

Exercise walk-through

2.1

Equations of motion

A-Level Physics

Sheet 2.1.3 – what you need to know

You must be able to:

- **derive** the four equations from a velocity–time graph
- state when they apply: **constant acceleration** in a straight line
- choose the equation, and give every vector a sign
- solve **two stages, two objects**, and a quadratic with two answers

What the exam asks

“State two conditions for this equation to apply.” [2]

“Calculate the acceleration.” [2]

multiple choice: a train between two stations

Practise on exercise sheet 2.1.3, then try the real exam questions on past-paper sheet 2.1.3.

The four SUVAT equations

Straight line, **uniform acceleration** : five symbols u , v , a , s , t – each equation drops one.

$$v = u + at$$

no s

$$s = ut + \frac{1}{2}at^2$$

no v

$$s = \frac{1}{2}(u + v)t$$

no a

$$v^2 = u^2 + 2as$$

no t

Pick the equation that holds your **three knowns** *and* your **unknown**. They hold only for **constant acceleration** in a straight line.

Where SUVAT comes from

Start from the **velocity–time graph** of the motion:

1 gradient: $a = \frac{v - u}{t}$, so $v = u + at$

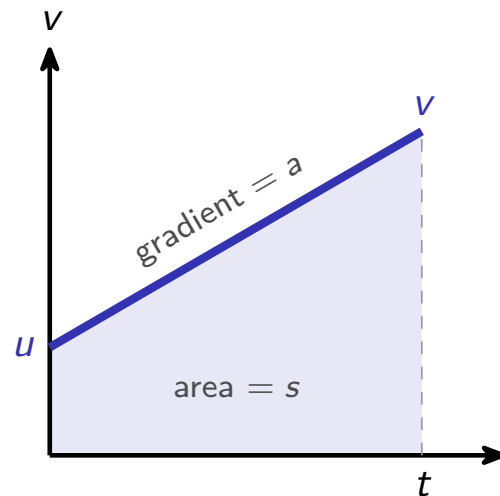
2 area of a trapezium : $s = \frac{1}{2}(u + v)t$

3 put $v = u + at$ into 2:

$$s = \frac{1}{2}(u + u + at)t = ut + \frac{1}{2}at^2$$

4 put $t = \frac{v - u}{a}$ into 2:

$$2as = (v + u)(v - u) = v^2 - u^2$$



A **derive** question wants exactly these steps, each starting from a definition or an equation already shown, with **no numbers**.

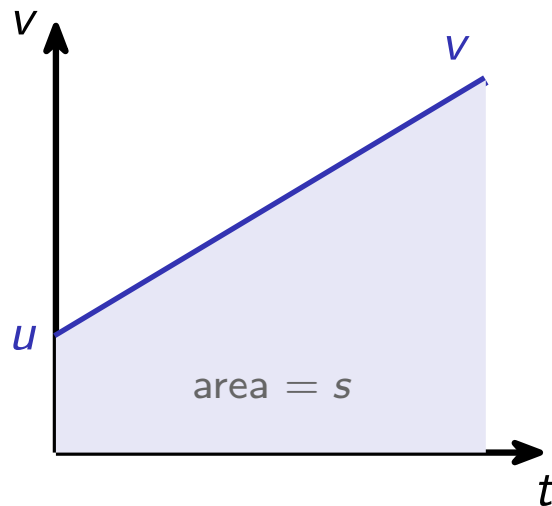
Worked example – find the acceleration

Car: $u = 8$, $v = 20 \text{ m s}^{-1}$ **over** $s = 56 \text{ m}$

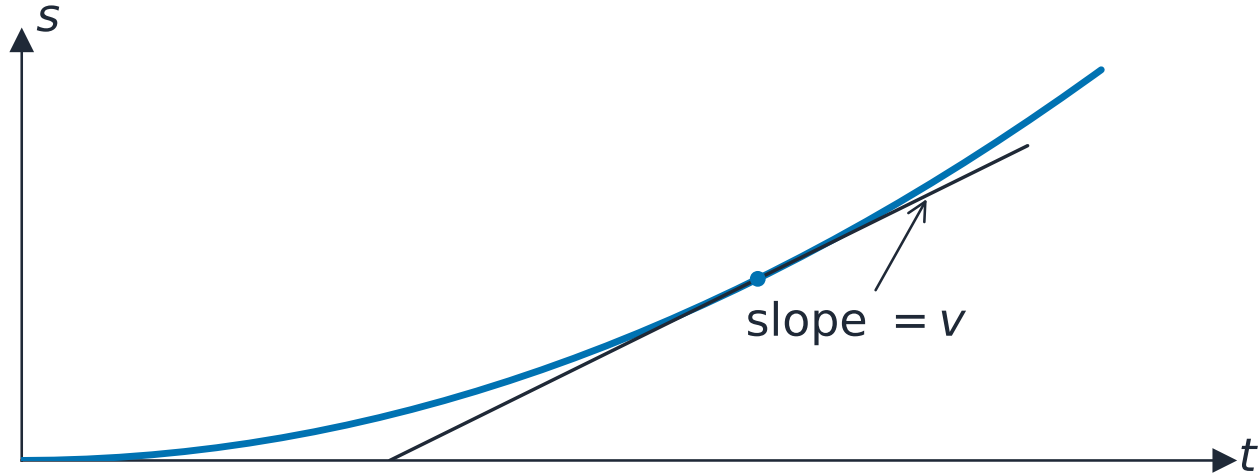
Knowns u, v, s ; **want** a , so use $v^2 = u^2 + 2as$:

$$20^2 = 8^2 + 2a(56)$$

$$336 = 112a \Rightarrow a = 3.0 \text{ m s}^{-2}.$$



The displacement–time graph is a parabola



constant $a \Rightarrow s-t$ is a parabola · gradient of tangent
= v

With uniform acceleration the $s-t$ graph is a **parabola**
the **tangent** at any point is the **velocity** at that instant.

: the gradient of

Exercise 2.1

2.1 For each quantity, write **scalar** or **vector**, and state its SI unit: (a) velocity (b) acceleration.



or **vector** , and state its SI unit: (a)
[2]

quantity	scalar or vector?	SI unit
velocity		
acceleration		

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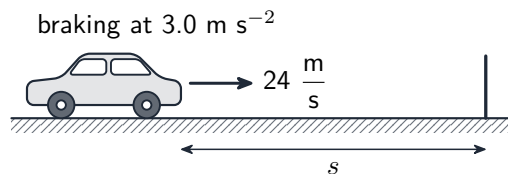
quantity	scalar or vector?	SI unit
velocity		
acceleration		

2.1

- velocity is a **vector**, unit $\frac{\text{m}}{\text{s}}$
- acceleration is a **vector**, unit m s^{-2}

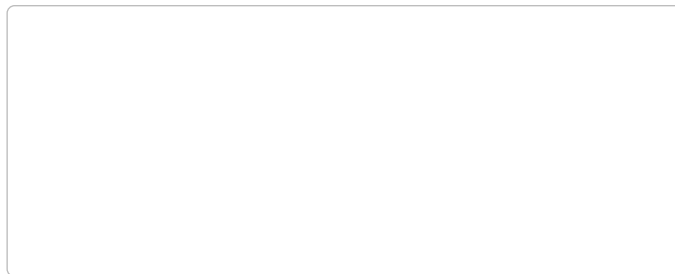
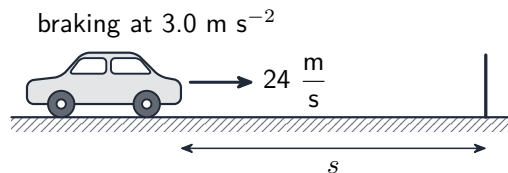
Exercise 2.2

2.2 A car travelling at $24 \frac{\text{m}}{\text{s}}$ brakes uniformly at 3.0 m s^{-2} until it stops. Find the braking distance. (*Hint: which SUVAT equation links u , v , a and s ?*) [3]



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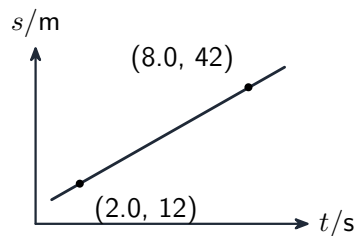
2.2

- known: $u = 24 \frac{\text{m}}{\text{s}}$, $v = 0$, $a = -3.0 \text{ m s}^{-2}$; want s (no t)
- use $v^2 = u^2 + 2as$, rearranged to $s = \frac{v^2 - u^2}{2a}$

$$s = \frac{v^2 - u^2}{2a} = \frac{0 - \left(24 \frac{\text{m}}{\text{s}}\right)^2}{2(-3.0 \text{ m s}^{-2})} = \frac{576 \text{ m}^2 \text{ s}^{-2}}{6.0 \text{ m s}^{-2}} = \mathbf{96 \text{ m}}$$

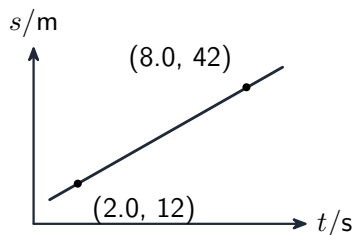
Exercise 2.3

2.3 The displacement–time graph of a cyclist is a straight line passing through (2.0 s, 12 m) and (8.0 s, 42 m). **Determine** her velocity, and state how you know her acceleration is zero. [3]



Exercise 2.3

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2.3

- velocity is the **gradient** of a displacement–time graph

$$v = \frac{\Delta s}{\Delta t} = \frac{42 \text{ m} - 12 \text{ m}}{8.0 \text{ s} - 2.0 \text{ s}} = \frac{30 \text{ m}}{6.0 \text{ s}} = \mathbf{5.0 \frac{m}{s}}$$

- the graph is a **straight** line, so v never changes; acceleration is the rate of change of velocity, so $a = 0$

Exercise 2.4

2.4 Describe an experiment to determine the acceleration of free fall using a falling object. State the measurements you would take, how you would use them, and one way to reduce the uncertainty. [4]



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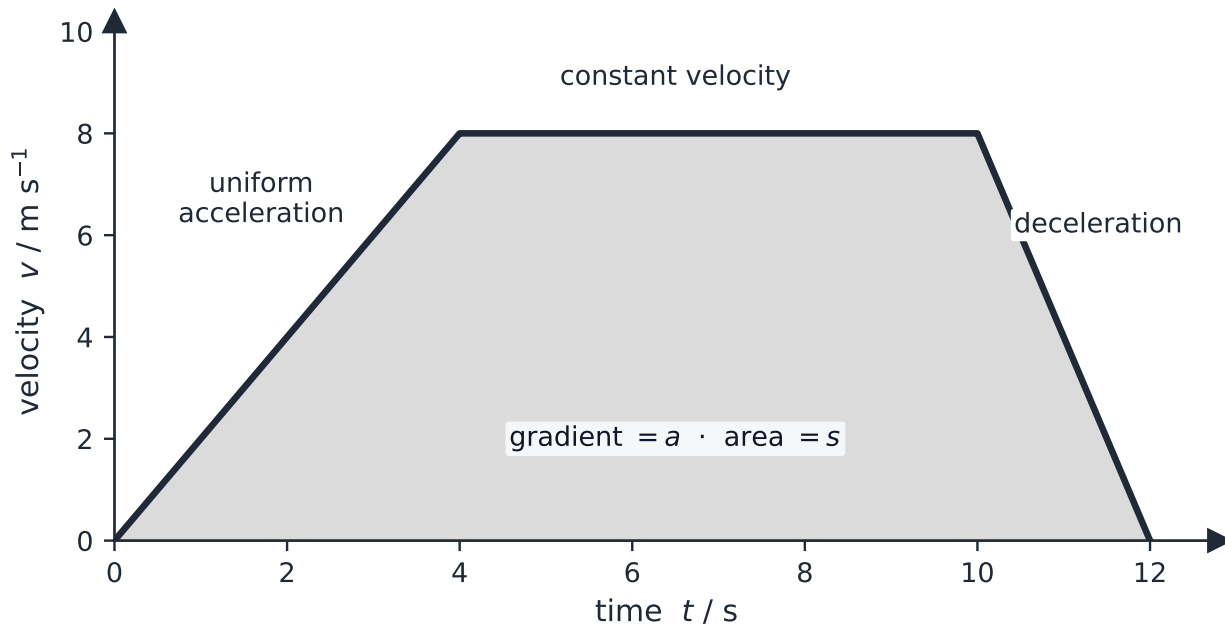


2.4

- hold a steel ball with an electromagnet a measured height h above a trapdoor
- switching the magnet off starts an electronic timer and releases the ball; the ball opens the trapdoor and stops the timer, giving t
- use $h = \frac{1}{2}gt^2$, or plot h against t^2 and take $g = 2 \times \text{gradient}$
- reduce uncertainty: repeat each timing, and use several heights so a graph averages the readings

Setup for exercise 2.5

2.5 The velocity–time graph shows a tram’s journey.



Velocity–time graph for the tram’s three-phase journey

Exercise 2.5 (a)

(a) Find the acceleration during the first 4 s.

[1]

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(a) acceleration = gradient of the v - t graph

$$a = \frac{\Delta v}{\Delta t} = \frac{8.0 \frac{\text{m}}{\text{s}} - 0}{4.0 \text{ s} - 0} = \mathbf{2.0 \text{ m s}^{-2}}$$

Exercise 2.5 (b)

(b) Find the total displacement of the tram. (*Hint: split the area into a triangle, a rectangle and a triangle.*) [3]

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(b) displacement = area under the line; split into triangle + rectangle + triangle

- 0–4 s, a triangle: $s_1 = \frac{1}{2}tv = \frac{1}{2} \times 4.0 \text{ s} \times 8.0 \frac{\text{m}}{\text{s}} = 16 \text{ m}$
- 4–10 s, a rectangle: $s_2 = tv = 6.0 \text{ s} \times 8.0 \frac{\text{m}}{\text{s}} = 48 \text{ m}$
- 10–12 s, a triangle: $s_3 = \frac{1}{2}tv = \frac{1}{2} \times 2.0 \text{ s} \times 8.0 \frac{\text{m}}{\text{s}} = 8 \text{ m}$
- total $s = s_1 + s_2 + s_3 = 16 \text{ m} + 48 \text{ m} + 8 \text{ m} = \mathbf{72 \text{ m}}$

Exercise 3.1

3.1 A stone is thrown vertically upwards at $15 \frac{\text{m}}{\text{s}}$. Take $g = 9.81 \text{ m s}^{-2}$ and ignore air resistance. Its height above the launch point after 2.0 s is closest to: [1]

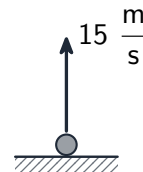
☐

A 10 m

B 20 m

C 30 m

D 50 m



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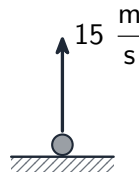
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A 10 m

B 20 m

C 30 m

D 50 m



3.1 A

- take up as positive: $u = 15 \frac{\text{m}}{\text{s}}$, $a = -9.81 \text{ m s}^{-2}$, $t = 2.0 \text{ s}$

$$s = ut + \frac{1}{2}at^2 = (15 \frac{\text{m}}{\text{s}})(2.0 \text{ s}) + \frac{1}{2}(-9.81 \text{ m s}^{-2})(2.0 \text{ s})^2 = 30 \text{ m} - 19.6 \text{ m} = 10.4 \text{ m}$$

- closest to **10 m**
- B** forgot the $\frac{1}{2}at^2$ term; **C** is ut alone

Exercise 3.2

3.2 The area under an **acceleration–time graph** gives which quantity?

[1]

☐

- A** displacement
- B** change in velocity
- C** distance
- D** force



Exercise 3.2

3.2 The area under an **acceleration–time** graph gives which quantity?

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☐

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- B change in velocity
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3.2 B

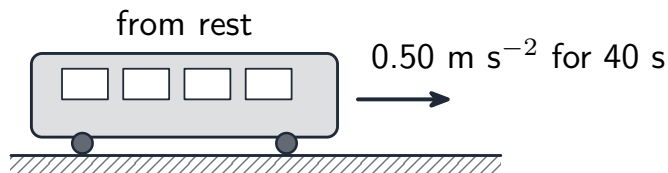
- the area under an **acceleration–time** graph is the **change in velocity**
- **A** / **C** are the area under a **velocity–time** graph

Exercise 3.3 (a)

3.3 A train starts from rest and accelerates uniformly at 0.50 m s^{-2} for 40 s.

(a) Calculate the speed it reaches.

[2]

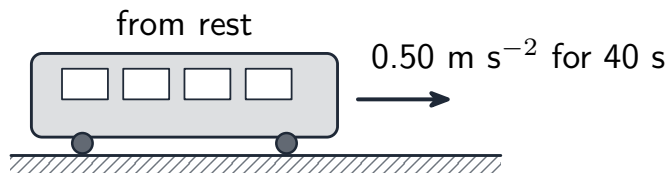


Exercise 3.3 (a)

3.3 A train starts from rest and accelerates uniformly at 0.50 m s^{-2} for 40 s.

(a) Calculate the speed it reaches.

[2]



(a) $u = 0$, $a = 0.50 \text{ m s}^{-2}$, $t = 40 \text{ s}$

$$v = u + at = 0 + (0.50 \text{ m s}^{-2})(40 \text{ s}) = \mathbf{20} \frac{\text{m}}{\text{s}}$$

Exercise 3.3 (b)

3.3 A train starts from rest and accelerates uniformly at 0.50 m s^{-2} for 40 s.

(b) Calculate the distance travelled during these 40 s.

[2]

Exercise 3.3 (b)

3.3 A train starts from rest and accelerates uniformly at 0.50 m s^{-2} for 40 s.

(b) Calculate the distance travelled during these 40 s. [2]

(b) no v needed: $s = ut + \frac{1}{2}at^2$

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}(0.50 \text{ m s}^{-2})(40 \text{ s})^2 = \mathbf{400 \text{ m}}$$

Exercise 3.3 (c)

3.3 A train starts from rest and accelerates uniformly at 0.50 m s^{-2} for 40 s.

(c) State one assumption you have made in your calculation.

[1]

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(c) State one assumption you have made in your calculation. [1]

(c) any one: the acceleration stays constant / the track is straight / no resistive forces

Exercise 3.4 (a)

3.4 A ball is kicked from level ground at $20 \frac{\text{m}}{\text{s}}$, at 30° above the horizontal. Take $g = 9.81 \text{ m s}^{-2}$ and ignore air resistance.

(a) Show that the vertical **component** of the initial velocity is about $10 \frac{\text{m}}{\text{s}}$. [1]

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(a) vertical component $= u \sin \theta$

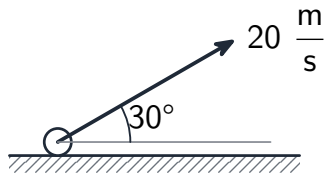
- $u_V = 20 \frac{\text{m}}{\text{s}} \times \sin 30^\circ = 20 \times 0.50 = \mathbf{10} \frac{\text{m}}{\text{s}}$

Exercise 3.4 (b)

3.4 A ball is kicked from level ground at $20 \frac{\text{m}}{\text{s}}$, at 30° above the horizontal. Take $g = 9.81 \text{ m s}^{-2}$ and ignore air resistance.

(b) Calculate the time the ball is in the air before it lands.

[2]

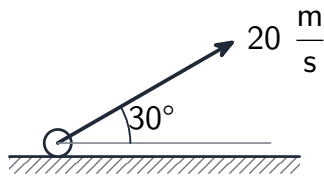


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[2]



(b) time to the top: $v_V = 0 = u_V - gt$

$$t_{\text{up}} = \frac{u}{g} = \frac{10 \frac{\text{m}}{\text{s}}}{9.81 \text{ m s}^{-2}} = 1.02 \text{ s}$$

- total time in the air $= 2 \times 1.02 \text{ s} = \mathbf{2.0 \text{ s}}$

Exercise 3.4 (c)

3.4 A ball is kicked from level ground at $20 \frac{\text{m}}{\text{s}}$, at 30° above the horizontal. Take $g = 9.81 \text{ m s}^{-2}$ and ignore air resistance.

(c) Calculate the horizontal distance it travels (its range). [2]

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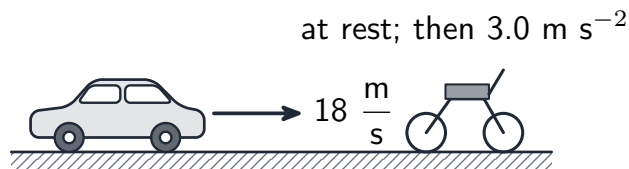
$$(c) u_H = 20 \frac{\text{m}}{\text{s}} \times \cos 30^\circ = 17.3 \frac{\text{m}}{\text{s}}$$

- $\text{range} = u_H \times t = (17.3 \frac{\text{m}}{\text{s}})(2.04 \text{ s}) = \mathbf{35 \text{ m}}$

Exercise 3.5 (a)

3.5 A car passes a stationary motorcyclist at a constant $18 \frac{\text{m}}{\text{s}}$. At that instant the motorcyclist sets off from rest with a uniform acceleration of 3.0 m s^{-2} .

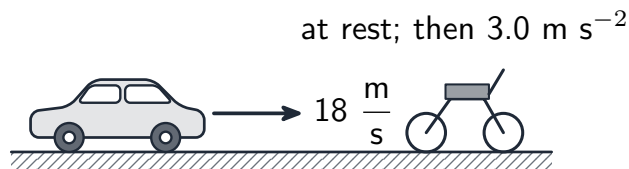
(a) Calculate the time the motorcyclist takes to catch up with the car. [3]



Exercise 3.5 (a)

3.5 A car passes a stationary motorcyclist at a constant $18 \frac{\text{m}}{\text{s}}$. At that instant the motorcyclist sets off from rest with a uniform acceleration of 3.0 m s^{-2} .

(a) Calculate the time the motorcyclist takes to catch up with the car. [3]



Full solution 3.5 (a)

(a) same start time and positive direction

- car (constant v): $s = (18 \frac{\text{m}}{\text{s}})t$
- motorcyclist (from rest): $s = \frac{1}{2}(3.0 \text{ m s}^{-2})t^2 = (1.5 \text{ m s}^{-2})t^2$
- they meet when the displacements are equal

$$(18 \frac{\text{m}}{\text{s}})t = (1.5 \text{ m s}^{-2})t^2 \Rightarrow t = \frac{18 \frac{\text{m}}{\text{s}}}{1.5 \text{ m s}^{-2}} = \mathbf{12 \text{ s}}$$

Exercise 3.5 (b)

3.5 A car passes a stationary motorcyclist at a constant $18 \frac{\text{m}}{\text{s}}$. At that instant the motorcyclist sets off from rest with a uniform acceleration of 3.0 m s^{-2} .

(b) Calculate how far each has travelled at that moment. [1]

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(b) Calculate how far each has travelled at that moment. [1]

$$(b) \ s = vt = \left(18 \frac{\text{m}}{\text{s}}\right)(12 \text{ s}) = \mathbf{216 \text{ m}}$$

Exercise 3.5 (c)

3.5 A car passes a stationary motorcyclist at a constant $18 \frac{\text{m}}{\text{s}}$. At that instant the motorcyclist sets off from rest with a uniform acceleration of 3.0 m s^{-2} .

(c) Calculate the motorcyclist's speed when she catches the car. [1]

Exercise 3.5 (c)

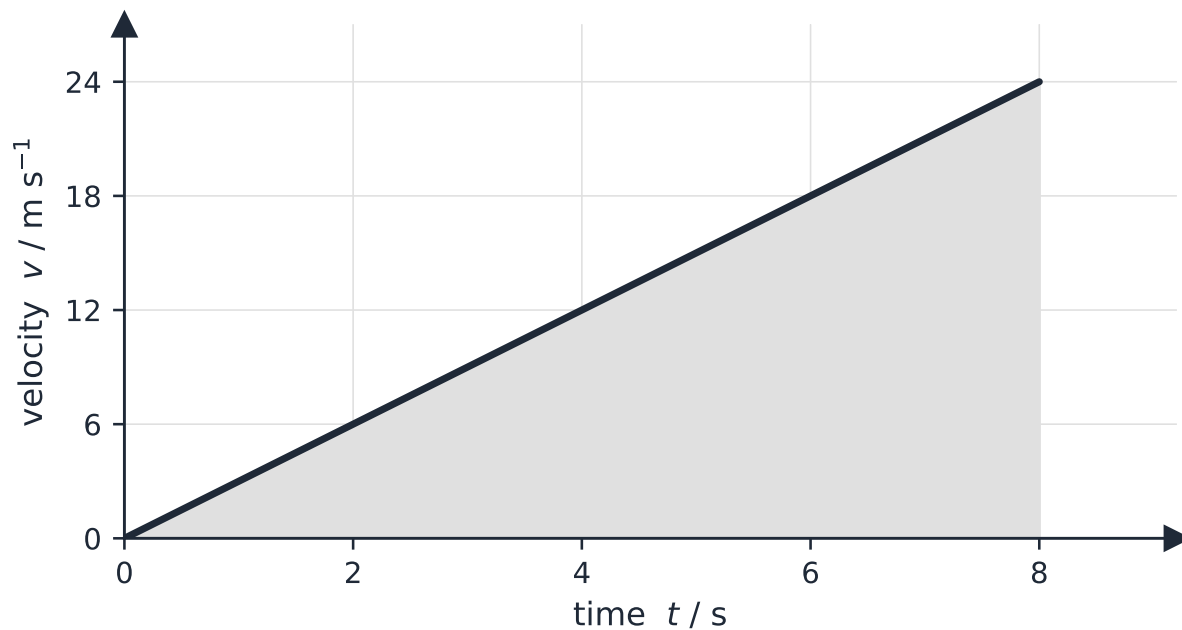
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m

Setup for exercise 3.6

3.6 A drone of mass 1.8 kg takes off from the ground and rises vertically. The graph shows its velocity for the first 8.0 s of the flight.



Exercise 3.6 (a)

(a) **Define** acceleration.

[1]

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[1]

(a) acceleration is the **rate of change of velocity**

Exercise 3.6 (b)

(b) **Determine** the acceleration of the drone.

[2]

Exercise 3.6 (b)

(b) **Determine** the acceleration of the drone.

[2]

(b) a = gradient of the v - t graph

$$a = \frac{\Delta v}{\Delta t} = \frac{24 \frac{\text{m}}{\text{s}} - 0}{8.0 \text{ s} - 0} = \mathbf{3.0 \text{ m s}^{-2}}$$

Exercise 3.6 (c)

(c) **Show that** the drone rises 96 m in these 8.0 s.

[2]

Exercise 3.6 (c)

(c) **Show that** the drone rises 96 m in these 8.0 s.

[2]

(c) height = area under the line (a triangle)

$$s = \frac{1}{2} t v = \frac{1}{2} \times 8.0 \text{ s} \times 24 \frac{\text{m}}{\text{s}} = \mathbf{96 \text{ m}}$$

Exercise 3.6 (d)

(d) Calculate the gain in **gravitational potential energy** of the drone.

[2]

Exercise 3.6 (d)

(d) Calculate the gain in **gravitational potential energy** of the drone.

[2]

(d)

$$\Delta E_P = mgh = (1.8 \text{ kg})(9.81 \text{ m s}^{-2})(96 \text{ m}) = \mathbf{1.7 \times 10^3 \text{ J}}$$

Exercise 3.6 (e)

(e) Calculate the gain in **kinetic energy** of the drone.

[2]

Exercise 3.6 (e)

(e) Calculate the gain in **kinetic energy** of the drone.

[2]

(e)

$$\Delta E_{\text{K}} = \frac{1}{2}mv^2 = \frac{1}{2}(1.8 \text{ kg})(24 \frac{\text{m}}{\text{s}})^2 = \mathbf{5.2 \times 10^2 \text{ J}}$$

Exercise 3.6 (f)

(f) **Determine** the average useful power output of the drone's motors during these 8.0 s.[2]

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(f) useful energy gained = $\Delta E_P + \Delta E_K$

$$P = \frac{1.7 \times 10^3 \text{ J} + 5.2 \times 10^2 \text{ J}}{8.0 \text{ s}} = \mathbf{2.8 \times 10^2 \text{ W}}$$

Exercise 3.6 (g)

(g) In the space below, **sketch** the variation with time of the **displacement** of the drone over the same 8.0 s. **Draw** and **label** both axes. [3]

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(g)

- axes drawn and labelled displacement / m (vertical) and time / s (horizontal)
- a curve starting at the origin
- getting **steeper** with time ($s \propto t^2$), reaching 96 m at 8.0 s

Exercise 3.6 (h)

(h) A second drone of mass 2.7 kg takes off and follows exactly the same velocity–time graph. **State and explain** whether the average useful power output of its motors is less than, the same as, or greater than your answer to (f). [2]

Exercise 3.6 (h)

(h) A second drone of mass 2.7 kg takes off and follows exactly the same velocity–time graph. **State and explain** whether the average useful power output of its motors is less than, the same as, or greater than your answer to (f). [2]

(h) **greater**

- the v – t graph is the same, so Δh and Δv are the same, but $\Delta E_P = mgh$ and $\Delta E_K = \frac{1}{2}mv^2$ both scale with **mass**, so more energy is gained in the same 8.0 s