

Exercise walk-through

## 2.1.1

# Describing motion and displacement–time graphs

---

A-Level Physics

## Sheet 2.1.2 – what you need to know

### You must be able to:

- **acceleration** = gradient of a velocity–time graph, also at one instant on a curve
- **displacement** = area; area below the time axis is negative
- tell distance from displacement; estimate an area by **counting squares**
- sketch one motion graph from another

### What the exam asks

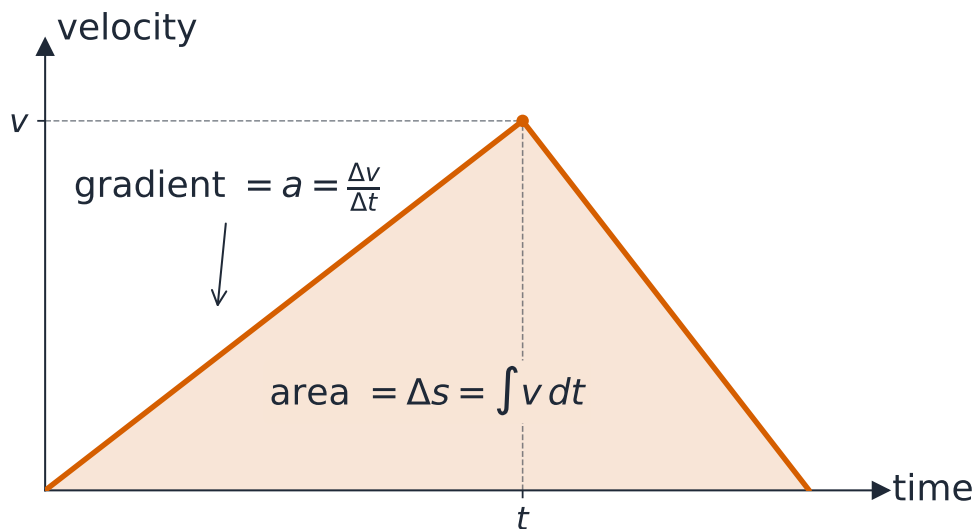
“Determine the acceleration at  $t = 4.0 \text{ s}$ .” [2]

“Determine the displacement at  $t = 12 \text{ s}$ .” [2]

“Sketch the variation with time of ...” [3]

*Practise on exercise sheet 2.1.2, then try the real exam questions on past-paper sheet 2.1.2.*

# Velocity–time graphs



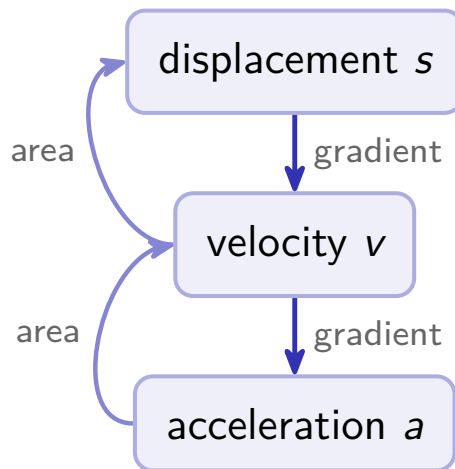
The **gradient** of a velocity–time graph = **acceleration**.

- flat line → **constant velocity**
- straight slope → **uniform acceleration**
- curve → **changing acceleration**

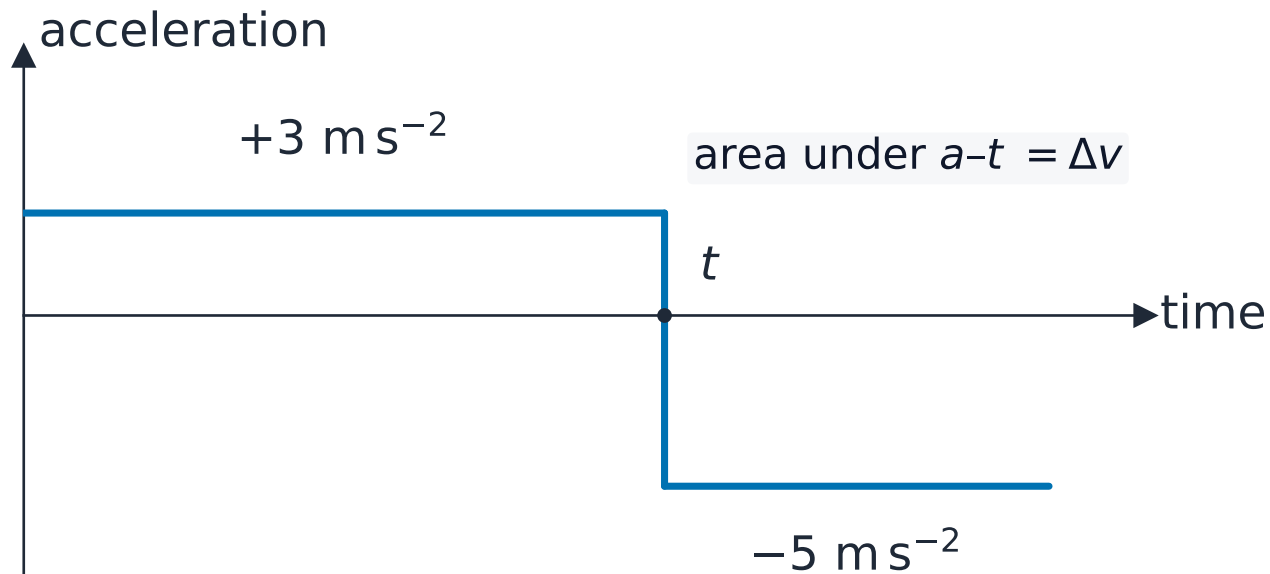
The **area** under the line = **displacement**. Area below the time axis is **negative**: the object moved backwards.

# Gradient and area – at a glance

These are the **graphical methods** the syllabus names. Two graphs carry most of the marks – displacement–time and velocity–time. Going **down** the chain you take a **gradient**; going **up** it you take an **area**.

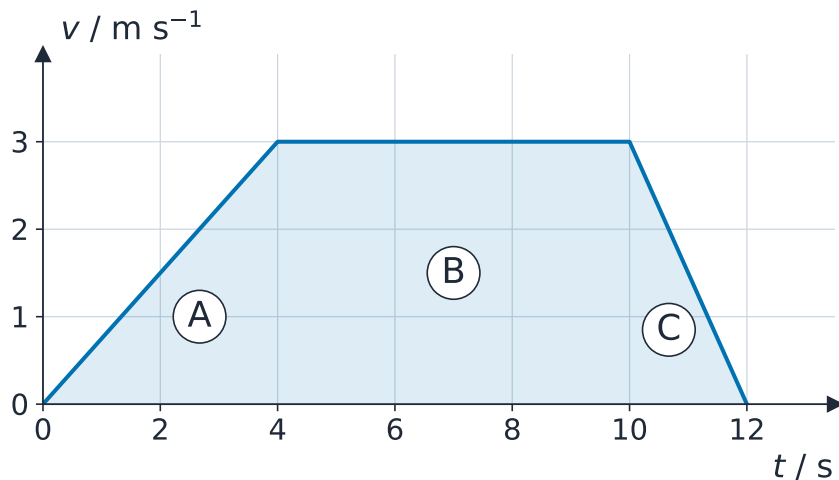


# Gradient and area – at a glance, in detail



Each step is a **gradient** of the  $v-t$  graph; the **area** under  $a-t$  is  $\Delta v$ .

## Worked example – a lift's velocity–time graph



### Accelerations and distance

**A:**  $a = (3.0 - 0)/4.0 = 0.75 \text{ m s}^{-2}$

**B:** a flat line, so  $a = 0$

**C:**  $a = (0 - 3.0)/2.0 = -1.5 \text{ m s}^{-2}$

**distance** = area under the line:

$$6.0 + 18 + 3.0 = 27 \text{ m}$$

Split the area into **triangles** ( $\frac{1}{2} \times \text{base} \times \text{height}$ ) and **rectangles**. Under a curve, **count the squares** (half or more counts as one) and multiply by one square's value.

## Exercise 2.1 (a)

---

**2.1** The examiner often starts a question with a definition.

(a) **Define** velocity.

[1]

velocity

speed

---

## Exercise 2.1 (a)

---

**2.1** The examiner often starts a question with a definition.

(a) **Define** velocity.

[1]

velocity

speed

---

(a) velocity is the **rate of change of displacement**

## Exercise 2.1 (b)

---

**2.1** The examiner often starts a question with a definition.

(b) State one way in which velocity is different from speed.

[1]

## Exercise 2.1 (b)

---

**2.1** The examiner often starts a question with a definition.

(b) State one way in which velocity is different from speed.

[1]

(b) velocity is a vector, so it has a direction; speed is a scalar, so it has none

## Exercise 2.2 (a)

---

**2.2** A cyclist rides 3.0 km due east and then 5.0 km due north. The whole ride takes 25 minutes.

(a) State the total distance travelled.

[1]

## Exercise 2.2 (a)

---

**2.2** A cyclist rides 3.0 km due east and then 5.0 km due north. The whole ride takes 25 minutes.

(a) State the total distance travelled.

[1]

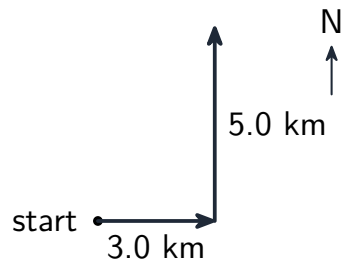
(a) total distance  $d = 3.0 \text{ km} + 5.0 \text{ km} = \mathbf{8.0 \text{ km}}$

## Exercise 2.2 (b)

---

**2.2** A cyclist rides 3.0 km due east and then 5.0 km due north. The whole ride takes 25 minutes.

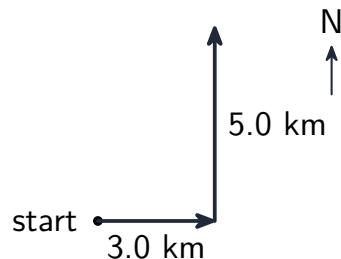
(b) **Determine** the size of the cyclist's displacement from the start. (*Hint: the two displacements are at right angles.*) [1]



## Exercise 2.2 (b)

**2.2** A cyclist rides 3.0 km due east and then 5.0 km due north. The whole ride takes 25 minutes.

(b) **Determine** the size of the cyclist's displacement from the start. (*Hint: the two displacements are at right angles.*) [1]



(b) the displacements are at right angles, so use Pythagoras

$$s = \sqrt{s_E^2 + s_N^2} = \sqrt{(3.0 \text{ km})^2 + (5.0 \text{ km})^2} = \mathbf{5.8 \text{ km}}$$

## Exercise 2.2 (c)

---

**2.2** A cyclist rides 3.0 km due east and then 5.0 km due north. The whole ride takes 25 minutes.

(c) Determine the direction of the displacement, as an angle measured from east towards north. [1]

## Exercise 2.2 (c)

---

**2.2** A cyclist rides 3.0 km due east and then 5.0 km due north. The whole ride takes 25 minutes.

(c) Determine the direction of the displacement, as an angle measured from east towards north. [1]

(c) opposite side north, adjacent side east

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{5.0 \text{ km}}{3.0 \text{ km}} \Rightarrow \theta = \mathbf{59^\circ}$$

north of east

## Exercise 2.2 (d)

---

**2.2** A cyclist rides 3.0 km due east and then 5.0 km due north. The whole ride takes 25 minutes.

(d) Calculate the average speed of the cyclist, in  $\frac{\text{m}}{\text{s}}$ . [2]

## Exercise 2.2 (d)

---

**2.2** A cyclist rides 3.0 km due east and then 5.0 km due north. The whole ride takes 25 minutes.

(d) Calculate the average speed of the cyclist, in  $\frac{\text{m}}{\text{s}}$ . [2]

(d) time =  $25 \times 60 \text{ s} = 1500 \text{ s}$  and distance = 8000 m

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{8000 \text{ m}}{1500 \text{ s}} = \mathbf{5.3} \frac{\text{m}}{\text{s}}$$

## Exercise 2.2 (e)

---

**2.2** A cyclist rides 3.0 km due east and then 5.0 km due north. The whole ride takes 25 minutes.

(e) Calculate the size of the average velocity of the cyclist, in  $\frac{\text{m}}{\text{s}}$ . [1]

## Exercise 2.2 (e)

---

**2.2** A cyclist rides 3.0 km due east and then 5.0 km due north. The whole ride takes 25 minutes.

(e) Calculate the size of the average velocity of the cyclist, in  $\frac{\text{m}}{\text{s}}$ . [1]

(e) use the displacement, not the distance

$$\text{average velocity} = \frac{\text{displacement}}{\text{total time}} = \frac{5830 \text{ m}}{1500 \text{ s}} = \mathbf{3.9} \frac{\text{m}}{\text{s}}$$

## Exercise 2.3

---

**2.3** Change the units. (*Hint: 1 km = 1000 m and 1 h = 3600 s.*) (a)  $72 \frac{\text{km}}{\text{h}}$  into  $\frac{\text{m}}{\text{s}}$ ; (b)  $25 \frac{\text{m}}{\text{s}}$  into  $\frac{\text{km}}{\text{h}}$ . [2]



72  
km/h

## Exercise 2.3

**2.3** Change the units. (*Hint: 1 km = 1000 m and 1 h = 3600 s.*) (a)  $72 \frac{\text{km}}{\text{h}}$  into  $\frac{\text{m}}{\text{s}}$ ; (b)  $25 \frac{\text{m}}{\text{s}}$  into  $\frac{\text{km}}{\text{h}}$ . [2]

$$72 \frac{\text{km}}{\text{h}}$$

**2.3**

(a) multiply by 1000 and divide by 3600

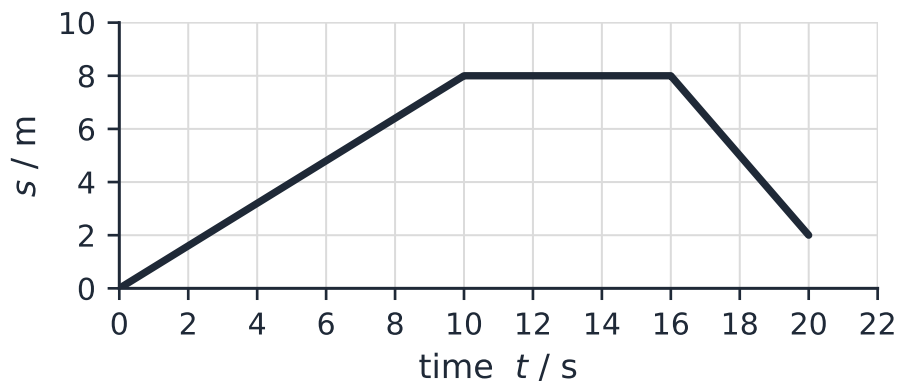
$$72 \frac{\text{km}}{\text{h}} = \frac{72 \times 1000 \text{ m}}{3600 \text{ s}} = \mathbf{20} \frac{\text{m}}{\text{s}}$$

(b) in one hour the car travels  $25 \text{ m} \times 3600 = 90000 \text{ m} = 90 \text{ km}$  so the speed is  $\mathbf{90} \frac{\text{km}}{\text{h}}$

## Setup for exercise 2.4

---

**2.4** The displacement–time graph shows a model train on a straight track.



## Exercise 2.4 (a)

---

(a) Determine the velocity of the train between  $t = 0$  and  $t = 10$  s.

[1]

## Exercise 2.4 (a)

---

(a) Determine the velocity of the train between  $t = 0$  and  $t = 10$  s. [1]

(a) velocity = gradient

$$v = \frac{\Delta s}{\Delta t} = \frac{8.0 \text{ m} - 0}{10 \text{ s} - 0} = \mathbf{0.80} \frac{\text{m}}{\text{s}}$$

## Exercise 2.4 (b)

---

(b) Describe the motion of the train between  $t = 10$  s and  $t = 16$  s.

[1]

## Exercise 2.4 (b)

---

- (b) Describe the motion of the train between  $t = 10$  s and  $t = 16$  s. [1]
- (b) the line is flat, so the displacement does not change: the train is **at rest**

## Exercise 2.4 (c)

---

(c) Determine the velocity of the train between  $t = 16$  s and  $t = 20$  s.

[1]

## Exercise 2.4 (c)

---

(c) Determine the velocity of the train between  $t = 16$  s and  $t = 20$  s. [1]

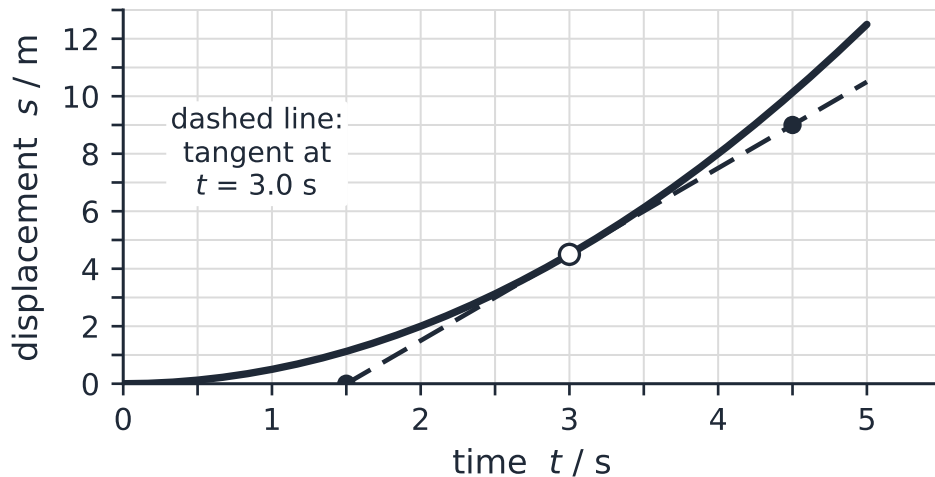
(c) gradient of the line from 16 s to 20 s

$$v = \frac{\Delta s}{\Delta t} = \frac{2.0 \text{ m} - 8.0 \text{ m}}{20 \text{ s} - 16 \text{ s}} = -\mathbf{1.5} \frac{\text{m}}{\text{s}}$$

- the minus sign means the train moves back towards its start

## Setup for exercise 2.5

**2.5** A trolley is released from rest at the top of a ramp. The graph shows its displacement down the ramp. A tangent has been drawn at  $t = 3.0$  s.



## Exercise 2.5 (a)

---

(a) Use the tangent to determine the velocity of the trolley at  $t = 3.0$  s. [2]

## Exercise 2.5 (a)

---

(a) Use the tangent to determine the velocity of the trolley at  $t = 3.0$  s. [2]

(a) the tangent passes through (1.5 s, 0) and (4.5 s, 9.0 m)

$$v = \frac{\Delta s}{\Delta t} = \frac{9.0 \text{ m} - 0}{4.5 \text{ s} - 1.5 \text{ s}} = \mathbf{3.0} \frac{\text{m}}{\text{s}}$$

## Exercise 2.5 (b)

---

(b) State how the graph shows that the trolley is speeding up.

[1]

## Exercise 2.5 (b)

---

- (b) State how the graph shows that the trolley is speeding up. [1]
- (b) the curve gets **steeper**: its gradient, the velocity, increases with time

## Exercise 3.1

---

**3.1** A runner completes 2.5 laps of a circular track of circumference 400 m in 300 s. What are her average speed and the size of her average velocity? [1]

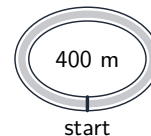
☐

**A** speed  $3.3 \frac{\text{m}}{\text{s}}$ , velocity 0

**B** speed  $3.3 \frac{\text{m}}{\text{s}}$ , velocity  $0.42 \frac{\text{m}}{\text{s}}$

**C** speed  $0.42 \frac{\text{m}}{\text{s}}$ , velocity  $3.3 \frac{\text{m}}{\text{s}}$

**D** speed  $3.3 \frac{\text{m}}{\text{s}}$ , velocity  $3.3 \frac{\text{m}}{\text{s}}$



## Exercise 3.1

**3.1** A runner completes 2.5 laps of a circular track of circumference 400 m in 300 s. What are her average speed and the size of her average velocity? [1]

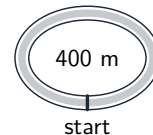


**A** speed  $3.3 \frac{\text{m}}{\text{s}}$ , velocity 0

**B** speed  $3.3 \frac{\text{m}}{\text{s}}$ , velocity  $0.42 \frac{\text{m}}{\text{s}}$

**C** speed  $0.42 \frac{\text{m}}{\text{s}}$ , velocity  $3.3 \frac{\text{m}}{\text{s}}$

**D** speed  $3.3 \frac{\text{m}}{\text{s}}$ , velocity  $3.3 \frac{\text{m}}{\text{s}}$



**3.1 B**

- distance =  $2.5 \times 400 \text{ m} = 1000 \text{ m}$

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{1000 \text{ m}}{300 \text{ s}} = 3.3 \frac{\text{m}}{\text{s}}$$

- after 2.5 laps the runner is half a lap from the start, so the displacement is one diameter

$$d = \frac{400 \text{ m}}{\pi} = 127 \text{ m}, \quad \text{average velocity} = \frac{127 \text{ m}}{300 \text{ s}} = \mathbf{0.42 \frac{\text{m}}{\text{s}}}$$

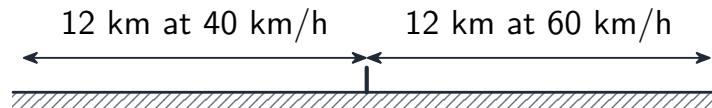
- A** is true only after whole laps; **C** swaps them; **D** uses the distance twice

## Exercise 3.2

---

**3.2** A bus travels 12 km at an average speed of  $40 \frac{\text{km}}{\text{h}}$ , then another 12 km at  $60 \frac{\text{km}}{\text{h}}$ .  
What is its average speed for the whole journey? [1]

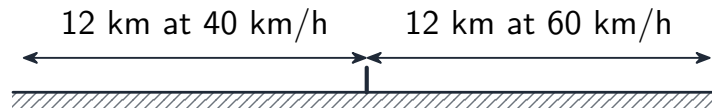
- ☐ **A**  $13 \frac{\text{m}}{\text{s}}$
- B**  $14 \frac{\text{m}}{\text{s}}$
- C**  $48 \frac{\text{m}}{\text{s}}$
- D**  $50 \frac{\text{m}}{\text{s}}$



## Exercise 3.2

**3.2** A bus travels 12 km at an average speed of  $40 \frac{\text{km}}{\text{h}}$ , then another 12 km at  $60 \frac{\text{km}}{\text{h}}$ .  
What is its average speed for the whole journey? [1]

- ☐ **A**  $13 \frac{\text{m}}{\text{s}}$
- B**  $14 \frac{\text{m}}{\text{s}}$
- C**  $48 \frac{\text{m}}{\text{s}}$
- D**  $50 \frac{\text{m}}{\text{s}}$



**3.2 A**

- time for each part

## Full solution 3.2

---

### 3.2 A

- time for each part

$$t = \frac{s}{v} : \quad t_1 = \frac{12 \text{ km}}{40 \frac{\text{km}}{\text{h}}} = 0.30 \text{ h}, \quad t_2 = \frac{12 \text{ km}}{60 \frac{\text{km}}{\text{h}}} = 0.20 \text{ h}$$

- average speed = total distance  $\div$  total time

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{24 \text{ km}}{0.50 \text{ h}} = 48 \frac{\text{km}}{\text{h}}$$

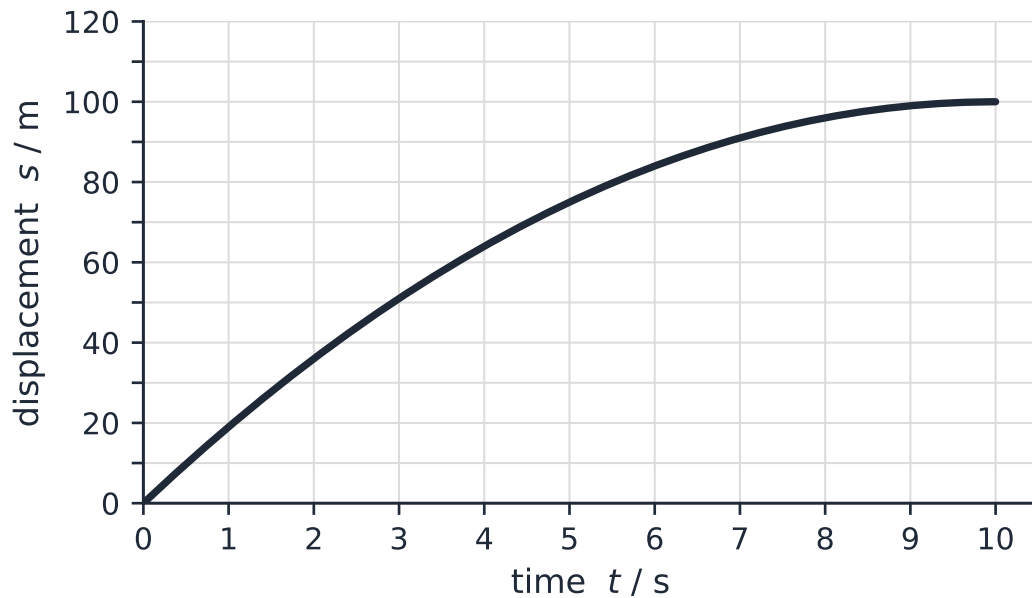
$$48 \frac{\text{km}}{\text{h}} = \frac{48 \times 1000 \text{ m}}{3600 \text{ s}} = \mathbf{13} \frac{\text{m}}{\text{s}}$$

- B** averages the two speeds, but the bus spends longer at the lower speed; **C** and **D** forget to change the units

## Setup for exercise 3.3

---

**3.3** The graph shows the displacement  $s$  of a car along a straight road as the driver brakes to a stop.



## Exercise 3.3 (a)

---

(a) **Describe** the motion of the car from  $t = 0$  to  $t = 10$  s.

[2]

## Exercise 3.3 (a)

---

(a) **Describe** the motion of the car from  $t = 0$  to  $t = 10$  s.

[2]

(a)

- the displacement increases, so the car moves forwards
- the gradient decreases to zero, so the car slows down and stops at  $t = 10$  s

## Exercise 3.3 (b)

---

(b) By drawing a tangent, determine the velocity of the car at  $t = 4.0$  s.

[3]

## Exercise 3.3 (b)

---

(b) By drawing a tangent, determine the velocity of the car at  $t = 4.0$  s. [3]

(b)

- tangent drawn at  $t = 4.0$  s
- gradient from a large triangle, for example through (0, 16 m) and (8.0 s, 112 m)

$$v = \frac{\Delta s}{\Delta t} = \frac{112 \text{ m} - 16 \text{ m}}{8.0 \text{ s} - 0} = \mathbf{12} \frac{\text{m}}{\text{s}}$$

(accept 11 to 13)

## Exercise 3.3 (c)

---

(c) Determine the average velocity of the car from  $t = 0$  to  $t = 10$  s.

[2]

## Exercise 3.3 (c)

---

(c) Determine the average velocity of the car from  $t = 0$  to  $t = 10$  s. [2]

(c) average velocity = displacement  $\div$  time

$$\text{average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{100 \text{ m}}{10 \text{ s}} = \mathbf{10} \frac{\text{m}}{\text{s}}$$

## Exercise 3.3 (d)

---

(d) Explain why your answers to (b) and (c) are different.

[1]

## Exercise 3.3 (d)

---

(d) Explain why your answers to (b) and (c) are different. [1]

(d) (b) is the velocity at one instant; (c) is the total displacement divided by the total time