

Exercise walk-through

23.1

Mass defect and nuclear binding energy

A-Level Physics

A nucleus weighs less than its parts

The mass of a **nucleus** is less than its Z **protons** plus N neutrons, $Zm_p + Nm_n$. The difference is the **mass defect** :

$$\Delta m = (Zm_p + Nm_n) - m_{\text{nucleus}}$$

That missing mass was released as **binding energy** when the nucleus formed:

$$B = \Delta m \cdot c^2$$

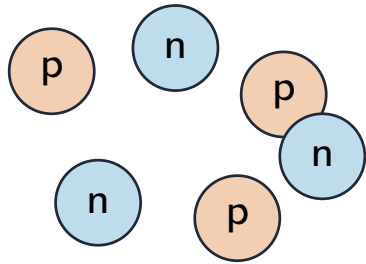
Helium-4, $\Delta m = 0.0304 \text{ u}$

$$B = 0.0304 \times 931 \approx 28 \text{ MeV}$$

The missing mass

Separated nucleons weigh **more** than the nucleus they make. That difference is the **mass defect**, and $E = mc^2$ turns it into the binding energy.

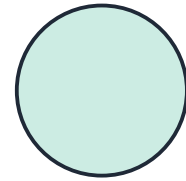
separate nucleons



$$\text{mass} = Zm_p + Nm_n$$

assemble
→

nucleus



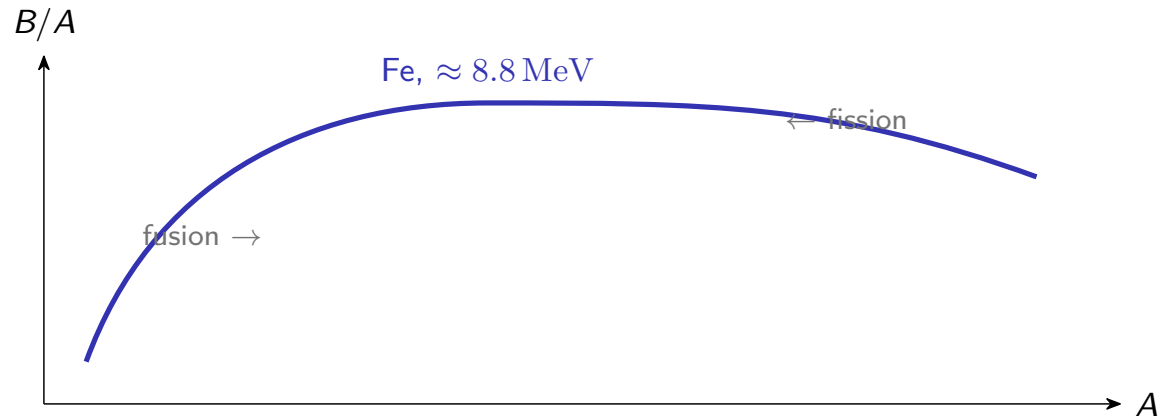
mass is *less*
(mass defect Δm)

binding
energy B



Binding energy per nucleon peaks at iron

B/A (MeV per nucleon) measures how tightly each nucleon is held.



Iron-56 is the most stable nucleus. Light nuclei sit to the left; heavy nuclei sit to the right.

Reading the binding-energy curve

the shape

dome-shaped: rising steeply from hydrogen, **peaking at iron**, then falling slowly

why it peaks

the **strong nuclear force** is short-range, so beyond about iron the electrostatic repulsion of the protons wins

to the left of iron

fusion moves nuclei up the curve and releases energy

to the right

fission moves them up too – both directions run downhill in energy

mass-energy equivalence

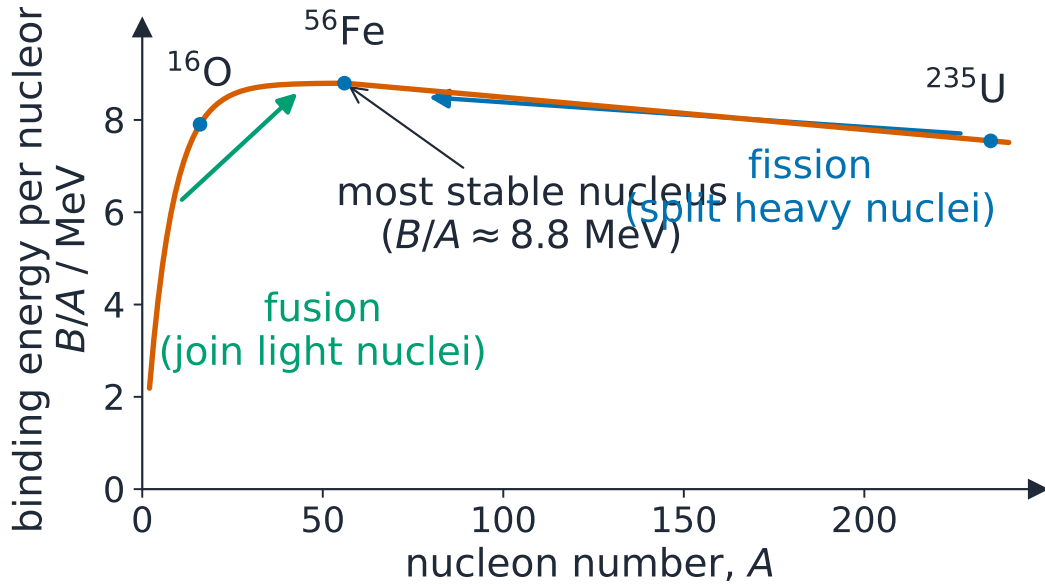
$E = \Delta mc^2$ converts the mass defect into the energy released

in every equation

nucleon number, **conservation of charge** and energy must all balance

Binding energy per nucleon measures **stability**, not total energy. Iron is the most stable nuclide, which is why stars stop there.

Binding energy per nucleon vs nucleon number



binding energy per nucleon peaks near ^{56}Fe

Binding energy per nucleon peaks near iron ($A \approx 56$); lighter and heavier nuclei are less tightly bound

Exercises 2.1-2.3

2.1 State **Einstein's** mass-energy equation.

[1]

2.2 Define **mass defect**.

[1]

2.3 Define **binding energy**.

[1]

Exercises 2.1-2.3

2.1 State **Einstein's** mass-energy equation. [1]

2.1

- $E = mc^2$

2.2 Define **mass defect**. [1]

2.2

- the difference between the **total mass of the separate nucleons** and the **mass of the nucleus**

2.3 Define **binding energy**. [1]

2.3

- the energy needed to **separate a nucleus** into its individual nucleons (equivalent to the mass defect)

Exercises 2.4·3.1

2.4 In ${}^{14}_7\text{N} + {}^4_2\text{He} \rightarrow {}^A_8\text{O} + {}^1_1\text{H}$, find the nucleon number A .

[1]

3.1 The **most stable** nuclei have:

[1]

☐

A the highest mass

B the highest binding energy per nucleon

C the lowest binding energy per nucleon

D zero binding energy

Exercises 2.4·3.1

2.4 In ${}^{14}_7\text{N} + {}^4_2\text{He} \rightarrow {}^A_8\text{O} + {}^1_1\text{H}$, find the nucleon number A . [1]

2.4

- $14 + 4 = A + 1 \Rightarrow A = 17$

3.1 The **most stable** nuclei have: [1]

☐

A the highest mass

B the highest binding energy per nucleon

C the lowest binding energy per nucleon

D zero binding energy

3.1 B

- the most stable nuclei have the **highest binding energy per nucleon**

Exercises 3.2-3.3

3.2 A reaction has a mass defect of 0.030 u. Calculate the energy released. (1 u = 931 MeV.) [2]

3.3 Explain how the binding-energy-per-nucleon curve shows that **both** fusion of light nuclei **and** fission of heavy nuclei can release energy. [3]

Exercises 3.2-3.3

3.2 A reaction has a mass defect of 0.030 u. Calculate the energy released. (1 u = 931 MeV.) [2]

3.2

- $E = \Delta m \times 931 \text{ MeV u}^{-1}$

$$E = \Delta mc^2 = (0.030 \text{ u})(931 \text{ MeV u}^{-1}) = \mathbf{28 \text{ MeV}}$$

3.3 Explain how the binding-energy-per-nucleon curve shows that **both** fusion of light nuclei **and** fission of heavy nuclei can release energy. [3]

3.3

- both light nuclei (fusing) and heavy nuclei (splitting) move **towards the peak** of the curve
- this **increases** the binding energy per nucleon
- the products are more tightly bound, so energy is **released**

Exercises 3.4-3.5

3.4 Calculate the energy equivalent of a mass change of 1.0×10^{-29} kg. [2]

3.5 In the decay ${}^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + {}^4_2\alpha$, the masses (u) are: Po = 209.98286, Pb = 205.97444, $\alpha = 4.00260$. Calculate the energy released, in MeV. (1 u = 931 MeV.) [3]

Exercises 3.4·3.5

3.4 Calculate the energy equivalent of a mass change of 1.0×10^{-29} kg. [2]

3.4

- $E = mc^2$

$$E = mc^2 = (1.0 \times 10^{-29} \text{ kg})(3.0 \times 10^8 \frac{\text{m}}{\text{s}})^2 = \mathbf{9.0 \times 10^{-13} \text{ J}}$$

3.5 In the decay ${}^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + {}^4_2\alpha$, the masses (u) are: Po = 209.98286, Pb = 205.97444, α = 4.00260. Calculate the energy released, in MeV. (1 u = 931 MeV.) [3]

3.5

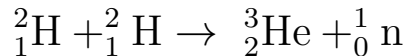
- mass lost, the mass before minus the mass after

$$\Delta m = m_{\text{Po}} - (m_{\text{Pb}} + m_{\alpha}) = 209.98286 - (205.97444 + 4.00260) = 0.00582 \text{ u}$$

$$Q = \Delta mc^2 = (0.00582 \text{ u})(931 \text{ MeV u}^{-1}) = \mathbf{5.4 \text{ MeV}}$$

Exercise 3.6 (a)

3.6 Deuterium nuclei fuse to form helium-3 in the reaction



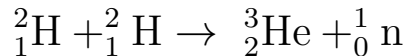
The masses are: deuterium 2.01410 u, helium-3 3.01603 u, neutron 1.00867 u. Take $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$, $c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$ and $1 \text{ MeV} = 1.60 \times 10^{-13} \text{ J}$.

(a) **State** what is meant by the **mass defect** of a nucleus.

[1]

Exercise 3.6 (a)

3.6 Deuterium nuclei fuse to form helium-3 in the reaction



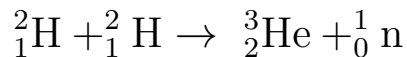
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(a) **State** what is meant by the **mass defect** of a nucleus. [1]

(a) the **difference** between the total mass of the separate nucleons and the mass of the assembled nucleus

Exercise 3.6 (b)

3.6 Deuterium nuclei fuse to form helium-3 in the reaction



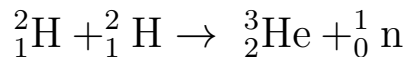
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(b) **Determine** the mass difference of this reaction, in u.

[2]

Exercise 3.6 (b)

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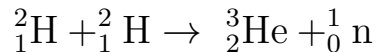
(b) **Determine** the mass difference of this reaction, in u. [2]

(b) left: $m_{\text{before}} = 2m_{\text{D}} = 2 \times 2.01410 \text{ u} = 4.02820 \text{ u}$

- right: $m_{\text{after}} = m_{\text{He}} + m_{\text{n}} = 3.01603 \text{ u} + 1.00867 \text{ u} = 4.02470 \text{ u}$
- difference = **0.00350 u**

Exercise 3.6 (c)

3.6 Deuterium nuclei fuse to form helium-3 in the reaction



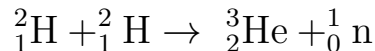
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(c) **Determine** the energy released, in joules and in MeV.

[3]

Exercise 3.6 (c)

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The masses are: deuterium 2.01410 u, helium-3 3.01603 u, neutron 1.00867 u. Take $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$, $c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$ and $1 \text{ MeV} = 1.60 \times 10^{-13} \text{ J}$.

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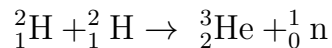
$$(c) \Delta m = (0.00350 \text{ u})(1.66 \times 10^{-27} \text{ kg u}^{-1}) = 5.81 \times 10^{-30} \text{ kg}$$

$$E = \Delta mc^2 = (5.81 \times 10^{-30} \text{ kg})(3.00 \times 10^8 \frac{\text{m}}{\text{s}})^2 = 5.2 \times 10^{-13} \text{ J}$$

$$\frac{5.2 \times 10^{-13} \text{ J}}{1.60 \times 10^{-13} \text{ J MeV}^{-1}} = \mathbf{3.3 \text{ MeV}}$$

Exercise 3.6 (d)

3.6 Deuterium nuclei fuse to form helium-3 in the reaction

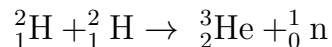


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(d) **Determine** the energy released, in MeV, when 1.0 g of deuterium is fused this way.[3]

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(d) 1.0 g of deuterium contains

$$N = \frac{m}{m_{\text{D}}} = \frac{1.0 \times 10^{-3} \text{ kg}}{(2.01410 \text{ u})(1.66 \times 10^{-27} \text{ kg u}^{-1})} = 3.0 \times 10^{23}$$

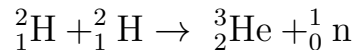
nuclei

- the reaction consumes **two** nuclei per event, so there are 1.5×10^{23} events

$$\text{total} = (\text{events}) \times (\text{energy per event}) = (1.5 \times 10^{23})(3.3 \text{ MeV}) = \mathbf{4.9 \times 10^{23} \text{ MeV}}$$

Exercise 3.6 (e)

3.6 Deuterium nuclei fuse to form helium-3 in the reaction

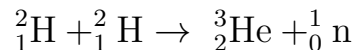


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(e) **Explain** why fusion releases energy for light nuclei but fission releases energy for heavy ones, referring to **binding energy per nucleon**. [3]

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(e) **Explain** why fusion releases energy for light nuclei but fission releases energy for heavy ones, referring to **binding energy per nucleon**. [3]

(e) binding energy per nucleon rises steeply from hydrogen to a maximum near iron, then falls slowly for heavier nuclei

- energy is released whenever the products have a **greater** binding energy per nucleon than the reactants
- so joining light nuclei moves up the curve and releases energy, while splitting a heavy nucleus moves up it from the other side and does the same