

Exercise walk-through

23.1.1

The binding-energy curve, and the energy bookkeeping of a nuclear reaction

A-Level Physics

A nucleus weighs less than its parts

The mass of a **nucleus** is less than its Z **protons** plus N neutrons, $Zm_p + Nm_n$. The difference is the **mass defect** :

$$\Delta m = (Zm_p + Nm_n) - m_{\text{nucleus}}$$

That missing mass was released as **binding energy** when the nucleus formed:

$$B = \Delta m \cdot c^2$$

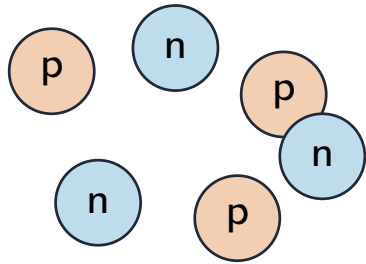
Helium-4, $\Delta m = 0.0304 \text{ u}$

$$B = 0.0304 \times 931 \approx 28 \text{ MeV}$$

The missing mass

Separated nucleons weigh **more** than the nucleus they make. That difference is the **mass defect**, and $E = mc^2$ turns it into the binding energy.

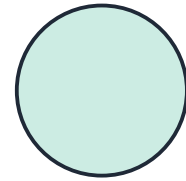
separate nucleons



$$\text{mass} = Zm_p + Nm_n$$

assemble
→

nucleus



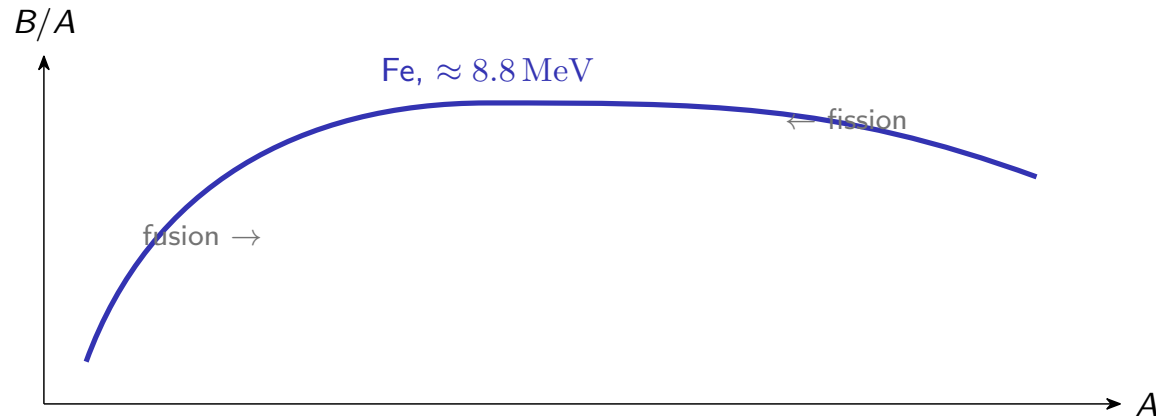
mass is *less*
(mass defect Δm)

binding
energy B



Binding energy per nucleon peaks at iron

B/A (MeV per nucleon) measures how tightly each nucleon is held.



Iron-56 is the most stable nucleus. Light nuclei sit to the left; heavy nuclei sit to the right.

Reading the binding-energy curve

the shape

dome-shaped: rising steeply from hydrogen, **peaking at iron**, then falling slowly

why it peaks

the **strong nuclear force** is short-range, so beyond about iron the electrostatic repulsion of the protons wins

to the left of iron

fusion moves nuclei up the curve and releases energy

to the right

fission moves them up too – both directions run downhill in energy

mass-energy equivalence

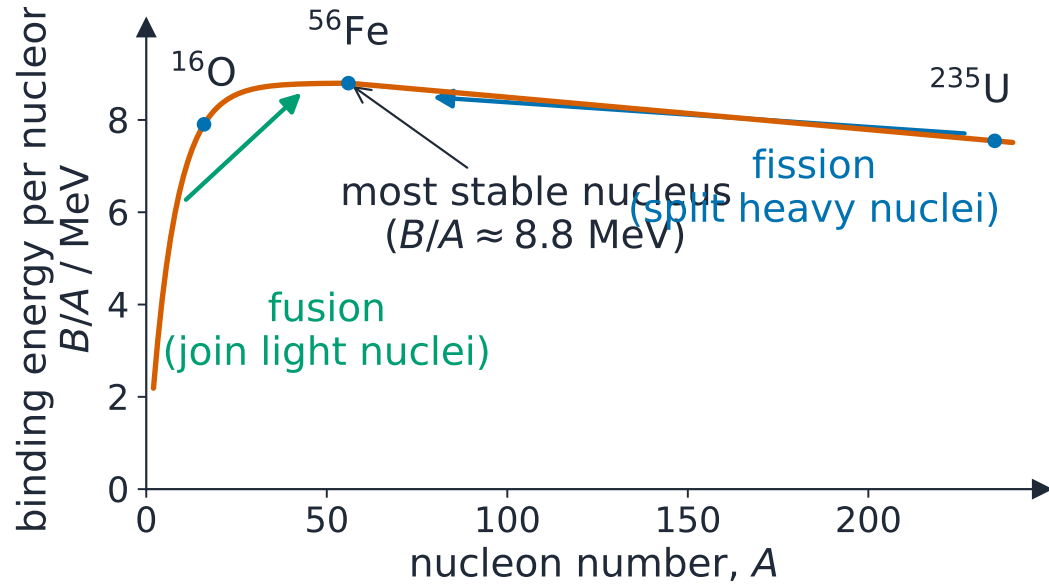
$E = \Delta mc^2$ converts the mass defect into the energy released

in every equation

nucleon number, **conservation of charge** and energy must all balance

Binding energy per nucleon measures **stability**, not total energy. Iron is the most stable nuclide, which is why stars stop there.

Binding energy per nucleon vs nucleon number



binding energy per nucleon peaks near ^{56}Fe

Binding energy per nucleon peaks near iron ($A \approx 56$); lighter and heavier nuclei are less tightly bound

Exercises 2.1·2.2

2.1 Define the **mass defect** of a nucleus, and state which is larger: the mass of the nucleus, or the total mass of its separate nucleons. [2]

2.2 Define the **binding energy** of a nucleus. [2]

Exercises 2.1·2.2

2.1 Define the **mass defect** of a nucleus, and state which is larger: the mass of the nucleus, or the total mass of its separate nucleons. [2]

2.1

- the mass defect is the **difference** between the total mass of the separate nucleons and the mass of the nucleus they form
- the **total mass of the separate nucleons is larger**; the nucleus is always lighter

2.2 Define the **binding energy** of a nucleus. [2]

2.2

- the **energy needed to separate a nucleus into its individual nucleons**
- equivalently, the energy released when those separate nucleons come together to form the nucleus

Exercise 2.3

2.3 Calculate the energy, in MeV, equivalent to a mass of 1.00 u. Take $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$, $c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$ and $1 \text{ MeV} = 1.60 \times 10^{-13} \text{ J}$. [3]

Exercise 2.3

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2.3

- use $E = mc^2$ in SI units

$$E = mc^2 = 1.66 \times 10^{-27} \text{ kg} \times (3.00 \times 10^8 \frac{\text{m}}{\text{s}})^2 = 1.49 \times 10^{-10} \text{ J}$$

- convert to MeV

$$E = \frac{1.49 \times 10^{-10} \text{ J}}{1.60 \times 10^{-13} \text{ J MeV}^{-1}} = \mathbf{934 \text{ MeV}}$$

- this is the tabulated 931.5 MeV, to the precision of the data given

Exercises 2.4-2.5

2.4 A nuclide of nucleon number 56 has a total binding energy of 492 MeV. Calculate its binding energy per nucleon. [2]

2.5 State what the position of the **maximum** of the binding-energy-per-nucleon curve tells you. [2]

Exercises 2.4·2.5

2.4 A nuclide of nucleon number 56 has a total binding energy of 492 MeV. Calculate its binding energy per nucleon. [2]

2.4

- divide the total binding energy by the number of nucleons

$$\frac{E_B}{A} = \frac{492 \text{ MeV}}{56} = \mathbf{8.8 \text{ MeV per nucleon}}$$

2.5 State what the position of the **maximum** of the binding-energy-per-nucleon curve tells you. [2]

2.5

- the maximum is at about **iron-56**, and it marks the **most stable** nuclide
- a higher binding energy per nucleon means more energy is needed to pull the nucleus apart; nothing beyond iron can be made with a release of energy

Exercise 3.1

3.1 Which statement about the binding energy of a nucleus is correct?

[1]

☐

A It is the energy stored inside the nucleus.

B It is the energy released when the nucleus decays.

C It is the energy needed to separate the nucleus into its individual nucleons.

D It is the energy equivalent of the total mass of the nucleus.

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3.1 C

- binding energy is the energy that must be **put in** to separate the nucleons
- **A** reverses the direction; **B** is the energy released in a particular decay, not the binding energy; **D** would be about 10^5 times too large

Exercise 3.2 (a)

3.2 The stability of a nuclide is described by its binding energy per nucleon.

(a) Sketch the variation of binding energy per nucleon with nucleon number, from hydrogen to uranium. Mark the position of the maximum and give its approximate value. [3]

Exercise 3.2 (a)

3.2 The stability of a nuclide is described by its binding energy per nucleon.

(a) Sketch the variation of binding energy per nucleon with nucleon number, from hydrogen to uranium. Mark the position of the maximum and give its approximate value. [3]

(a)

- a **steep rise** from hydrogen through the light nuclei
- a **maximum** at about nucleon number **56**, at roughly 8.8 MeV per nucleon
- a **gentle fall** from there to about 7.6 MeV per nucleon at uranium

Exercise 3.2 (b)

3.2 The stability of a nuclide is described by its binding energy per nucleon.

(b) Use your sketch to explain why energy is released **both** when light nuclei fuse **and** when heavy nuclei undergo fission. [3]

Exercise 3.2 (b)

3.2 The stability of a nuclide is described by its binding energy per nucleon.

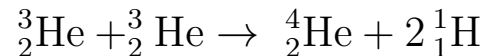
(b) Use your sketch to explain why energy is released **both** when light nuclei fuse **and** when heavy nuclei undergo fission. [3]

(b)

- energy is released whenever the products have a **higher binding energy per nucleon** than the reactants, that is, whenever the nucleons move **up** the curve
- **fusion** of two light nuclei moves far up the steep left-hand side, towards the maximum
- **fission** of a heavy nucleus produces two nuclei nearer the maximum, moving back up the gentle right-hand side

Exercise 3.3 (a)

3.3 Deep inside the Sun, two helium-3 nuclei fuse:

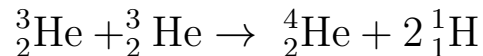


The masses, in u, are ${}^3\text{He} = 3.01603$, ${}^4\text{He} = 4.00260$ and ${}^1\text{H} = 1.00783$.

(a) Show that both nucleon number and proton number are conserved in this reaction.[2]

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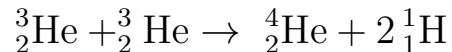
(a) Show that both nucleon number and proton number are conserved in this reaction.[2]

(a)

- nucleon number: $3 + 3 = 6$ on the left, and $4 + 2 \times 1 = 6$ on the right
- proton number: $2 + 2 = 4$ on the left, and $2 + 2 \times 1 = 4$ on the right

Exercise 3.3 (b)

3.3 Deep inside the Sun, two helium-3 nuclei fuse:



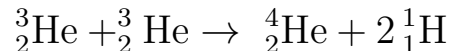
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(b) Calculate the mass defect of the reaction, in u.

[2]

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(b)

- total masses before and after, keeping all the decimal places

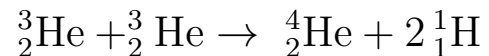
$$m_{\text{before}} = 2m_{\text{He-3}} = 2 \times 3.01603 \text{ u} = 6.03206 \text{ u}$$

$$m_{\text{after}} = m_{\text{He-4}} + 2m_{\text{H}} = 4.00260 + 2 \times 1.00783 = 6.01826 \text{ u}$$

$$\Delta m = m_{\text{before}} - m_{\text{after}} = 6.03206 \text{ u} - 6.01826 \text{ u} = \mathbf{1.380 \times 10^{-2} \text{ u}}$$

Exercise 3.3 (c)

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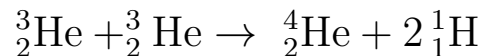
The masses, in u, are ${}^3\text{He} = 3.01603$, ${}^4\text{He} = 4.00260$ and ${}^1\text{H} = 1.00783$.

(c) Calculate the energy released, in MeV. Take $1 \text{ u} = 931.5 \text{ MeV}$.

[2]

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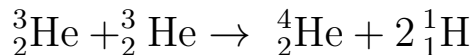
(c)

- multiply by the conversion factor

$$E = 931.5 \text{ MeV u}^{-1} \times 1.380 \times 10^{-2} \text{ u} = \mathbf{12.9 \text{ MeV}}$$

Exercise 3.3 (d)

3.3 Deep inside the Sun, two helium-3 nuclei fuse:



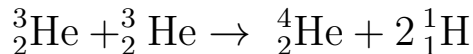
The masses, in u, are ${}^3\text{He} = 3.01603$, ${}^4\text{He} = 4.00260$ and ${}^1\text{H} = 1.00783$.

(d) Explain why this reaction takes place only at a very high temperature.

[3]

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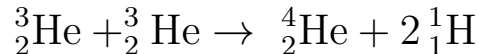
(d) Explain why this reaction takes place only at a very high temperature. [3]

(d)

- both nuclei carry a **positive charge**, so they **repel** each other electrostatically
- to fuse they must come close enough for the short-range **strong nuclear force** to act, which needs a very large kinetic energy to overcome that repulsion
- a very high temperature is what gives the nuclei kinetic energy of that size, since the mean kinetic energy of a particle is proportional to the thermodynamic temperature

Exercise 3.3 (e)

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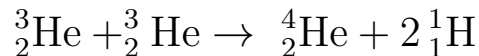
The masses, in u, are ${}^3\text{He} = 3.01603$, ${}^4\text{He} = 4.00260$ and ${}^1\text{H} = 1.00783$.

(e) In a different nuclear reaction the total mass of the products is **greater** than the total mass of the reactants. State what this means about the energy of the reaction, and explain your answer.

[2]

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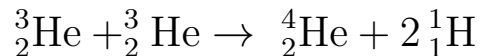
(e) In a different nuclear reaction the total mass of the products is **greater** than the total mass of the reactants. State what this means about the energy of the reaction, and explain your answer. [2]

(e)

- the reaction **absorbs** energy; it cannot happen spontaneously
- mass has increased, so by $E = \Delta mc^2$ energy of at least $|\Delta m|c^2$ must have been **supplied** from outside to create it

Exercise 3.3 (f)

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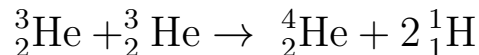


The masses, in u, are ${}^3\text{He} = 3.01603$, ${}^4\text{He} = 4.00260$ and ${}^1\text{H} = 1.00783$.

(f) Explain why fusing light nuclei releases much more energy **per nucleon** than the fission of a heavy nucleus. [2]

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(f) Explain why fusing light nuclei releases much more energy **per nucleon** than the fission of a heavy nucleus. [2]

(f)

- the binding-energy-per-nucleon curve **rises very steeply** on the light side but **falls only gently** beyond the maximum
- so fusing two light nuclei moves each nucleon much further up the curve than splitting a heavy nucleus does, and the energy released per nucleon is correspondingly larger