Exercise walk-through

18.4
Electric field of a point charge

A-Level Physics

Sheet 18.4 – what you need to know

You must be able to:

- use $E = Q/4\pi\varepsilon_0 r^2$, measuring r from the centre of a sphere
- sketch E against distance, with its sign
- link V, E, Q and C for a charged sphere

What the exam asks

"Explain why the electric potential near an isolated proton is positive." **[3]**

"Sketch the variation of the electric field E at point P with x for values of x between $x = -3R$ and $x = 3R$." **[3]**

"Determine the capacitance of the sphere." **[2]**

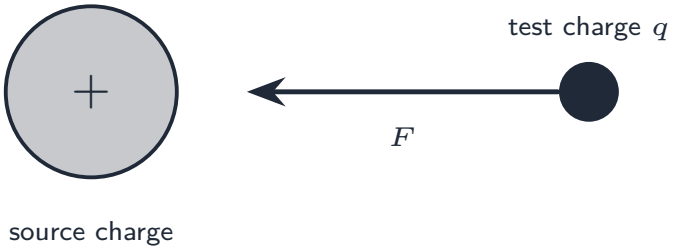
Practise on exercise sheet 18.4, then try the real exam questions on past-paper sheet 18.4.

Spheres as point charges

A charged spherical **conductor** of total charge Q:

- outside: identical to a point charge Q at its centre (measure r from the centre)
- inside a hollow one: the field is zero

> Zero field inside makes the whole conductor one **equipotential** – and is why a car shields you from lightning.



$$E = F/q \quad (\mathrm{N\,C^{-1}})$$

The field of a point charge

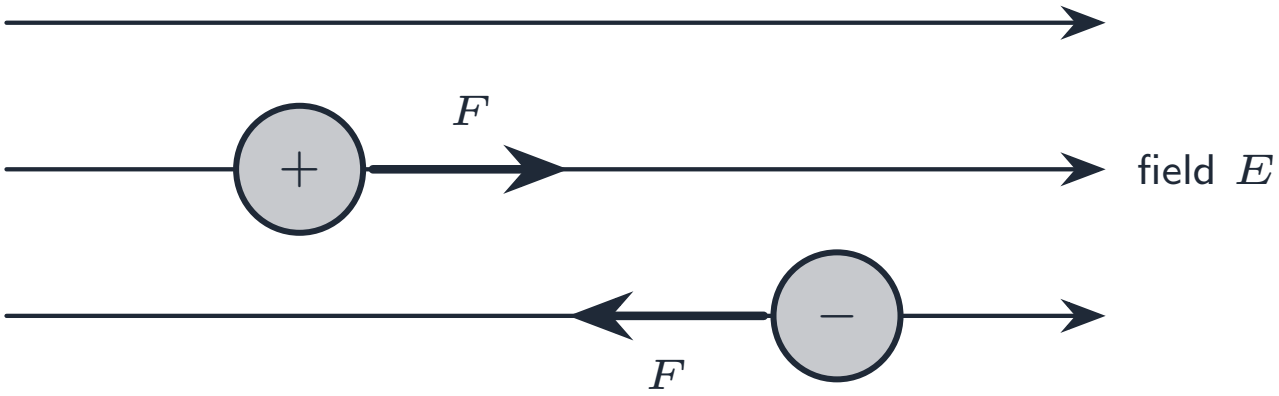
Coulomb's law gives the force between two charges. Divide by the test charge and only the source is left:

$$E = \frac{F}{q} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Out of a positive Q, into a negative one, falling as $1/r^2$.

> The same shape as $g = GM/r^2$ – but charges can cancel, so for several of them add the fields as a vector sum.

The field of a point charge, in detail



The field of a point charge Coulomb's law gives the force between two charges.

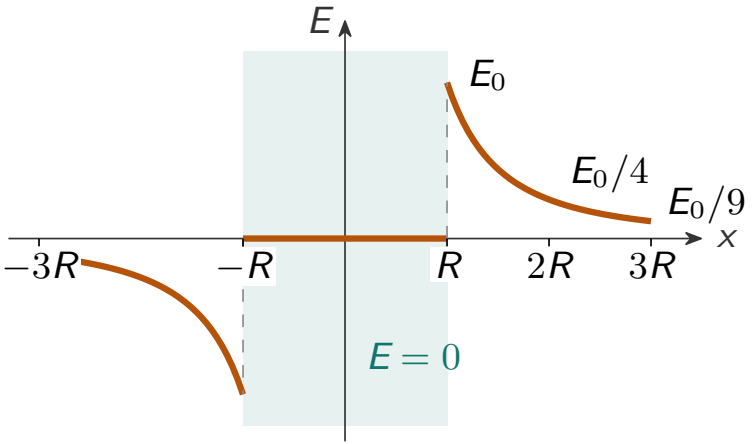
Signs: why $V > 0$ near a proton, and E either side of a sphere

"Explain why the electric potential near an isolated proton is positive." [3]

Potential is the work done per unit positive charge bringing it from infinity.

A positive test charge is repelled by the proton,

so work must be done on it: V is positive.



Left of the sphere the field points the other way, so E is negative there.

Exercises 2.1·2.3

2.1 Write the equation for the field strength E due to a **point charge**. [1]

2.2 State how E varies with distance r from a point charge. [1]

2.3 Find the field strength at 0.10 m from a +5.0 nC point charge. ($\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9$.) [2]

Exercises 2.1–2.3

2.1 Write the equation for the field strength E due to a **point charge**. [1]

2.1

- $E = Q/(4\pi\varepsilon_0 r^2)$

2.2 State how E varies with distance r from a point charge. [1]

2.2

- $E \propto 1/r^2$ (inverse-square)

2.3 Find the field strength at 0.10 m from a +5.0 nC point charge. ($\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9$.) [2]

2.3

- $E = Q/(4\pi\varepsilon_0 r^2)$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{(9.0\times10^{9}\ \text{N m}^2\,\text{C}^{-2})(5.0\times10^{-9}\ \text{C})}{(0.10\ \text{m})^2} = \mathbf{4.5\times10^{3}}\ \text{N C}^{-1}$$

Exercises 3.1·3.2

3.1 Both the electric field of a point charge and the gravitational field of a point mass vary with distance as: [1]

- ☐ **A** $1/r$
- **B** $1/r^2$
- **C** r
- **D** r^2

3.2 Calculate the electric field strength at 2.0 cm from a point charge of +8.0 nC. [3]

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Exercise 3.3

3.3 State **one** similarity and **one** difference between an electric field and a gravitational field. [2]

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- similarity: both follow an **inverse-square** law (radial field around a point source)
- difference: the electric force can be **attractive or repulsive**, but gravity is **always attractive**

Exercise 3.4 (a)

3.4 An isolated solid metal sphere of radius R carries a positive charge $+Q$. Point Q lies outside the sphere, a distance $2R$ from its centre.

(a) **Explain** why the electric potential everywhere near an isolated **positive** charge is positive. [2]

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3.4 An isolated solid metal sphere of radius R carries a positive charge $+Q$. Point Q lies outside the sphere, a distance $2R$ from its centre.

(a) **Explain** why the electric potential everywhere near an isolated **positive** charge is positive. [2]

(a) potential is the work done per unit positive charge bringing a test charge from infinity

- a positive charge **repels** the test charge, so external work must be done **on** it to bring it closer, making the potential positive everywhere around the charge

Exercise 3.4 (b)

3.4 An isolated solid metal sphere of radius R carries a positive charge $+Q$. Point Q lies outside the sphere, a distance $2R$ from its centre.

(b) **State** the direction of the electric field at point Q, and **state** how the field inside the metal sphere compares with it. [2]

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(b) **State** the direction of the electric field at point Q, and **state** how the field inside the metal sphere compares with it. [2]

(b) at Q the field points **radially away** from the sphere, i.e. directly outwards along the line from the centre

- inside the metal the field is **zero** – the charge resides on the surface and the interior is field-free

Exercise 3.4 (c)

3.4 An isolated solid metal sphere of radius R carries a positive charge $+Q$. Point Q lies outside the sphere, a distance $2R$ from its centre.

(c) On the axes below, **sketch** the variation of the electric field strength E with distance x from the centre of the sphere, for x from 0 to $4R$. [3]

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3.4 An isolated solid metal sphere of radius R carries a positive charge $+Q$. Point Q lies outside the sphere, a distance $2R$ from its centre.

(c) On the axes below, **sketch** the variation of the electric field strength E with distance x from the centre of the sphere, for x from 0 to $4R$. [3]

(c) $E = 0$ from $x = 0$ to $x = R$, drawn along the axis

- a **discontinuous jump** to the maximum E_0 at $x = R$
- then an inverse-square decay for $x > R$, falling to $E_0/4$ at $2R$ and $E_0/16$ at $4R$

Exercise 3.4 (d)

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(d) The field strength at the surface of the sphere is E_0. **Determine**, in terms of E_0, the field strength at point Q. [2]

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(d) outside the sphere it behaves as a point charge at its centre, so $E \propto 1/x^2$

- at $2R$ the field is $E_0/4$

Exercise 3.4 (e)

3.4 An isolated solid metal sphere of radius R carries a positive charge $+Q$. Point Q lies outside the sphere, a distance $2R$ from its centre.

(e) A second identical sphere carrying the same charge is now placed with its centre $6R$ from the centre of the first. **Determine** the position on the line joining their centres at which the resultant field is zero, and **state** what the potential is there compared with the potential at the surface of either sphere. [3]

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(e) by symmetry the fields cancel at the **midpoint**, $3R$ from each centre

- the potentials, being scalars, **add** there, so the potential is not zero – it is smaller than at either surface but still positive