

Exercise walk-through

17.2

Energy in simple harmonic motion

A-Level Physics

Energy in SHM

An oscillator keeps swapping energy between two forms: **kinetic energy** $\frac{1}{2}mv^2$, and potential energy (elastic for a spring, gravitational for a pendulum).

$$E_K = \frac{1}{2}m\omega^2(x_0^2 - x^2)$$

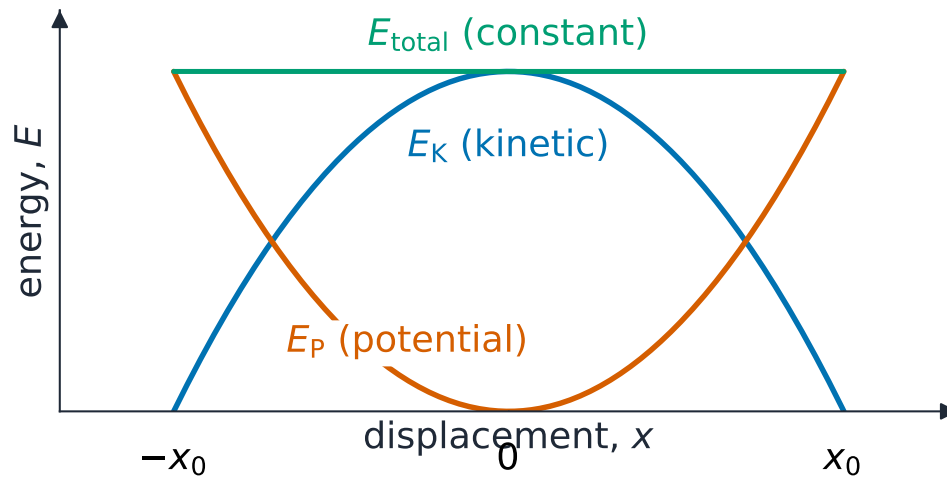
$$E_P = \frac{1}{2}m\omega^2x^2$$

E_K is a downward parabola, E_P an upward one – and their **sum is flat**:

$$E_{\text{total}} = \frac{1}{2}m\omega^2x_0^2$$

Kinetic energy and potential energy trade places while the total stays put – **conservation of energy** for a mass on a **spring**. $E \propto x_0^2$: **double** the amplitude for **four times** the energy.

Energy in SHM, in detail



SHM: $E_k + E_p = \text{const}$ · max E_k at centre

Kinetic and potential energy swap over a cycle while the total energy stays constant

Worked example – total energy

A 0.20 kg mass oscillates with $x_0 = 0.050$ m and $\omega = 15.7 \text{ rad s}^{-1}$. Find the total energy.

The total equals the maximum KE, as it passes $x = 0$ at $v_{\text{max}} = \omega x_0$:

$$E = \frac{1}{2} m \omega^2 x_0^2 = \frac{1}{2} (0.20) (15.7)^2 (0.050)^2 \\ \approx 0.062 \text{ J}$$

Evaluate the total where one form vanishes – at $x = 0$ it is all kinetic, at $x = x_0$ all potential. Either gives the same number.

Exercises 2.1-2.3

2.1 Describe the **energy interchange** during SHM.

[2]

2.2 State where the **kinetic energy** is maximum and where the **potential energy** is maximum.

[2]

2.3 Write the formula for the **total energy** of a simple harmonic oscillator.

[1]

Exercises 2.1-2.3

2.1 Describe the **energy interchange** during SHM.

[2]

2.1

- energy is continually transferred between **kinetic** and **potential** forms
- with no damping the **total** stays constant

2.2 State where the **kinetic energy** is maximum and where the **potential energy** is maximum.

[2]

2.2

- KE is maximum at the **equilibrium** ($x = 0$)
- PE is maximum at the **extremes** ($x = \pm x_0$)

2.3 Write the formula for the **total energy** of a simple harmonic oscillator.

[1]

2.3

- $E = \frac{1}{2}m\omega^2x_0^2$

Exercises 3.1·3.2

3.1 In SHM, the total energy is proportional to:

[1]

☐

A the amplitude x_0

B the amplitude squared x_0^2

C $1/x_0$

D the period T

3.2 A 0.20 kg mass oscillates with $\omega = 10 \text{ rad s}^{-1}$ and amplitude 0.050 m. Calculate the **total energy**.

[3]

Exercises 3.1·3.2

3.1 In SHM, the total energy is proportional to:

[1]

☐

A the amplitude x_0

B the amplitude squared x_0^2

C $1/x_0$

D the period T

3.1 B

- $E \propto x_0^2$, so energy is proportional to the **amplitude squared**

3.2 A 0.20 kg mass oscillates with $\omega = 10 \text{ rad s}^{-1}$ and amplitude 0.050 m. Calculate the **total energy**. [3]

3.2

- $E = \frac{1}{2}m\omega^2x_0^2$

$$E = \frac{1}{2}m\omega^2x_0^2 = \frac{1}{2}(0.20 \text{ kg})(10 \text{ rad s}^{-1})^2(0.050 \text{ m})^2 = \mathbf{0.025 \text{ J}}$$

Exercises 3.3-3.4

3.3 Sketch how the **kinetic energy** and **potential energy** vary with displacement in SHM. [2]

3.4 State what happens to the total energy if the **amplitude is doubled**. [1]

Exercises 3.3-3.4

3.3 Sketch how the **kinetic energy** and **potential energy** vary with displacement in SHM. [2]

3.3

- KE: a **downward** parabola, maximum at $x = 0$, zero at $\pm x_0$
 - PE: an **upward** parabola, zero at $x = 0$, maximum at $\pm x_0$; their sum is constant
-

3.4 State what happens to the total energy if the **amplitude is doubled**. [1]

3.4

- it becomes **four times** larger ($E \propto x_0^2$)

Exercise 3.5 (a)

3.5 A graph of the **kinetic energy** of an oscillator against **time** repeats every 0.40 s.

(a) State the **period** of the oscillation, explaining your answer. [2]

Exercise 3.5 (a)

3.5 A graph of the **kinetic energy** of an oscillator against **time** repeats every 0.40 s.

(a) State the **period** of the oscillation, explaining your answer. [2]

(a) the KE varies at **twice** the oscillation frequency (the object passes the centre twice per cycle)

- so the oscillation period is **twice** the KE-graph period: $T = 2T_{\text{KE}} = 2 \times 0.40 \text{ s} = \mathbf{0.80 \text{ s}}$

Exercise 3.5 (b)

3.5 A graph of the **kinetic energy** of an oscillator against **time** repeats every 0.40 s.

(b) Calculate the **angular frequency** ω of the oscillation. [2]

Exercise 3.5 (b)

3.5 A graph of the **kinetic energy** of an oscillator against **time** repeats every 0.40 s.

(b) Calculate the **angular frequency** ω of the oscillation. [2]

(b) $\omega = 2\pi/T$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.80 \text{ s}} = \mathbf{7.9 \text{ rad s}^{-1}}$$

Exercise 3.6 (a)

3.6 A mass hangs on a spring and oscillates vertically with simple harmonic motion. A data logger records its **kinetic energy** against time, giving a curve that repeats every 0.40 s. The mass is 0.25 kg and the amplitude of the oscillation is 4.0 cm.

(a) **Determine** the period of the oscillation of the **mass**, and explain why it is not 0.40 s. [2]

Exercise 3.6 (a)

3.6 A mass hangs on a spring and oscillates vertically with simple harmonic motion. A data logger records its **kinetic energy** against time, giving a curve that repeats every 0.40 s. The mass is 0.25 kg and the amplitude of the oscillation is 4.0 cm.

(a) **Determine** the period of the oscillation of the **mass**, and explain why it is not 0.40 s. [2]

(a) the kinetic energy is zero at **both** ends of the swing and maximum at the centre twice per cycle, so the energy curve repeats **twice** per oscillation

- the period of the mass is therefore $T = 2T_{\text{KE}} = 2 \times 0.40 \text{ s} = 0.80 \text{ s}$

Exercise 3.6 (b)

3.6 A mass hangs on a spring and oscillates vertically with simple harmonic motion. A data logger records its **kinetic energy** against time, giving a curve that repeats every 0.40 s. The mass is 0.25 kg and the amplitude of the oscillation is 4.0 cm.

(b) **Determine** the angular frequency of the oscillation.

[2]

Exercise 3.6 (b)

3.6 A mass hangs on a spring and oscillates vertically with simple harmonic motion. A data logger records its **kinetic energy** against time, giving a curve that repeats every 0.40 s. The mass is 0.25 kg and the amplitude of the oscillation is 4.0 cm.

(b) **Determine** the angular frequency of the oscillation. [2]

(b) $\omega = 2\pi/T$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.80 \text{ s}} = \mathbf{7.9 \text{ rad s}^{-1}}$$

Exercise 3.6 (c)

3.6 A mass hangs on a spring and oscillates vertically with simple harmonic motion. A data logger records its **kinetic energy** against time, giving a curve that repeats every 0.40 s. The mass is 0.25 kg and the amplitude of the oscillation is 4.0 cm.

(c) **Determine** the total energy of the oscillation.

[3]

Exercise 3.6 (c)

3.6 A mass hangs on a spring and oscillates vertically with simple harmonic motion. A data logger records its **kinetic energy** against time, giving a curve that repeats every 0.40 s. The mass is 0.25 kg and the amplitude of the oscillation is 4.0 cm.

(c) **Determine** the total energy of the oscillation. [3]

(c) the total energy equals the maximum kinetic energy, $E = \frac{1}{2}m\omega^2x_0^2$

$$E = \frac{1}{2}m\omega^2x_0^2 = \frac{1}{2}(0.25 \text{ kg})(7.85 \text{ rad s}^{-1})^2(0.040 \text{ m})^2 = \mathbf{1.2 \times 10^{-2} \text{ J}}$$

Exercise 3.6 (d)

3.6 A mass hangs on a spring and oscillates vertically with simple harmonic motion. A data logger records its **kinetic energy** against time, giving a curve that repeats every 0.40 s. The mass is 0.25 kg and the amplitude of the oscillation is 4.0 cm.

(d) **Determine** the speed of the mass when its displacement is 2.0 cm. [3]

Exercise 3.6 (d)

3.6 A mass hangs on a spring and oscillates vertically with simple harmonic motion. A data logger records its **kinetic energy** against time, giving a curve that repeats every 0.40 s. The mass is 0.25 kg and the amplitude of the oscillation is 4.0 cm.

(d) **Determine** the speed of the mass when its displacement is 2.0 cm. [3]

$$(d) \ v = \omega \sqrt{x_0^2 - x^2}$$

$$v = \omega \sqrt{x_0^2 - x^2} = (7.85 \text{ rad s}^{-1}) \sqrt{(0.040 \text{ m})^2 - (0.020 \text{ m})^2}$$

$$v = \omega \sqrt{x_0^2 - x^2} = (7.85 \text{ rad s}^{-1})(0.0346 \text{ m}) = \mathbf{0.27} \frac{\text{m}}{\text{s}}$$

Exercise 3.6 (e)

3.6 A mass hangs on a spring and oscillates vertically with simple harmonic motion. A data logger records its **kinetic energy** against time, giving a curve that repeats every 0.40 s. The mass is 0.25 kg and the amplitude of the oscillation is 4.0 cm.

(e) The oscillation is **lightly damped**. **State and explain** what happens to the time between successive maxima of the kinetic-energy curve, and to the height of those maxima. [3]

Exercise 3.6 (e)

3.6 A mass hangs on a spring and oscillates vertically with simple harmonic motion. A data logger records its **kinetic energy** against time, giving a curve that repeats every 0.40 s. The mass is 0.25 kg and the amplitude of the oscillation is 4.0 cm.

(e) The oscillation is **lightly damped**. **State and explain** what happens to the time between successive maxima of the kinetic-energy curve, and to the height of those maxima. [3]

(e) light damping barely changes the period, so the time between maxima stays about 0.40 s

- the maxima get **lower** with time
- because the amplitude decays and the total energy is proportional to the **square** of the amplitude, so the energy falls faster than the amplitude does