

Exercise walk-through

18.3

Electric force between point charges

A-Level Physics

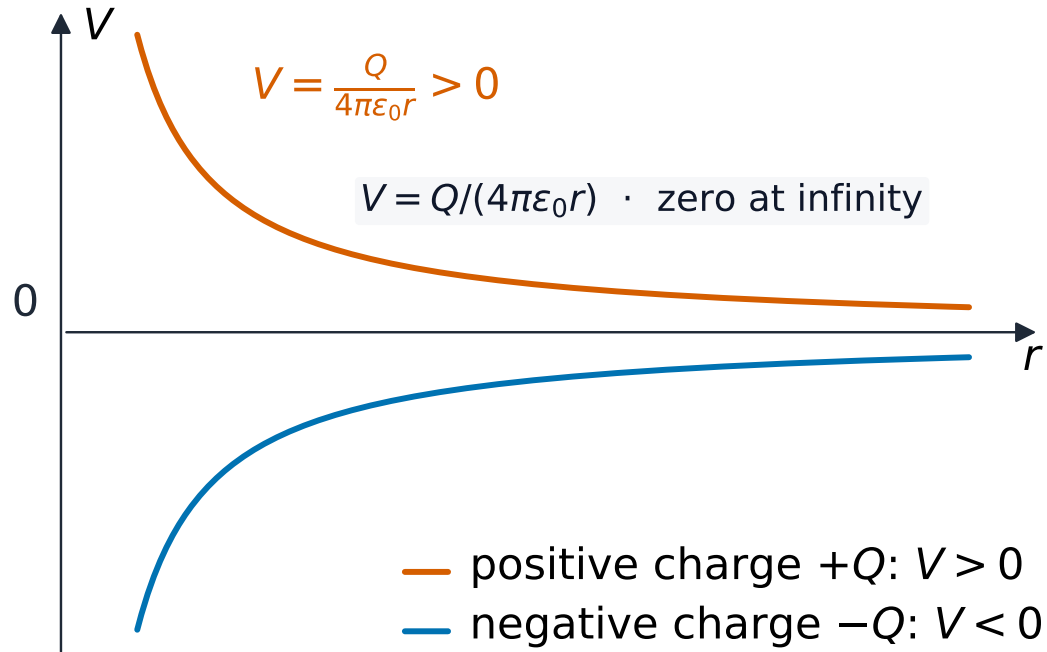
Electric potential

Electric potential is the work done per unit **positive** charge bringing a test charge in from **infinity** :

$$V = \frac{W}{q} = \frac{Q}{4\pi\epsilon_0 r}$$

Note $1/r$, **not** $1/r^2$. V is a **scalar** and takes the sign of Q – so several charges just **add**, signs and all, with no directions to resolve.

Electric potential, in detail



Potential falls as $1/r$, **not** $1/r^2$, and takes the sign of its charge – so several charges simply add, signs and all.

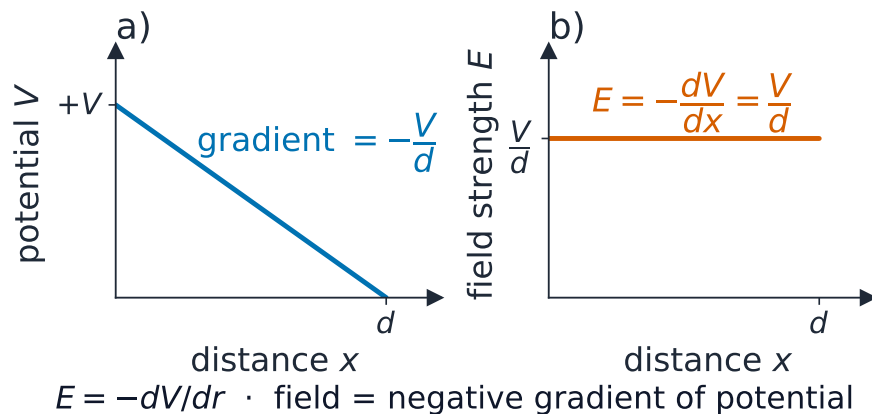
Worked example – potential near a charge

Find V at 4.0 cm from a point charge of +3.0 nC.

$$V = \frac{Q}{4\pi\epsilon_0 r} = \frac{(9.0 \times 10^9)(3.0 \times 10^{-9})}{0.040}$$
$$\approx 6.8 \times 10^2 \text{ V}$$

Same charge, same distance as the Coulomb example – but r appears to the **first** power here. Mixing up $1/r$ and $1/r^2$ is the standard slip.

Field is the potential gradient



The field is the **negative potential gradient**:

$$E = -\frac{dV}{dx}$$

- between plates V falls evenly $\Rightarrow E = V/d$, constant
- for a point charge

$$-\frac{dV}{dr} = \frac{Q}{4\pi\epsilon_0 r^2} = E$$

The minus sign says the field points **downhill** in potential – from high V to low.

Electric potential energy

A charge q where the potential is V has $E_P = qV$; for two point charges

$$E_P = \frac{Qq}{4\pi\epsilon_0 r}$$

- like: $E_P > 0$ – energy released if they fly apart
- opposite: $E_P < 0$ – a **bound** well

Both tend to **zero** as $r \rightarrow \infty$ – that is the choice of zero, exactly as for gravity.

Exercises 2.1-2.3

2.1 State Coulomb's law.

[2]

2.2 State the value of $\frac{1}{4\pi\epsilon_0}$.

[1]

2.3 State whether **like** charges attract or repel.

[1]

Exercises 2.1·2.3

2.1 State **Coulomb's law**.

[2]

2.1

- the force between two **point charges** is **proportional to the product of the charges**
- and **inversely proportional to the square of their separation**

2.2 State the value of $\frac{1}{4\pi\epsilon_0}$.

[1]

2.2

- $1/(4\pi\epsilon_0) = 9.0 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$

2.3 State whether **like** charges attract or repel.

[1]

2.3

- they **repel**

Exercises 3.1-3.2

3.1 If the distance between two charges is **doubled**, the Coulomb force becomes: [1]

☐

A twice as large

B half as large

C one quarter as large

D four times as large

3.2 Calculate the force between point charges of +2.0 nC and +3.0 nC placed 4.0 cm apart. [3]

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3.1 C

- $F \propto 1/r^2$, so doubling r makes F **one quarter**

3.2 Calculate the force between point charges of +2.0 nC and +3.0 nC placed 4.0 cm apart. [3]

3.2

- $F = Q_1 Q_2 / (4\pi\epsilon_0 r^2)$

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} = \frac{(9.0 \times 10^9)(2.0 \times 10^{-9})(3.0 \times 10^{-9})}{(0.040)^2} = \mathbf{3.4 \times 10^{-5} \text{ N}}$$

Exercise 3.3

3.3 Two charges of $+5.0 \text{ nC}$ are 2.0 cm apart. Calculate the force between them, and state whether it is attractive or repulsive. [3]

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- $F = Q^2 / (4\pi\epsilon_0 r^2)$

$$F = \frac{Q^2}{4\pi\epsilon_0 r^2} = \frac{(9.0 \times 10^9 \text{ N m}^2 \text{ C}^{-2})(5.0 \times 10^{-9} \text{ C})^2}{(0.020 \text{ m})^2} = \mathbf{5.6 \times 10^{-4} \text{ N}}$$

- repulsive** (like charges)

Exercise 3.4 (a)

3.4 A simplified model of a helium atom has a nucleus of charge $+2e$ with two electrons in a circular orbit of radius 2.6×10^{-11} m around it. Take $e = 1.60 \times 10^{-19}$ C, $\frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9$ m F⁻¹ and the electron mass as 9.11×10^{-31} kg. Ignore the force between the two electrons.

(a) **State** Coulomb's law.

[2]

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(a) **State** Coulomb's law. [2]

(a) the electric force between two **point charges** is proportional to the **product of the charges**

- and inversely proportional to the **square of their separation**

Exercise 3.4 (b)

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(b) **Show that** the electric force between the nucleus and one electron is about 1.4×10^{-7} N. [2]

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(b) **Show that** the electric force between the nucleus and one electron is about 1.4×10^{-7} N. [2]

(b) $F = Q_1Q_2/(4\pi\epsilon_0r^2)$

$$F = \frac{Q_1Q_2}{4\pi\epsilon_0r^2} = (8.99 \times 10^9) \frac{(2 \times 1.60 \times 10^{-19})(1.60 \times 10^{-19})}{(2.6 \times 10^{-11})^2} \approx \mathbf{1.4 \times 10^{-7} \text{ N}}$$

Exercise 3.4 (c)

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(c) This force provides the centripetal force on the electron. **Calculate** the speed of the orbiting electron. [3]

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(c) This force provides the centripetal force on the electron. **Calculate** the speed of the orbiting electron. [3]

(c) the electric force is the centripetal force: $mv^2/r = F$

$$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{(1.36 \times 10^{-7} \text{ N})(2.6 \times 10^{-11} \text{ m})}{9.11 \times 10^{-31} \text{ kg}}} = \mathbf{2.0 \times 10^6 \frac{m}{s}}$$

Exercise 3.4 (d)

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(d) **Determine** the period of the orbit.

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[2]

(d) $T = 2\pi r/v$

$$T = \frac{2\pi r}{v} = \frac{2\pi(2.6 \times 10^{-11} \text{ m})}{2.0 \times 10^6 \frac{\text{m}}{\text{s}}} = \mathbf{8.2 \times 10^{-17} \text{ s}}$$

Exercise 3.4 (e)

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(e) **Determine** the ratio $\frac{\text{electric force on the electron}}{\text{gravitational force on the electron}}$, given that the nucleus has mass 6.6×10^{-27} kg and $G = 6.67 \times 10^{-11}$ N m ^{2} kg $^{-2}$, and **suggest** what your answer means for a model of the atom. [3]

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(e) $F_{\text{grav}} = GMm/r^2$

$$F_{\text{grav}} = \frac{GMm}{r^2} = \frac{(6.67 \times 10^{-11})(6.6 \times 10^{-27})(9.11 \times 10^{-31})}{(2.6 \times 10^{-11})^2} = 5.9 \times 10^{-46} \text{ N}$$

$$\frac{F_{\text{elec}}}{F_{\text{grav}}} = \frac{1.4 \times 10^{-7} \text{ N}}{5.9 \times 10^{-46} \text{ N}} = \mathbf{2 \times 10^{38}}$$

- gravity is utterly negligible on this scale, so a model of the atom can ignore it and keep only the electric force