60} = \mathbf{270 \text{ s}^{-1}}$$

Exercise 3.3 (e)

3.3 A student measured the count rate C from a radioactive source at intervals, subtracted the background count, and plotted $\ln(C/\text{s}^{-1})$ against time t . The points lay on a straight line passing through $(0, 5.60)$ and $(900 \text{ s}, 3.44)$.

(e) Suggest and explain the effect on the shape of the graph if the student had **not** subtracted the background count. [2]

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(e) Suggest and explain the effect on the shape of the graph if the student had **not** subtracted the background count. [2]

(e)

- the background count is a **constant** added to every reading, and it does **not** decay
- so at long times the measured count rate tends towards the background instead of towards zero, $\ln C$ levels off, and the graph **curves away** from the straight line rather than continuing down