

Exercise walk-through

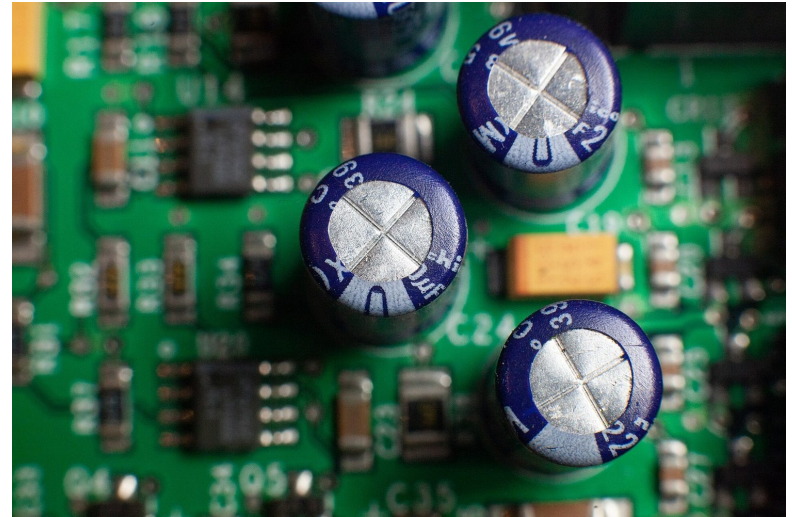
19.3

Discharging a capacitor

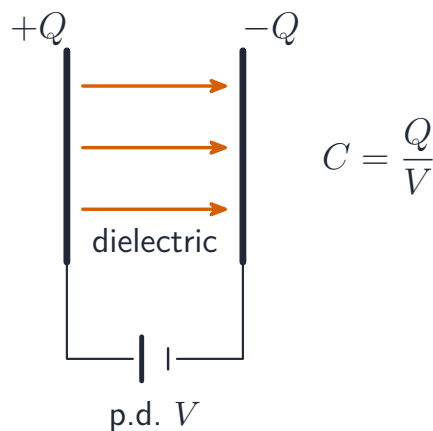
A-Level Physics

What we will cover

- 1 Capacitance $C = Q/V$
- 2 Combining capacitors
- 3 Energy stored
- 4 Discharge through a resistor
- 5 The time constant RC
- 6 Recap



What a capacitor does



A **capacitor** stores charge:
 two **conductor** plates with an
insulator between them – a
dielectric, or just a
vacuum – with a **potential**
difference V across the gap.

- connected to a battery, $+Q$
 builds on one plate, $-Q$ on the
 other

The two charges are **equal and opposite** – the capacitor stores charge **separation**, not net charge.

Capacitance $C = Q/V$

$$C = \frac{Q}{V}, \quad [C] = \text{F} = \text{C V}^{-1}$$

- set by size and dielectric, so C is **constant**
- double Q and V doubles too – the **ratio** holds
- an isolated sphere: $C = 4\pi\epsilon_0 r$

A farad is **huge** – real capacitors run from pF to mF.



Exercises 2.1-2.3

2.1 Write the formula for the **time constant**.

[1]

2.2 A $100\ \mu\text{F}$ capacitor discharges through a $10\ \text{k}\Omega$ resistor. Find the **time constant**. [2]

2.3 State what happens to the **charge** as a capacitor discharges.

[1]

Exercises 2.1·2.3

2.1 Write the formula for the **time constant**.

[1]

2.1

- $\tau = RC$

2.2 A $100\ \mu\text{F}$ capacitor discharges through a $10\ \text{k}\Omega$ resistor. Find the **time constant**. [2]

2.2

- $\tau = RC$

$$\tau = RC = (10 \times 10^3\ \Omega)(100 \times 10^{-6}\ \text{F}) = \mathbf{1.0\ s}$$

2.3 State what happens to the **charge** as a capacitor discharges.

[1]

2.3

- it **decreases exponentially** towards zero

Exercises 2.4·3.1

2.4 Write the discharge equation for **charge**.

[1]

3.1 The time constant RC is the time for the charge to fall to:

[1]

☐

A zero

B half its initial value

C $1/e$ (about 37%) of its initial value

D $1/2e$ of its initial value

Exercises 2.4·3.1

2.4 Write the discharge equation for **charge**.

[1]

2.4

- $Q = Q_0 e^{-t/RC}$

3.1 The time constant RC is the time for the charge to fall to:

[1]

☐

A zero

B half its initial value

C $1/e$ (about 37%) of its initial value

D $1/2e$ of its initial value

3.1 C

- after one time constant, $Q = Q_0/e$ (about 37%)

Exercises 3.2-3.3

3.2 A $470\ \mu\text{F}$ capacitor charged to $6.0\ \text{V}$ discharges through a $2.0\ \text{k}\Omega$ resistor. Calculate
(a) the time constant, (b) the p.d. after $1.0\ \text{s}$. [4]

3.3 Sketch a graph of **charge against time** for a discharging capacitor. [2]

Exercises 3.2-3.3

3.2 A $470\ \mu\text{F}$ capacitor charged to $6.0\ \text{V}$ discharges through a $2.0\ \text{k}\Omega$ resistor. Calculate (a) the time constant, (b) the p.d. after $1.0\ \text{s}$. [4]

3.2

(a) $\tau = RC$

$$\tau = RC = (2.0 \times 10^3\ \Omega)(470 \times 10^{-6}\ \text{F}) = \mathbf{0.94\ \text{s}}$$

(b) $V = V_0 e^{-t/RC}$

$$V = (6.0\ \text{V}) e^{-1.0/0.94} = (6.0\ \text{V})(0.345) = \mathbf{2.1\ \text{V}}$$

3.3 Sketch a graph of **charge against time** for a discharging capacitor. [2]

3.3

- a curve starting at the initial charge and **decaying exponentially** towards zero
- with the same shape (never quite reaching zero)

Exercises 3.4·3.5

3.4 State what happens to the time constant if the **resistance** is doubled. [1]

3.5 A $220\ \mu\text{F}$ capacitor charged to $9.0\ \text{V}$ discharges through a $15\ \text{k}\Omega$ resistor. Calculate the **time** for the p.d. to fall to $3.0\ \text{V}$. [4]

Exercises 3.4·3.5

3.4 State what happens to the time constant if the **resistance** is doubled. [1]

3.4

- it **doubles** ($\tau = RC$)

3.5 A $220\ \mu\text{F}$ capacitor charged to $9.0\ \text{V}$ discharges through a $15\ \text{k}\Omega$ resistor. Calculate the **time** for the p.d. to fall to $3.0\ \text{V}$. [4]

3.5

- $\tau = RC$

$$\tau = RC = (15 \times 10^3\ \Omega)(220 \times 10^{-6}\ \text{F}) = 3.3\ \text{s}$$

- $t = -RC \ln(V/V_0)$

$$t = -RC \ln \frac{V}{V_0} = -3.3\ \text{s} \times \ln \frac{3.0\ \text{V}}{9.0\ \text{V}} = -3.3\ \text{s} \times (-1.10) = \mathbf{3.6\ \text{s}}$$

Exercise 3.6 (a)

3.6 A capacitor C is charged and then discharged through a resistor R . A data logger records the p.d. V_R across the resistor and the p.d. V_C across the capacitor. At $t = 0$ the p.d. across the capacitor is 9.0 V, and the initial current is 1.8 mA. The p.d. across the capacitor falls to 3.3 V after 12 s.

(a) **State** the relationship between V_C and V_R during the discharge, and give a reason.[2]

Exercise 3.6 (a)

3.6 A capacitor C is charged and then discharged through a resistor R . A data logger records the p.d. V_R across the resistor and the p.d. V_C across the capacitor. At $t = 0$ the p.d. across the capacitor is 9.0 V, and the initial current is 1.8 mA. The p.d. across the capacitor falls to 3.3 V after 12 s.

(a) **State** the relationship between V_C and V_R during the discharge, and give a reason.[2]

(a) they are **equal** at every instant, $V_C = V_R$

- the capacitor and resistor form a single loop with no other source, so the p.d. driving the current is the capacitor's own p.d.

Exercise 3.6 (b)

3.6 A capacitor C is charged and then discharged through a resistor R . A data logger records the p.d. V_R across the resistor and the p.d. V_C across the capacitor. At $t = 0$ the p.d. across the capacitor is 9.0 V, and the initial current is 1.8 mA. The p.d. across the capacitor falls to 3.3 V after 12 s.

(b) **Determine** the resistance R .

[2]

Exercise 3.6 (b)

3.6 A capacitor C is charged and then discharged through a resistor R . A data logger records the p.d. V_R across the resistor and the p.d. V_C across the capacitor. At $t = 0$ the p.d. across the capacitor is 9.0 V, and the initial current is 1.8 mA. The p.d. across the capacitor falls to 3.3 V after 12 s.

(b) **Determine** the resistance R .

[2]

(b) $R = V/I$

$$R = \frac{V}{I} = \frac{9.0 \text{ V}}{1.8 \times 10^{-3} \text{ A}} = \mathbf{5.0 \times 10^3 \Omega}$$

Exercise 3.6 (c)

3.6 A capacitor C is charged and then discharged through a resistor R . A data logger records the p.d. V_R across the resistor and the p.d. V_C across the capacitor. At $t = 0$ the p.d. across the capacitor is 9.0 V, and the initial current is 1.8 mA. The p.d. across the capacitor falls to 3.3 V after 12 s.

(c) **Determine** the time constant τ of the circuit.

[3]

Exercise 3.6 (c)

3.6 A capacitor C is charged and then discharged through a resistor R . A data logger records the p.d. V_R across the resistor and the p.d. V_C across the capacitor. At $t = 0$ the p.d. across the capacitor is 9.0 V, and the initial current is 1.8 mA. The p.d. across the capacitor falls to 3.3 V after 12 s.

(c) **Determine** the time constant τ of the circuit. [3]

(c) $V = V_0 e^{-t/\tau}$, so $3.3/9.0 = e^{-12/\tau}$

$$\ln(0.367) = -\frac{12 \text{ s}}{\tau} \Rightarrow \tau = \mathbf{12 \text{ s}}$$

- the p.d. has fallen to $1/e$ of its start, which is what the time constant means

Exercise 3.6 (d)

3.6 A capacitor C is charged and then discharged through a resistor R . A data logger records the p.d. V_R across the resistor and the p.d. V_C across the capacitor. At $t = 0$ the p.d. across the capacitor is 9.0 V, and the initial current is 1.8 mA. The p.d. across the capacitor falls to 3.3 V after 12 s.

(d) Hence **determine** the capacitance C , in μF .

[2]

Exercise 3.6 (d)

3.6 A capacitor C is charged and then discharged through a resistor R . A data logger records the p.d. V_R across the resistor and the p.d. V_C across the capacitor. At $t = 0$ the p.d. across the capacitor is 9.0 V, and the initial current is 1.8 mA. The p.d. across the capacitor falls to 3.3 V after 12 s.

(d) Hence **determine** the capacitance C , in μF . [2]

(d) $\tau = RC$, so $C = \tau/R$

$$C = \frac{\tau}{R} = \frac{12 \text{ s}}{5.0 \times 10^3 \Omega} = 2.4 \times 10^{-3} \text{ F} = \mathbf{2.4 \times 10^3 \mu\text{F}}$$

Exercise 3.6 (e)

3.6 A capacitor C is charged and then discharged through a resistor R . A data logger records the p.d. V_R across the resistor and the p.d. V_C across the capacitor. At $t = 0$ the p.d. across the capacitor is 9.0 V, and the initial current is 1.8 mA. The p.d. across the capacitor falls to 3.3 V after 12 s.

(e) **Explain**, in terms of the current in the circuit, why the decay of the charge on the capacitor is **exponential**. [3]

Exercise 3.6 (e)

3.6 A capacitor C is charged and then discharged through a resistor R . A data logger records the p.d. V_R across the resistor and the p.d. V_C across the capacitor. At $t = 0$ the p.d. across the capacitor is 9.0 V, and the initial current is 1.8 mA. The p.d. across the capacitor falls to 3.3 V after 12 s.

(e) **Explain**, in terms of the current in the circuit, why the decay of the charge on the capacitor is **exponential**. [3]

(e) the current is $I = V_C/R$, and $V_C \propto Q$, so the current – the rate at which charge leaves – is **proportional to the charge still on the capacitor**

- a quantity whose rate of decrease is proportional to itself decays exponentially
- so as the charge falls the current falls with it, the discharge slows, and Q never quite reaches zero