

Exercise walk-through

# 2.1.5

## Projectile motion

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A-Level Physics

## Sheet 2.1.3 – what you need to know

### You must be able to:

- **derive** the four equations from a velocity–time graph
- state when they apply: **constant acceleration** in a straight line
- choose the equation, and give every vector a sign
- solve **two stages**, **two objects**, and a quadratic with two answers

### What the exam asks

“State two conditions for this equation to apply.” [2]

“Calculate the acceleration.” [2]

multiple choice: a train between two stations

*Practise on exercise sheet 2.1.3, then try the real exam questions on past-paper sheet 2.1.3.*

# The four SUVAT equations

Straight line, **uniform acceleration** : five symbols  $u$ ,  $v$ ,  $a$ ,  $s$ ,  $t$  – each equation drops one.

$$v = u + at$$

no  $s$ 

$$s = ut + \frac{1}{2}at^2$$

no  $v$ 

$$s = \frac{1}{2}(u + v)t$$

no  $a$ 

$$v^2 = u^2 + 2as$$

no  $t$ 

Pick the equation that holds your **three knowns** *and* your **unknown**. They hold only for **constant acceleration** in a straight line.

# Where SUVAT comes from

Start from the **velocity–time graph** of the motion:

1 gradient:  $a = \frac{v - u}{t}$ , so  $v = u + at$

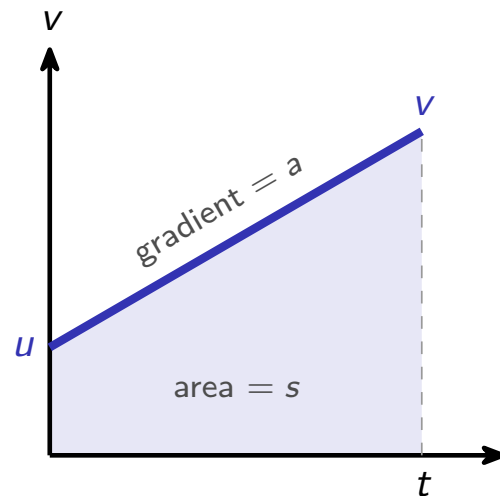
2 area of a trapezium :  $s = \frac{1}{2}(u + v)t$

3 put  $v = u + at$  into 2:

$$s = \frac{1}{2}(u + u + at)t = ut + \frac{1}{2}at^2$$

4 put  $t = \frac{v - u}{a}$  into 2:

$$2as = (v + u)(v - u) = v^2 - u^2$$



A **derive** question wants exactly these steps, each starting from a definition or an equation already shown, with **no numbers**.

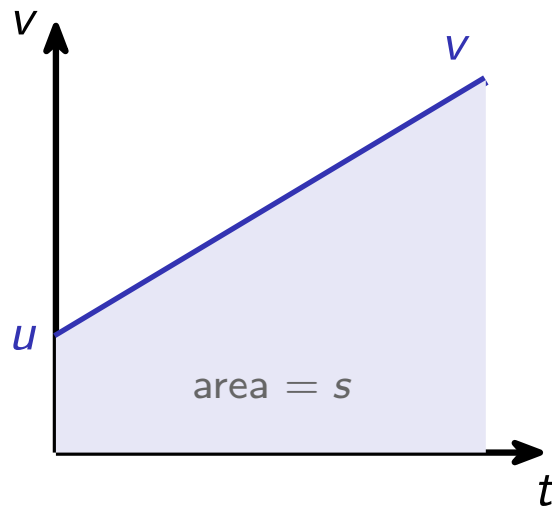
## Worked example – find the acceleration

**Car:**  $u = 8$ ,  $v = 20 \text{ m s}^{-1}$  **over**  $s = 56 \text{ m}$

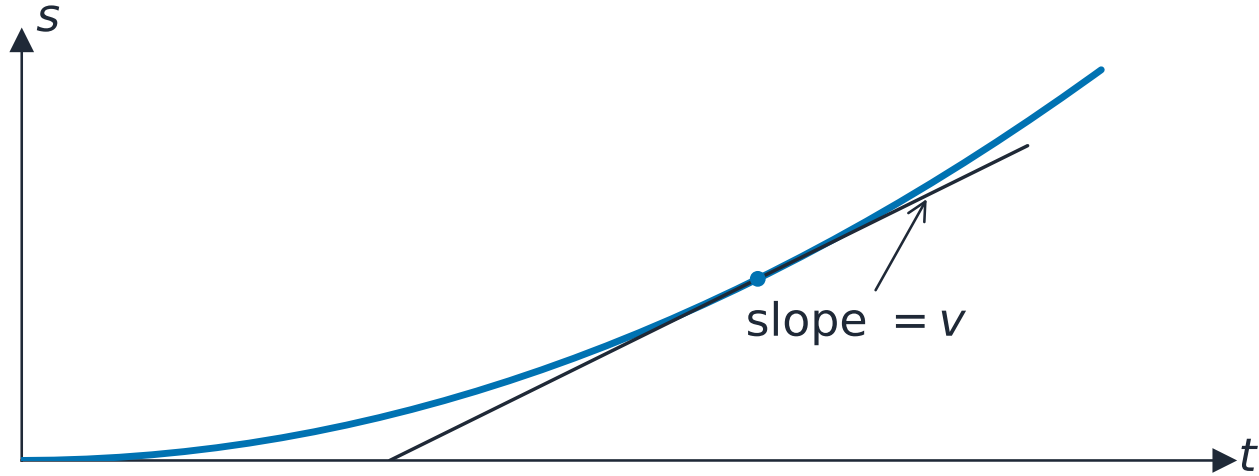
**Knowns**  $u, v, s$ ; **want**  $a$ , so use  $v^2 = u^2 + 2as$ :

$$20^2 = 8^2 + 2a(56)$$

$$336 = 112a \Rightarrow a = 3.0 \text{ m s}^{-2}.$$



# The displacement–time graph is a parabola



constant  $a \Rightarrow s-t$  is a parabola · gradient of tangent  
=  $v$

With uniform acceleration the  $s-t$  graph is a **parabola**  
the **tangent** at any point is the **velocity** at that instant.

: the gradient of

## Exercise 2.1 (a)

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**2.1** A delivery drone flies horizontally at  $12 \frac{\text{m}}{\text{s}}$ , 45 m above flat ground, and releases a parcel. Air resistance is negligible.

(a) State the horizontal and vertical components of the parcel's velocity just after it is released. [1]

## Exercise 2.1 (a)

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**2.1** A delivery drone flies horizontally at  $12 \frac{\text{m}}{\text{s}}$ , 45 m above flat ground, and releases a parcel. Air resistance is negligible.

(a) State the horizontal and vertical components of the parcel's velocity just after it is released. [1]

— — —

(a) horizontal  $12 \frac{\text{m}}{\text{s}}$  (the drone's velocity) and vertical 0

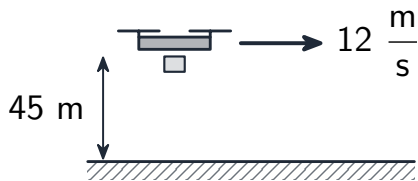
## Exercise 2.1 (b)

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**2.1** A delivery drone flies horizontally at  $12 \frac{\text{m}}{\text{s}}$ , 45 m above flat ground, and releases a parcel. Air resistance is negligible.

(b) Calculate the time taken for the parcel to reach the ground.

[2]

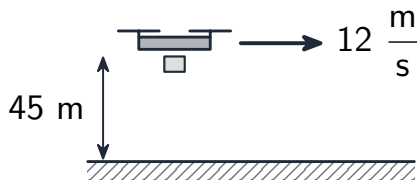


## Exercise 2.1 (b)

**2.1** A delivery drone flies horizontally at  $12 \frac{\text{m}}{\text{s}}$ , 45 m above flat ground, and releases a parcel. Air resistance is negligible.

(b) Calculate the time taken for the parcel to reach the ground.

[2]



(b) vertical motion from rest, down positive; use  $s = \frac{1}{2}at^2$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 45 \text{ m}}{9.81 \text{ m s}^{-2}}} = \mathbf{3.0 \text{ s}}$$

## Exercise 2.1 (c)

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**2.1** A delivery drone flies horizontally at  $12 \frac{\text{m}}{\text{s}}$ , 45 m above flat ground, and releases a parcel. Air resistance is negligible.

(c) Calculate the speed of the parcel as it reaches the ground.

[2]

## Exercise 2.1 (c)

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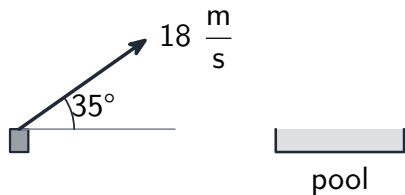
(c) vertically  $v = u + at$  gives  $v_V = (9.81 \text{ m s}^{-2})(3.03 \text{ s}) = 29.7 \frac{\text{m}}{\text{s}}$ ; then combine the two components

$$v = \sqrt{v_H^2 + v_V^2} = \sqrt{(12 \frac{\text{m}}{\text{s}})^2 + (29.7 \frac{\text{m}}{\text{s}})^2} = \mathbf{32 \frac{\text{m}}{\text{s}}}$$

## Exercise 2.2 (a)

**2.2** Water leaves the **nozzle** of a fountain at  $18 \frac{\text{m}}{\text{s}}$  at  $35^\circ$  above the horizontal, and lands in a pool at the same height as the nozzle. Treat each drop of water as a projectile, and ignore the effect of the air.

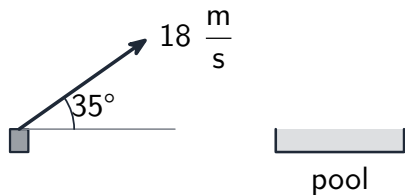
(a) Calculate the horizontal and vertical components of the velocity of the water as it leaves the nozzle. [2]



## Exercise 2.2 (a)

**2.2** Water leaves the **nozzle** of a fountain at  $18 \frac{\text{m}}{\text{s}}$  at  $35^\circ$  above the horizontal, and lands in a pool at the same height as the nozzle. Treat each drop of water as a projectile, and ignore the effect of the air.

(a) Calculate the horizontal and vertical components of the velocity of the water as it leaves the nozzle. [2]



(a) resolve:  $u_H = u \cos \theta$  and  $u_V = u \sin \theta$

$$u_H = \left(18 \frac{\text{m}}{\text{s}}\right) \cos 35^\circ = \mathbf{14.7 \frac{\text{m}}{\text{s}}}, \quad u_V = \left(18 \frac{\text{m}}{\text{s}}\right) \sin 35^\circ = \mathbf{10.3 \frac{\text{m}}{\text{s}}}$$

## Exercise 2.2 (b)

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**2.2** Water leaves the **nozzle** of a fountain at  $18 \frac{\text{m}}{\text{s}}$  at  $35^\circ$  above the horizontal, and lands in a pool at the same height as the nozzle. Treat each drop of water as a projectile, and ignore the effect of the air.

(b) Calculate the greatest height of the water above the nozzle. [2]

## Exercise 2.2 (b)

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**2.2** Water leaves the **nozzle** of a fountain at  $18 \frac{\text{m}}{\text{s}}$  at  $35^\circ$  above the horizontal, and lands in a pool at the same height as the nozzle. Treat each drop of water as a projectile, and ignore the effect of the air.

(b) Calculate the greatest height of the water above the nozzle. [2]

(b) at the top  $v_V = 0$ ; use  $v^2 = u^2 + 2as$  vertically

$$H = \frac{v_V^2 - u_V^2}{2a} = \frac{0 - (10.3 \frac{\text{m}}{\text{s}})^2}{2(-9.81 \text{ m s}^{-2})} = \mathbf{5.4 \text{ m}}$$

## Exercise 2.2 (c)

---

**2.2** Water leaves the **nozzle** of a fountain at  $18 \frac{\text{m}}{\text{s}}$  at  $35^\circ$  above the horizontal, and lands in a pool at the same height as the nozzle. Treat each drop of water as a projectile, and ignore the effect of the air.

(c) Calculate the horizontal distance from the nozzle to where the water lands. [2]

## Exercise 2.2 (c)

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**2.2** Water leaves the **nozzle** of a fountain at  $18 \frac{\text{m}}{\text{s}}$  at  $35^\circ$  above the horizontal, and lands in a pool at the same height as the nozzle. Treat each drop of water as a projectile, and ignore the effect of the air.

(c) Calculate the horizontal distance from the nozzle to where the water lands. [2]

(c) time to the top from  $v = u + at$  is  $10.3/9.81 = 1.05$  s, so the flight takes 2.10 s; then  $x = u_H t$

$$R = u_H t = (14.7 \frac{\text{m}}{\text{s}})(2.10 \text{ s}) = \mathbf{31 \text{ m}}$$

## Exercise 2.3

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**2.3** For a drop of the water in **2.3**, sketch on one set of axes how the horizontal and vertical components of its velocity change with time, from the nozzle until it lands. Take up as positive, label each line, and mark the values at the start and at the end. [3]

## Exercise 2.3

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### 2.3

- horizontal component: a horizontal line at  $14.7 \frac{\text{m}}{\text{s}}$  from 0 to 2.1 s
- vertical component: a straight line from  $+10.3 \frac{\text{m}}{\text{s}}$  at  $t = 0$
- through zero at 1.05 s to  $-10.3 \frac{\text{m}}{\text{s}}$  at 2.1 s, with both lines labelled

## Exercise 2.4 (a)

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**2.4** A basketball player throws the ball at  $7.3 \frac{\text{m}}{\text{s}}$  at  $50^\circ$  above the horizontal. The ball leaves her hands 2.0 m above the floor, and the hoop is a horizontal distance of 4.2 m away. Air resistance is negligible.

(a) **Show that** the ball takes 0.90 s to reach the hoop. (*Hint: the distance to the hoop is horizontal.*) [2]

## Exercise 2.4 (a)

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(a) **Show that** the ball takes 0.90 s to reach the hoop. (*Hint: the distance to the hoop is horizontal.*) [2]

— — —

(a) the horizontal velocity is constant,  $u_H = (7.3 \frac{\text{m}}{\text{s}}) \cos 50^\circ = 4.69 \frac{\text{m}}{\text{s}}$ ; use  $x = u_H t$

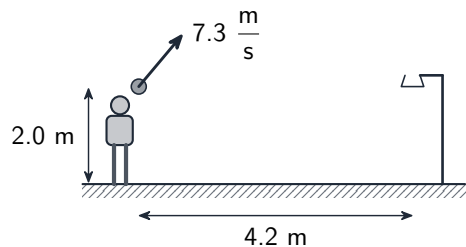
$$t = \frac{x}{u_H} = \frac{4.2 \text{ m}}{4.69 \frac{\text{m}}{\text{s}}} = 0.895 \text{ s} \approx 0.90 \text{ s}$$

## Exercise 2.4 (b)

**2.4** A basketball player throws the ball at  $7.3 \frac{\text{m}}{\text{s}}$  at  $50^\circ$  above the horizontal. The ball leaves her hands  $2.0 \text{ m}$  above the floor, and the hoop is a horizontal distance of  $4.2 \text{ m}$  away. Air resistance is negligible.

(b) Calculate the height of the ball above the floor at that moment.

[3]

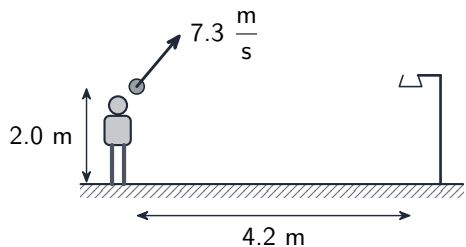


## Exercise 2.4 (b)

**2.4** A basketball player throws the ball at  $7.3 \frac{\text{m}}{\text{s}}$  at  $50^\circ$  above the horizontal. The ball leaves her hands  $2.0 \text{ m}$  above the floor, and the hoop is a horizontal distance of  $4.2 \text{ m}$  away. Air resistance is negligible.

(b) Calculate the height of the ball above the floor at that moment.

[3]



(b) vertical motion, up positive, with  $u_v = (7.3 \frac{\text{m}}{\text{s}}) \sin 50^\circ = 5.59 \frac{\text{m}}{\text{s}}$ ; use  $y = u_v t + \frac{1}{2} a t^2$

$$y = u_v t + \frac{1}{2} a t^2 = (5.59 \frac{\text{m}}{\text{s}})(0.90 \text{ s}) + \frac{1}{2}(-9.81 \text{ m s}^{-2})(0.90 \text{ s})^2 = 1.06 \text{ m}$$

- height above the floor  $h = h_{\text{table}} + y = 2.0 \text{ m} + 1.06 \text{ m} = \mathbf{3.1 \text{ m}}$

## Exercise 3.1

---

**3.1** A ball is thrown at an angle above horizontal ground. Air resistance is negligible. Which row describes the ball at the highest point of its path? [1]

|          | velocity   | acceleration                      |
|----------|------------|-----------------------------------|
| <b>A</b> | zero       | zero                              |
| <b>B</b> | zero       | $9.81 \text{ m s}^{-2}$ downwards |
| <b>C</b> | horizontal | zero                              |
| <b>D</b> | horizontal | $9.81 \text{ m s}^{-2}$ downwards |



## Exercise 3.1

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|          | velocity   | acceleration                      |
|----------|------------|-----------------------------------|
| <b>A</b> | zero       | zero                              |
| <b>B</b> | zero       | $9.81 \text{ m s}^{-2}$ downwards |
| <b>C</b> | horizontal | zero                              |
| <b>D</b> | horizontal | $9.81 \text{ m s}^{-2}$ downwards |



### 3.1 D

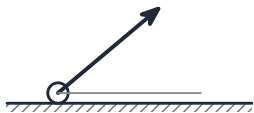
- at the top the vertical component is zero, but the horizontal component is not, so the velocity is horizontal
- the acceleration is  $g$  downwards during the whole flight, including at the top
- **A** and **B** forget the horizontal velocity; **C** thinks the acceleration stops at the top

## Exercise 3.2 (a)

---

**3.2** A golf ball is hit from level ground at an angle above the horizontal.

(a) A ball of greater mass is hit with the same velocity. Air resistance is still negligible. State and explain the effect, if any, on the time the ball spends in the air. [2]

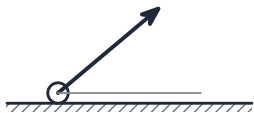


## Exercise 3.2 (a)

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**3.2** A golf ball is hit from level ground at an angle above the horizontal.

(a) A ball of greater mass is hit with the same velocity. Air resistance is still negligible. State and explain the effect, if any, on the time the ball spends in the air. [2]



(a)

- the time in the air does not change
- the acceleration of free fall is the same for every mass, and the vertical component of the launch velocity is the same

## Exercise 3.2 (b)

---

**3.2** A golf ball is hit from level ground at an angle above the horizontal.

(b) In practice, air resistance is not negligible. State and explain the effect of air resistance on the horizontal distance the ball travels. [2]

## Exercise 3.2 (b)

---

**3.2** A golf ball is hit from level ground at an angle above the horizontal.

(b) In practice, air resistance is not negligible. State and explain the effect of air resistance on the horizontal distance the ball travels. [2]

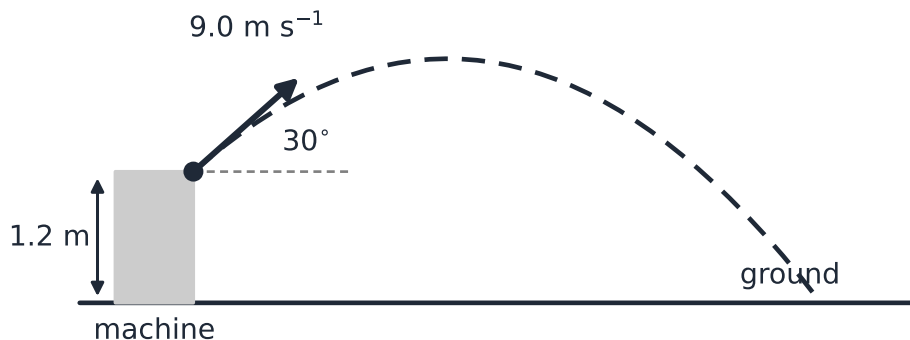
(b)

- the horizontal distance is smaller
- air resistance acts against the motion, so the horizontal component of the velocity decreases during the flight

## Setup for exercise 3.3

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**3.3** A ball-launching machine is 1.2 m above level ground. It fires a ball at  $9.0 \frac{\text{m}}{\text{s}}$  at  $30^\circ$  above the horizontal. Air resistance is negligible.



## Exercise 3.3 (a)

---

- (a) Calculate the horizontal and vertical components of the ball's initial velocity. [2]

## Exercise 3.3 (a)

---

(a) Calculate the horizontal and vertical components of the ball's initial velocity. [2]

(a) resolve:  $u_H = u \cos \theta$  and  $u_V = u \sin \theta$

$$u_H = \left(9.0 \frac{\text{m}}{\text{s}}\right) \cos 30^\circ = \mathbf{7.8 \frac{m}{s}}, \quad u_V = \left(9.0 \frac{\text{m}}{\text{s}}\right) \sin 30^\circ = \mathbf{4.5 \frac{m}{s}}$$

## Exercise 3.3 (b)

---

(b) Calculate the greatest height of the ball above the ground.

[2]

## Exercise 3.3 (b)

---

(b) Calculate the greatest height of the ball above the ground.

[2]

(b) rise above the machine, with  $v_V = 0$  at the top; use  $v^2 = u^2 + 2as$

$$s = \frac{v^2 - u^2}{2a} = \frac{0 - (4.5 \frac{\text{m}}{\text{s}})^2}{2(-9.81 \text{ m s}^{-2})} = 1.03 \text{ m}$$

- height above the ground  $h = s + h_{\text{machine}} = 1.03 \text{ m} + 1.2 \text{ m} = \mathbf{2.2 \text{ m}}$

## Exercise 3.3 (c)

---

(c) **Show that** the ball takes about 1.1 s to reach the ground.

[3]

## Exercise 3.3 (c)

---

(c) **Show that** the ball takes about 1.1 s to reach the ground.

[3]

(c) up positive:  $s = -1.2 \text{ m}$ ,  $u = +4.5 \frac{\text{m}}{\text{s}}$ ,  $a = -9.81 \text{ m s}^{-2}$  in  $s = ut + \frac{1}{2}at^2$

$$-1.2 = 4.5t - 4.905t^2 \quad \Rightarrow \quad 4.905t^2 - 4.5t - 1.2 = 0$$

- quadratic formula, values in m and s

$$t = \frac{4.5 + \sqrt{20.25 + 23.5}}{9.81} = 1.13 \text{ s} \approx 1.1 \text{ s}$$

## Exercise 3.3 (d)

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(d) Calculate the horizontal distance from the machine to where the ball lands. [1]

## Exercise 3.3 (d)

---

(d) Calculate the horizontal distance from the machine to where the ball lands. [1]

$$(d) \ x = u_H t = \left(7.8 \frac{\text{m}}{\text{s}}\right)(1.13 \text{ s}) = \mathbf{8.8 \text{ m}}$$

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## Exercise 3.3 (e)

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(e) Calculate the speed of the ball as it lands, and the angle its velocity makes with the ground. [3]

## Exercise 3.3 (e)

---

(e) Calculate the speed of the ball as it lands, and the angle its velocity makes with the ground. [3]

(e)  $v = u + at$  gives  $v_V = 4.5 \frac{\text{m}}{\text{s}} + (-9.81 \text{ m s}^{-2})(1.13 \text{ s}) = -6.6 \frac{\text{m}}{\text{s}}$ ; then combine the components

$$v = \sqrt{v_H^2 + v_V^2} = \sqrt{(7.8 \frac{\text{m}}{\text{s}})^2 + (6.6 \frac{\text{m}}{\text{s}})^2} = \mathbf{10 \frac{\text{m}}{\text{s}}}$$

- $\tan \theta = 6.6/7.8$ , so the velocity is at **40°** below the horizontal